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An Affirmative Fixed Point Result on b-Metric Spaces using (CLR) Property

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Abstract

This study aims to show that four maps in a b-metric space that satisfy pairwise weak compatibility have common fixed points under certain conditions. In the main results of this paper, (CLR) property is employed, and common fixed points for four weakly compatible mappings are established. All our findings are backed up by befitting examples. Our results generalize and extend certain previous findings in the literature.

Keywords: Coincidence points; Common fixed points; b-Metric space; Weakly compatible; (CLR) property.

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1. Introduction

The idea of b-metric was first proposed by Bakhtin [1] and Czerwik [2]. Bakhtin [1], proved the Banach fixed point theorem in the setting of b-metric space. On the other hand, Jungck [3] proposed the idea of compatibility of two mappings in 1986, and later in 1998, Jungck and Rhodes [4] proposed the idea of weak compatibility. Subsequently, in 2002, Aamri and Moutawakil [5] established the (E.A) property, which has been widely used by the authors to establish fixed points. Later on, in 2011, the common limit range (CLR) property was introduced by Sintunavarat *et al*. [6]. Using this property it was proved that the closed range of any of the underlying mappings is not necessary for having fixed points. And later, Chauhan [7] came up with a new property, the common limit range property concerning the maps S and T (briefly, (CLR_{ST}) property), which allows us to relax the containment conditions to obtain fixed points.

In 2016, certain fixed point theorems for four maps in b-metric spaces were proved by Ozturk and Radenović [8] by applying the b-(E.A) property. After that, various researchers worked in different directions, and several remarkable results about the presence of common fixed points for single-valued and multi-valued mappings in b-metric spaces were obtained ([9-16]). In a recent development, common fixed points for four self maps satisfying (CLR) Property in b metric space was obtained [17-19].

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Inspired by the work done by Ozturk and Radenović [8], in this paper, certain fixed point theorems in b-metric spaces are proved by replacing the b-(EA) property with (CLR) property for which the closed property of the range of any map is not required. The (CLR_{ST}) property is employed here, which allows us to relax the containment conditions. Our findings generalize and extend certain previous findings in the literature.

2. Preliminaries

Definition 2.1. (Bakhtin [1], Czerwik [2]) Let X be a nonempty set and $s \geq 1$ be a given real number. A function $d: X^2 \to [0, \infty)$ is said to be a b-metric on X if for all $u, v, w \in X$,
(b1) $d(u, v) = 0$ if and only if $u = v$,
(b2) $d(u, v) = d(v, u)$,
(b3) $d(u, v) \leq s[d(u, w) + d(w, v)]$ (b - triangular inequality).
(X,d) is said to be a b-metric space in this case. Every metric is a b-metric with $s = 1$. But every b-metric need not be a metric.
Example 2.2. [20] Define $\rho(u, v) = (d(u, v))^p$, where $p > 1$ be a real number and *(X, d)* be a metric space. Then ρ is a b-metric on X with $s = 2^{p-1}$. It is clear that ρ is not a metric on X.
Example 2.3. [20] Define $\rho: R^2 \to [0, \infty)$ by $\rho(u, v) = (u - v)^2$. Then ρ is a b-metric on R with $s = 2$.
Definition 2.4. (Boriceanu, [21]) A sequence $\{u_n\}$ in a b-metric space *(X,d)* is said to be

(i) b-convergent to some $a \in X$ if and only if $d(u_n, a) \to 0$ as $n \to \infty$. We write $\lim_{n\to\infty} u_n = a$ in this case.

(ii) b-Cauchy if and only if $d(u_n, u_m) \to 0$ as $n, m \to \infty$.

If every b-Cauchy sequence is b-convergent, then the b-metric space is said to be complete.
Remark 2.5. (Boriceanu, [21]) *The following assertions hold in a b-metric space (X, d):*

1) *If a sequence is b-convergent, then it has a unique limit.*
2) *If a sequence is b-convergent, then it is b-Cauchy.*
3) *A b-metric is not continuous, in general.*

Definition 2.6. A pair *(P, Q)* of mappings of a b-metric space X is said to

(i) be compatible [3] if $\lim_{n\to\infty} d(PQu_n, QPu_n) = 0$, whenever $\{u_n\}$ is a sequence in X and $\lim_{n\to\infty} Pu_n = \lim_{n\to\infty} Qu_n = t, t \in X$.

