DEPARTMENT OF MATHEMATICS

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A Report on

Student Study Project

Importance of Vedic Mathematics

Vedic Mathematics is an Indian ancient system of mathematical calculations or operations techniques developed in the year of 1957 with 16-word formulae and some sub-formulae. In competitive examinations, students find difficult to solve the aptitude questions effectively with very less or small-time durations. Even though students are able to understand the problem, they are not able to speedup calculation process. In this Project some basic mathematical calculations, multiplication, square root, cube root and subtraction of fractional decimal numbers are distributed to a group of 5 students, using Vedic methods techniques. The time taken to complete the calculations are taken in terms of minutes before and after adopting Vedic method's techniques. This Project could able to find that Vedic method significantly improves the speed of calculations while performing some basic mathematical operations. Wish this Project could play an active and supportive role in actual research of Vedic mathematics and techniques to improve the speed of calculations especially while writing any competitive examinations. Five Students participated in this project.

Importance Of Vedic Mathematics

Student study project

Academic Year 2020-21

Submitted by

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Abstract: **Vedic Mathematics** is a collection of Techniques/Sutras to solve mathematical arithmetic's in easy and faster way. It consists of 16 Sutras (Formulae) and 13 sub-sutras (Sub Formulae) which can be used for problems involved in arithmetic, algebra, geometry, calculus, conics.

In this project we study the importance and benefits of Vedic mathematics.

How Vedic Mathematics is Beneficial and What are the Advantages of Vedic Mathematics

Vedic Mathematics can definitely solve mathematical numerical calculations in faster way. Some Vedic Math Scholars mentioned that Using Vedic Maths tricks you can do calculations 10-15 times faster than our usual methods. I agree this to some extent because some methods in Vedic Mathematics are really very fast. But some of these methods are dependent on the specific numbers which are to be calculated. They are called specific methods.

- Eradicates fear of Math completely. So If your child has Math-Phobia High Speed Vedic More than 1700% times faster than normal Math: this makes it the World's Fastest.
- Math is a Fun-Filled way to do Math and arises interest in your child.
- Much Improved Academic Performance in School and Instant Results. Just see the first exercise and believe it for yourself. Go over the examples given in the tutorials you would be amazed.
- Sharpens your mind, increases mental agility and intelligence.
- Increases your speed and accuracy. Become a Mental Calculator yourself.
- Improves memory and boosts self confidence.
- Cultivates an Interest in your for numbers.
- Develops your left and right sides of your brain hence using intuition and innovation. It has been noted that Geniuses have been using the right side of the brain to achieve exceptional results.
- Easy to master and apply. You just need the knowledge of tables to learn this
- Vedic Maths Techniques/Sutras have the maths tricks for fast calculation and can be used in exams like CAT, CET, SAT, Banking Exams, etc.

Conclusions:

- It helps a person to solve problems 10-15 times faster.

- It reduces burden (Need to learn tables up to nine only).
- It provides one line answer.
- It is a magical tool to reduce scratch work and finger counting.
- It increases concentration.
- Logical thinking process gets enhanced.

Benefits to STUDENTS:

- Better calculation skills.
- Better logical and anaytical skills.
- Phobia of mathematics in a very practical way.
- Helpful for future if the students have to appear for IIT,IAS etc.
- Usegul for students appearing for NTSE or board examination.
- Gain IT helps students solve mathematical problems about 15 times faster.
- Aids intelligent guestimation(Knowing the answer without solving the problem).
- Reduces finger countconting risk and improve mental calculation

Benefits to PARENTS:

- A child doing well in mathematics and reasoning at a very young age stands a great chance of getting in to a better career.
- Mathematics phobia is eradicated.
- A successful child always paves a path for a great career and gives the parents immense pleasure.
- It generates the interest of child towords mathematics and science.
- It increases the confidence of children as they are able to better in mathematics and science.
- Reduces the dependency of children on calculators and computes.
- Rduces the dependency of children on tutions.

Benefits for SCHOOLS:

- A school offering this program will definitely gain popularity and name.
- A school can use this program as one of the USP's of the school for advertisement purpoose.
- Weaker studens will ger extra support from this program and thus results would be better.
- High scalability for the school in terms of student traffic.
- Better results in subjects like mathematics and science.
- Better results in subjecs like mathematics and science.
- Better results in comoetitions likeNTSE, AIEEE, IIT, PMT etc.

Study of Different Methods on Check digits an Application of Division Algorithm

Project submitted to the Department of Mathematics for the Student Study Project

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ABSTRACT:

In this project we are going to see different schemes or methods to find out cheque digit in Postal money orders, Air tickets, Universal product code, International standard book number, Cheque number, Bank note serial numbers and we will be deriving conditions when error go un detected. Most of these methods are based on modular arithmetic which is mainly dependent on division algorithm. We have been using this division algorithm in our daily life for example if it is now Wednesday we know that in 23 days it will be Friday because $23 = 7.3 + 2$ so we have added 2 days to the Wednesday instead of counting 23 days. Surprisingly, this simple idea has numerous important applications in mathematics and computer science. There is a method which was developed by J.Verhoeff a German software developer this is relied on dihedral group D_5. This method detects all single digit errors and all transposition errors. Aadhaar a Unique identity number is a 12 digit number based on biometric related information. A digit will be appended to a 11 digit number as a check which can be calculated by Verhoeff scheme. Using these methods we found out check digits of various identifiers. At the end we form a table of formulas of check digits and condition when errors detection is not caught.

