GOVERNMENT DEGREE COLLEGE, YELLANDU

(Affiliated to Kakatiya University, Warangal)

Re-Accredited by NAAC with "B" Grade

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STUDENT STUDY PROJECT

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DEPARTMENTOF MATHEMATICS

COMMISSIONERIATE OF COLLEGIATE EDUCATION-HYDERABAD-TS.

GOVERNMENT DEGREE COLLEGE YELLANDU
BHADRADRI KOTHAGUDEM DT.

DEPARTMENT OF MATHEMATICS

CERTIFICATE

This is to certify that this is a Bonafide study project of the students from Department of Mathematics, Government Degree college Yellandu. I congratulate the Students for carrying out a wonderful study project.

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* THE METHOD OF VARIATION OF PARAMETERS *

⇒ Abstract :

⇒ In the present study project, the method of variation of parameters is studied in detail with illustration we obtain the particular solution to non-homogeneous differential equation using the method of variation parameters. Some of applications of the method are given.

⇒ Introduction :

⇒ Analysis has been the dominant branch of mathematics for 300 years. and differential equations are the heart of analysis This subset is the natural goal of elementary caculs and the most important part of mathematics for understanding the physical science.

⇒ The primary purpose of differential equations is to serve as a tool for the study of change in the physical world. There is an old Amenian saying He who locks a sence of the past is condemned to live in the harrow darkness of his own generations.

⇒ Mathematics without history is mathematics stripped of its greatness. for like the othere arts of civilization. it derives its and grandeur from the fact of being a human creation.

=> An equation involving one dependent varible and its derivatives with respect to one or more independent varibles is called a differential equation many of the genaral laws of nature - in physics chemistry, biology, and astronomy - find their most natural expression in the language of differential equations. Applications also abound in mathematics it self, especially in geomentry, and in enginee ring, economics, and many othere fields of applied science..

=> An ordinary differential equation is one in which there is only one independent varible. so that all the derivatives occurring in it are ordinary derivatives. each of these equations is ordinary, the order of a diffential equation is the order of the highest derivation present.

=> A partial differential equation is one involving more than one independent varible. so that the derivatics occurring in it are partial derivatives.

=> The aim of current proJect is to solve non-homogeneous differential equation using the method of variation of parameters" Some of the application so the method are illustrated in this project."

The technique described in section 18 for determining a particular solution of the non-homogeneous equation

$$y'' + P(x)y' + Q(x)y = R(x)$$

⇒ has two severe limitations: it can be used only when the cofficients $P(x)$ and $Q(x)$ are constants and even then it works only when the right hand term $R(x)$ has a particalary simple form with in these limitations. however this procedure is usally the easiest to apply.

⇒ We how develop a more powerful method that alway works - regardless of the nature of P, Q, R, - provided only that the general solution of the corresponding homogeneous equation.

$$y'' + P(x)y' + Q(x)y = 0$$

⇒ is already known. we assume. them. that is some way the general solution.

$$y(x) = c_1 y_1(x) + c_2 y_2(x)$$

⇒ OF(2) has been found the method is similar to that discussed in section 16; that is, we replace the constants c_1 and c_2 by unkown functions $v_1(x)$ and $v_2(x)$, and atteempt to determine v_1 and v_2 in such a manner that

$$y = v_1 y_1 + v_2 y_2$$

⇒ Will be a Solution of (1) with two unkonown functions to find, it will be becessary to have two equations relating These functions, we obtain one of These by requiring That (4) be a Solution of (1). It will soon be clear what the second equation should be. we begin by computing the derivative of (4). arranged as follows.

$$\boxed{y' = (v_1 y_1' + v_2 y_2') + (v_1' y_1 + v_2' y_2)}$$

⇒ Another differentiation will introduce second derivations of The unknowns v_1 and v_2, we avoid This complications by requiring The second expression in paren These to Vanish.

$$\boxed{v_1' y_1 + v_2' y_2 = 0}$$

⇒ This gives.

$$\boxed{y' = v_1 y_1' + v_2 y_2'}$$

$$\boxed{y'' = v_1 y_1'' + v_1' y_1' + v_2 y_2'' + v_2' y_2'}$$

So on substituting (4), (7), (8), into (1), and rearranging we get.

$$\boxed{v_1(y_1'' + P y_1' + Q y_1) + v_2(y_2'' + P y_2' + Q y_2) + v_1' y_1' + v_2' y_2' = R(x)}$$

since y_1 and y_2 are Solutions of (2) The two expressions in parenTheses are equal to, 0, and (9) collapses to.

$$\boxed{v_1' y_1' + v_2' y_2' = R(x)}$$

Taking (6) and (10) together, we have two equations in two unknows v_1 and v_2

$$v_1' y_1 + v_2' y_2 = 0$$
$$v_1' y_1' + v_2' y_2' = R(x)$$

These can be Solved at once giving.

$$v_1' = -y_2 R(x) / W(y_1 \cdot y_2)$$
$$v_2' = y_1 R(x) / W(y_1 \cdot y_2)$$

=> It should be hoted That These formulas are legitimate for The wronskian in The denominators is honzero by The linear independence of y_1 and y_2. All That remains is to integrate formulas (11) to find v_1 and v_2.

$$v_1 = \int -y_2 R(x)\, dx \quad \text{and} \quad v_2 = \int y_1 R(x)\, dx$$

We can now put everything together and assert That

$$y = y_1 \int -y_2 R(x)\, dx + y_2 \int y_1 R(x)\, dx$$

is The particular Solution of (1) we are seeking.

=> The reader will see That This method has disadvantages of its own. In particular, The integrals in (12) many be diffcult or impossible to work out. Also of course it is necessary to work know The general Solution of (2) before The process can even be started but This objection is really immaterial because we are unlikely to came a bout finding a particular solution of (1) unless the general solution of (2) is already at hand. The method variation of parameters was invented by the French mathematician Lagrange in connection with his epoch-marking work in analytical mechanics.

Example 1 :-

⇒ Find a particular solution of $y'' + y = \cos x$.

⇒ The corresponding homogeneous equation $y'' + y = 0$. has $y(x) = c_1 \sin x + c_2 \cos x$ as its general solution, so $y_1 = \sin x$, $y_1' = \cos x$, $y_2 = \cos x$. and $y_2' = -\sin x$ The wronskian of y_1 and y_2 is.

$$\boxed{W(y_1 y_2) = y_1 y_2' - y_2 y_1' = -\sin^2 x - \cos^2 x = -1}$$

So by (12) we have.

$$v_1 = \int -\cos x \cdot \cos x / - dx.$$

$$v_1 = \int \cos x / \sin x \, dx.$$

$$v_1 = \log(\sin x)$$

$$v_2 = \int \sin x \cdot \csc x / -1 \, dx.$$

$$v_2 = -x.$$

Accordingly;

$$\boxed{y = \sin x \log(\sin x) - x \cos x.}$$

Is The desired particular Solution.

Application of The method

=> The method has so many advantages in solving the differential equations in

* vibration in mechanical and eletrical systems.
* un damped simple harmonic vibrations.
* Damped vibrations.
* Forced vibrations.

Results and discussion

=> The method of variation of parameters is discussed in detail, The Solution to a non-homogeneous differential equation is obtainned in this study project, Applications are discussed.

References

1, G.F. simmons, "Differential equation with application and historical notes", TATA MCGRAW-HILL EDITION.

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