(ii) satisfies (E.A) property [5] if there exists a sequence $\{u_n\}$ in X such that $\lim_{n\to\infty} Pu_n = \lim_{n\to\infty} Qu_n = t, t \in X$.

Definition 2.7. Let P and Q be two self maps of a nonempty set X. Then
(i) a point $u \in X$ is said to be a coincidence point of P and Q if $Pu = Qu$ and $C(P, Q) = \{u \in X: Pu = Qu\}$.
(ii) the pair *(P,Q)* is called weakly compatible [4] if $PQu = QPu$ for every $u \in C(P, Q)$.

Definition 2.8. (Sintunavarat and Kumam, [6]) The pair *(P, Q)* of self mappings of *X*, where *X* is a b-metric space, is said to satisfy the common limit in the range of *Q* (CLR_Q)-property if there exists a sequence $\{u_n\}$ in X such that $\lim_{n\to\infty} Pu_n = \lim_{n\to\infty} Qu_n = Qu, u \in X$.

Example 2.9. Consider the b-metric $d: X^2 \to [0,\infty)$ given by $d(u,v) = (u-v)^2$ with *s = 2* where *X=R*. Define $P, Q: X \to X$ by $P(u) = e^u$ and $Q(u) = 1 - u^2$.

Then for the sequence $u_n = \frac{1}{n}, n = 1,2,\ldots\ldots$ in *X*.

$$d(Qu_n, 1) = \frac{1}{n^4} \to 0 \text{ as } n \to \infty \text{ and } d(Pu_n, 1) = \left(e^{\frac{1}{n}} - 1\right)^2 \to 0 \text{ as } n \to \infty.$$

$\lim_{n\to\infty} Pu_n = \lim_{n\to\infty} Qu_n = 1 = Q(0)$. The pair *(P, Q)* satisfy (CLR_Q) property.

Chauhan [7] introduced (CLR_{ST}) property in the setting of fuzzy metric spaces. Analogously this property can be defined in b-metric spaces as shown below.

Definition 2.10. (Chauhan, [7]) Two pairs *(A, P)* and *(B, Q)* of self maps of *X*, where (*X, d)* is a b-metric space, is said to satisfy the common limit range property with respect to *P* and *Q* (briefly, (CLR_{PQ}) property) if there exist sequences $\{u_n\}$ and $\{v_n\}$ in *X* such that $\lim_{n\to\infty} Au_n = \lim_{n\to\infty} Pu_n = \lim_{n\to\infty} Bv_n = \lim_{n\to\infty} Qv_n = p,$

where $p \in P(X) \cap Q(X)$.

Example 2.11. Consider the b-metric $d: X^2 \to [0,\infty)$ given by $d(u,v) = (u-v)^2$ with *s = 2* where *X=R*.

Let the maps *A*, *B*, *P*, and *Q* of *X* be defined by $A(u) = e^u$, $P(u) = 1 - u^2$, $B(u) = u^2$ and $Q(u) = 2 - u^2$.

Consider the sequences $u_n = \frac{1}{n}$ and $v_n = 1 - \frac{1}{\sqrt{n}}, n = 1, 2, \ldots\ldots$ in *X*.

Then $d(Au_n, 1) = \left(e^{\frac{1}{n}} - 1\right)^2 \to 0$ as $n \to \infty$, $d(Pu_n, 1) = \frac{1}{n^4} \to 0$ as $n \to \infty$,

$d(Bv_n, 1) = \left(\left\{1 - \frac{1}{\sqrt{n}}\right\}^2 - 1\right)^2 \to 0$ as $n \to \infty$ and $d(Qv_n, 1) = \left(1 - \left\{1 - \frac{1}{\sqrt{n}}\right\}^2\right)^2 \to 0$ as $n \to \infty$, $\lim_{n\to\infty} Au_n = \lim_{n\to\infty} Pu_n = \lim_{n\to\infty} Bv_n = \lim_{n\to\infty} Qv_n = 1 = P(0) = Q(1)$.

The pair *(A, P)* and *(B, Q)* satisfy (CLR_{PQ}) property.

Remark 2.12. If the (E.A) property is replaced with the (CLR) property, then without the closedness property of the range of any underlying map, the existence of fixed points can be derived. This statement is supported by the following theorems.

3. Main Results

Theorem 3.1*: Let A, B, P, and Q be self maps of a b-metric space (X,d) with s >1 such that* $s^k d(Au, Bv) \le M_s(u,v)$ *for all* $u, v \in X$*, where k > 1 is a constant and*

$$M_s(u,v) = max\{ d(Pu,Qv), d(Au,Pu), d(Bv,Qv), \frac{d(Au,Qv)+d(Pu,Bv)}{2s}\}. \quad (3.1)$$

If either

(i) The pair (A, P) satisfy CLR_P*-property and* $A(X) \subseteq Q(X)$*; or*

(ii) The pair (B, Q) satisfy CLR_Q*- property and* $B(X) \subseteq P(X)$*,*

Then $C(A,P) \neq \phi and C(B,Q) \neq \phi$.
Furthermore, if the pairs (A, P) and (B, Q) are weakly compatible, then the maps A, B, P, and Q have a unique common fixed point.
Proof: Firstly, we consider assumption (i).
From the (CLR_P) property of the pair *(A, P)*, we can see that there must be a sequence $\{u_n\}$ in *X* such that $\lim_{n\to\infty} Au_n = \lim_{n\to\infty} Pu_n = Pz = q$, for some $z, q \in X$. (3.2)
Since $A(X) \subseteq Q(X)$, $Au_n = Qv_n$ for a sequence $\{v_n\}$in *X*.
Hence $\lim_{n\to\infty} Qv_n = q$. (3.3)
Now, we claim that $\lim_{n\to\infty} Bv_n = q$.
By replacing *u*, *v* with u_n, v_n respectively in (3.1) and using $Qv_n = Au_n$, we get

$$s^k d(Au_n, Bv_n) \leq M_s(u_n, v_n) \tag{3.4}$$

where,$M_s(u_n, v_n) =$

$$max\left\{d(Pu_n, Qv_n), d(Au_n, Pu_n), d(Bv_n, Qv_n), \frac{d(Au_n,Qv_n)+d(Pu_n,Bv_n)}{2s}\right\}$$

$$= max\left\{d(Pu_n, Au_n), d(Au_n, Pu_n), d(Bv_n, Au_n), \frac{d(Pu_n, Bv_n)}{2s}\right\}$$

$$\leq max\left\{d(Au_n, Pu_n), d(Bv_n, Au_n), \frac{s[d(Pu_n, Au_n) + d(Au_n, Bv_n)]}{2s}\right\}$$

$$= max\{d(Au_n, Pu_n), d(Bv_n, Au_n)\}.$$

On taking limit superior in (3.4), we get

$$\limsup_{n\to\infty} s^k d(Au_n, Bv_n) \leq \limsup_{n\to\infty} M_s(u_n, v_n)$$

$$\leq \lim_{n\to\infty} sup max\{d(Au_n, Pu_n), d(Bv_n, Au_n)\}$$

$$= \lim_{n\to\infty} sup\ d\,(Bv_n, Au_n), \text{since} d(Au_n, Pu_n) \to 0 as n \to \infty, \text{ from (3.2).}$$

Since $s^k > s > 1$, we get $\lim_{n\to\infty} sup d(Bv_n, Au_n) = 0$ and hence $\lim_{n\to\infty} d(Bv_n, Au_n) = 0$.
Also, $d(q, Bv_n) \leq s[d(q, Au_n) + d(Au_n, Bv_n)] \to 0asn \to \infty$.
Thus, $\lim_{n\to\infty} Bv_n = q$. (3.5)
Now, we prove that $Az = q$.
By taking $u = z, v = v_n$ in (3.1) and using $Pz = q$, we get
$s^k d(Az, Bv_n) \leq M_s(z, v_n)$, where (3.6)

$$M_s(z, v_n) = max\left\{d(Pz, Qv_n), d(Az, Pz), d(Bv_n, Qv_n), \frac{d(Az, Qv_n) + d(Pz, Bv_n)}{2s}\right\}$$

$$= max\left\{d(q, Qv_n), d(Az, q), d(Bv_n, Qv_n), \frac{d(Az, Qv_n) + d(q, Bv_n)}{2s}\right\}$$

$$\leq max\left\{d(q, Qv_n), d(Az, q), s[d(Bv_n, q) + d(q, Qv_n)], \frac{sd(Az, q) + sd(q, Qv_n) + d(q, Bv_n)}{2s}\right\}.$$

On taking limit as$n \to \infty$ and using (3.3) and (3.5), we have $\lim_{n\to\infty} M_s(z, v_n) = d(Az, q)$.

Therefore (3.6) implies, $\lim_{n\to\infty} s^k d(Az, Bv_n) \leq \lim_{n\to\infty} M_s(z, v_n) = d(Az, q)$.

That is, $s^k d(Az, q) \leq d(Az, q)$, by (3.5).

Hence $d(Az, q) = 0$, because $s^k > s > 1$. Then $Az = q$. (3.7)

From (3.2) and (3.7), $q = Az = Pz$ and hence $C(A, P) \neq \phi$.

As $A(X) \subseteq Q(X)$, we have $q = Az = Qw$ for some point $w \in X$. (3.8)

We claim that $q = Bw$.

By taking $u = z, v = w$ in (3.1), we have $s^k d(q, Bw) = s^k d(Az, Bw) \leq M_s(z, w)$ and

$$M_s(z,w) = \max\left\{d(Pz,Qw), d(Az,Pz), d(Bw,Qw), \frac{d(Az,Qw)+d(Pz,Bw)}{2s}\right\}$$

$$= \max\left\{d(Bw,q), \frac{d(q,Bw)}{2s}\right\}, \text{ by using (3.2), (3.7) and (3.8)}$$

$$= d(Bw, q).$$

Then $s^k d(q, Bw) \leq d(Bw, q)$.

It follows that $d(q, Bw) = 0$, because $s^k > s > 1$. Hence $q = Bw$. (3.9)

From (3.8) and (3.9), we have $q = Bw = Qw$ and therefore $C(B, Q) \neq \phi$.

Also, $Az = Pz = Bw = Qw = q$. (3.10)

From a weak compatible property of the pairs *(A, P)* and *(B, Q)*, we get

$Aq = Pq$ and $Bq = Qq$ (3.11)

Now, we will show that $Aq = q$.

From (3.1), we have $s^k d(Aq, q) = s^k d(Aq, Bw) \leq M_s(q, w)$,

where $M_s(q, w) = max\left\{d(Pq, Qw), d(Aq, Pq), d(Bw, Qw), \frac{d(Aq,Qw)+d(Pq,Bw)}{2s}\right\}$

$= max\left\{d(Aq, q), d(Aq, Aq), d(q, q), \frac{d(Aq,q)+d(Aq,q)}{2s}\right\}$, by (3.10), (3.11)

$= d(Aq, q)$, since $s > 1$.

Hence $s^k d(Aq, q) \leq d(Aq, q)$, which follows that $q = Aq = Pq$ because $s^k > s > 1$.

Similarly, we can prove that $Bq = q$.

Therefore $Aq = Pq = q = Bq = Qq$.

To prove the uniqueness of q, if possible, suppose that $q^*(q \neq q^*)$ be another common fixed
of $A, B, P,$ and Q . Then $Aq^* = Pq^* = q^* = Bq^* = Qq^*$.

From (3.1), $s^k d(q, q^*) = s^k d(Aq, Bq^*) \leq M_s(q, q^*)$

and $M_s(q, q^*) = max\left\{d(Pq, Qq^*), d(Aq, Pq), d(Bq^*, Qq^*), \frac{d(Aq,Qq^*)+d(Pq,Bq^*)}{2s}\right\}$

$$= max\left\{d(q, q^*), d(q, q), d(q^*, q^*), \frac{d(q, q^*) + d(q, q^*)}{2s}\right\}$$

$= d(q, q^*)$ since $s > 1$.

Hence, $s^k d(q, q^*) \leq d(q, q^*)$, a contradiction to our supposition $q \neq q^*$.

Therefore $q = q^*$, because $s^k > s > 1$.

Similarly, the proof follows under the assumption (ii).

Example 3.2. Take $X = [0,1)$.

Consider the function $d: X \times X \to [0,\infty)$, $d(u,v) = \begin{cases} 0, & if u = v \\ (u+v)^2, & if u \neq v \end{cases}$

Then it is clear that *(X, d)* is a b- metric space with *s = 2*.

We define the mappings *A, B, P,* and *Q* on *X* by

$$A(u) = \frac{u}{10}, \quad P(u) = \begin{cases} u, & 0 \leq u \leq \frac{1}{3} \\ \frac{u}{2}, & \frac{1}{3} < u < 1 \end{cases}, \quad B(u) = 0, \quad Q(u) = \frac{u}{2}$$

Let k = 2.

If $u \in \left[0, \frac{1}{3}\right]$, $v \in (0,1)$, then $s^k d(Au, Bv) = 2^2 \left(\frac{u}{10}\right)^2 \leq \left(\frac{11u}{10}\right)^2 = d(Au, Pu) \leq M_s(u,v)$.

If $u \in \left(\frac{1}{3}, 1\right)$, $v \in (0,1)$, then $s^k d(Au, Bv) = 2^2 \left(\frac{u}{10}\right)^2 \leq \left(\frac{3u}{5}\right)^2 = d(Au, Pu) \leq M_s(u,v)$.

Then the contractive condition (3.1) holds.

It is clear that neither *P(X)* nor *Q(X)* are closed and$A(X) \subseteq Q(X)$. .

Now for the sequence $u_n = \frac{1}{n+3}, n = 1,2,\ldots.. in X$,

$$d(Au_n, 0) = \frac{1}{100(n+3)^2} \to 0 \text{ and } d(Pu_n, 0) = \frac{1}{(n+3)^2} \to 0 \text{ as } n \to \infty.$$

Therefore $\lim_{n\to\infty} Au_n = \lim_{n\to\infty} Pu_n = 0 = P(0)$. The pair *(A, P)* satisfies CLR_P - property.

We observe that *(A, P)* and *(B, Q)* are weakly compatible.

Also, it can be noted that *A, B, P,* and *Q* has a common fixed point ‘0’and clearly, it is unique.

Corollary 3.3. *Let A and P be self maps of a b-metric space (X, d) with s >1 such that* $s^k d(Au, Av) \leq M_s(u,v)$ *for all* $u, v \in X$, *where* $k > 1$ *is a constant and* $M_s(u,v) = max\left\{d(Pu,Pv), d(Au,Pu), d(Av,Pv), \frac{d(Au,Pv)+d(Pu,Av)}{2s}\right\}$.

If the pair (A, P) satisfy $CLR_P -$ *property, then* $C(A,P) \neq \phi$.

Furthermore, if the pair (A, P) is weakly compatible, then the maps A and P have a unique common fixed point.

Proof: The proof follows by taking *B = A* and *Q = P* in Theorem 3.1.

Remark 3.4. By applying *(CLR_{PQ})* property, we can relax the containment conditions. The following theorem affirms this statement.

Theorem 3.5: *Let A, B, P, and Q be self maps of a b-metric space (X,d) with s >1 such that* $s^k d(Au, Bv) \leq M_s(u,v)$ *for all* $u, v \in X$, *where k > 1 is a constant and*

$$M_s(u,v) = max\{d(Pu,Qv), d(Au,Pu), d(Bv,Qv), \frac{d(Au,Qv)+d(Pu,Bv)}{2s}\}. \tag{3.12}$$

If the pairs (A,P) and (B, Q) satisfy (CLR_{PQ})*- property, then* $C(A,P) \neq \phi and C(B,Q) \neq \phi$.

Furthermore, if the pairs (A, P) and (B, Q) are weakly compatible, then the maps A, B, P, and Q have a unique common fixed point.

Proof: From the (CLR_{PQ}) - property of *(A, P)* and *(B, Q)*, there exist two sequences $\{u_n\}$ and $\{v_n\}$ in X such that

$$\lim_{n\to\infty} Au_n = \lim_{n\to\infty} Pu_n = \lim_{n\to\infty} Bv_n = \lim_{n\to\infty} Qv_n = p, \text{ where } p \in P(X) \cap Q(X). \quad (3.13)$$

Then $p = Pr = Qt$ for some $r, t \in X$. (3.14)

We now prove that $p = Ar$.
From (3.12), $s^k d(Ar, Bv_n) \leq M_s(r, v_n)$,
and $M_s(r, v_n) = max\left\{d(Pr, Q\,v_n), d(Ar, Pr)\,, d(Bv_n, Qv_n), \frac{d(Ar,Qv_n)+d(Pr,Bv_n)}{2s}\right\}$.
On letting $n \to \infty$ and using (3.13) and (3.14), we will have

$$\lim_{n\to\infty} M_s(r, v_n) = max\left\{d(p,p), d(Ar,p), d(p,p), \frac{d(Ar,p) + d(p,p)}{2s}\right\}$$

$= d(Ar, p)$, since $s > 1$.

Therefore $s^k d(Ar, p) = \lim_{n\to\infty} s^k d(Ar, Bv_n) \leq d(Ar, p)$,

which follows that $p = Ar$ because $s^k > s > 1$. (3.15)
From (3.14) and (3.15), $p = Ar = Pr$ and hence $C(A, P) \neq \phi$.
We now prove that $p = Bt$.
From (3.12), $s^k d(Ar, Bt) \leq M_s(r, t)$,
where $M_s(r, t) = max\left\{d(Pr, Qt), d(Ar, Pr)\,, d(Bt, Qt), \frac{d(Ar,Qt)+d(Pr,Bt)}{2s}\right\}$
By using (3.14) and (3.15),
we get $M_s(r, t) = max\left\{d(p,p), d(p,p), d(Bt,p), \frac{d(p,p)+d(p,Bt)}{2s}\right\}$

$= d(Bt, p)$, since $s > 1$.

Then, $s^k d(p, Bt) = s^k d(Ar, Bt) \leq d(Bt, p)$,
Which follows that $p = Bt$, because $s^k > s > 1$. (3.16)
From (3.14) and (3.16), we have $p = Bt = Qt$ and hence $C(B, Q) \neq \phi$.
From the weak compatibility of the pairs *(A, P)* and *(B, Q)*, the unique common fixed point of *A*, *B*, *P* and *Q* can be established the same as in the proof of theorem 3.1.

Example 3.6. Take $X = [0,1)$.
Let the *b*-metric and *A*, *B*, *P*, and *Q* on *X* be defined as in Example 3.2 and let *k = 2*
Then it is shown in Example 3.2 that the contractive condition (3.12) is satisfied and neither
P(X) nor *Q(X)* are closed.

Now, consider the sequences, $u_n = \frac{1}{n+3}$, $v_n = \frac{1}{n}, n = 1, 2, ...$

Then $d(Au_n, 0) = \frac{1}{100(n+3)^2} \to 0$ and $d(Pu_n, 0) = \frac{1}{(n+3)^2} \to 0$ as $n \to \infty$.

Also $d(Bv_n, 0) = d(0,0) \to 0$ and $d(Qv_n, 0) = \frac{1}{4n^2} \to 0$ as $n \to \infty$.

Therefore, $\lim_{n\to\infty} Au_n = \lim_{n\to\infty} Pu_n = \lim_{n\to\infty} Bv_n = \lim_{n\to\infty} Qv_n = 0 = P(0) = Q(0)$.

This implies that *(A, P)* and *(B, Q)* satisfy (CLR_{PQ}) - property.
We observe that *(A, P)* and *(B, Q)* are weakly compatible.
Also, it can be noted that *A*, *B*, *P*, and *Q* has a common fixed point '0' and clearly, it is unique.

4. Conclusion

In Theorem 3.1, common fixed points for four mappings are established by using the notion of the (CLR) property without assuming the closedness of any of the ranges. Theorem 3.5 applies (CLR_{PQ}) property, and consequently, the containment conditions are relaxed to prove the existence of common fixed points. Using the notion of (CLR) property, the pre-existing results in b-metric spaces have been improvised. In addition to that, appropriate examples are provided to back up our findings.

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