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Introduction

Numbers are integral part of our daily life. This world is filled with numbers .If we buy any product from supermarket it is identified by a twelve digit number. If we issue a bank cheque to someone it contains a nine digit number at bottom of the cheque If we purchase a book it has a ten digit number called ISBN number our Aadhaar card contains a twelve digit number there are so many other things on which we see a string of digits these things are identified by Unique numbers. The last digit of these big numbers are called check digit. Generally we did not give much attention to it this digit has specific purpose while feeding these numbers into computers through keyboard some typing mistakes may occur these mistakes are like single digit errors to multi digit errors happen which leads to a wrong check hence operator can easily identify the mistake and he or she can reenter that number.

Objectives

- To Understand Division algorithm and its applications
- To Understand Various schemes for finding check digits
- Finding check digits of UPC, ISBN,etc
- To understand Dihedral groups
- To understand how Algebraic structures like groups can be used in real life

Methodology

Division Algorithm

Let a and b be integers with $b > 0$ then there exist unique integers q and r with the property that $a = bq + r$ where $0 \leq r < b$

Example

$a = 18, b = 4$Then$18 = 4.4 + 2$

$a = -25, b = 7$Then$-25 = 7(-4) + 3$

Modular Arithmetic

This is an application of Division Algorithm. Knowingly or unknowingly we use this in our day to daylife.

For example

If it is now October, What month it will be 27 months from now the answer is JanuaryLogic behind this answer is D.A Since $27 = 2.12 + 3$ just add 3 months to October

If $a = qn + r$

$q = quotient, r = remainder\ upon\ dividing\ a\ by\ n$

We write it as $\boldsymbol{a\ mod\ n = r}$ **or** $\boldsymbol{a = r\ mod\ n}$

Thus 3 mod $2 = 1$

$11\ mod\ 3 = 2\ i.e\ 11 = 3.3 + 2$

$62\ mod\ 85 = 62\ i.e\ 62 = 0.85 + 62$

Simple Properties

$(ab)\ mod\ n = ((a\ mod\ n)(b\ mod\ n))mod\ n$

$(a + b)\ mod\ n = ((a\ mod\ n) + (b\ mod\ n))\ mod\ n$

Applications of Modular Arithmetic

Identification of numbers for the purpose of detecting forgery or errors.

When human beings entering identification numbers such as Money orders,ISBN, UPC, Air ticket, Cheque ,Aadhaar numbers into computers for specific purposes some errors may occur they are of following type

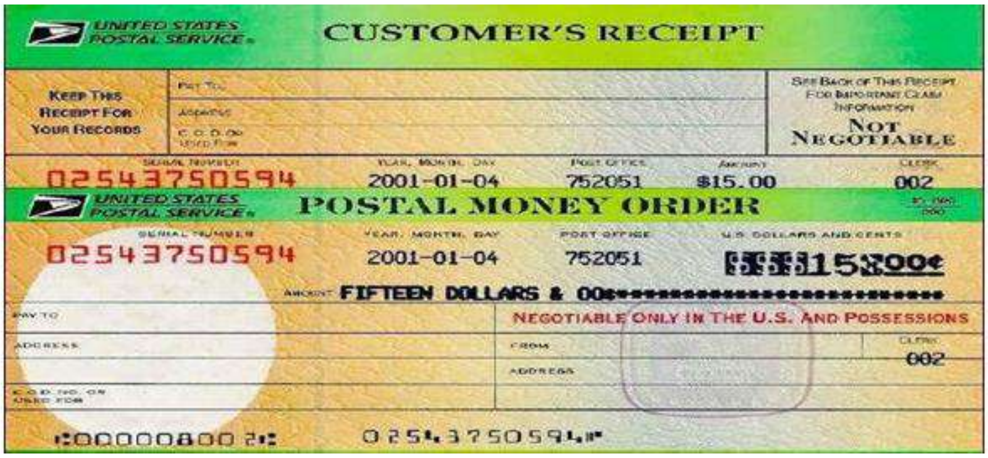
- Single digit errors, such as 2 →3
- Transposition errors, such as 32 → 23
- Twin errors, such as 22 → 33
- Jump transpositions errors, such as 134 → 431

- Jump twin errors, such as 131 →232
- Phonetic errors, such as 60 → 16 ("sixty" to "sixteen")

For simplicity, of above we confine ourselves to Single digit errors and Transposition errors to avoidthese errors one digit is appended to identification number. For different identifications different formulasare used. In this project we will see various check digit formulas, how they works, merits and demerits **POSTAL SERVICE MONEY ORDER**

The United States Postal Service money order shown in the following figure has an identification number consists of 10 digits together with an extra digit called CHECK**.** The check digit is the sum of the digits modulo 9.Thus the number **02543750594** has check digit 4 since

$0+2+5+4+3+7+5+0+5+9=40 \, mod \, 9=4$



If the number 0254375059 were incorrectly entered into a computer as say 02**3**4375059 the computerwould calculate the check as 2 since

$0+2+\mathbf{3}+4+3+7+5+0+5+9=38 \, mod \, 9=2$

Where as the check digit would be 4.Thus the error would be detected but is it detects all single digit errors?

If the number 024375059 were incorrectly entered into a computer as say 0234375050 the computerwould calculate the check as 4 since

$0+2+5+4+3+7+5+0+5+0=31 \, mod \, 9=4$

But error is not detected because $0 \, mod \, 9=9 \, mod \, 9$

Let us formulate above concept

The last digit is the check digit given by the following formula

$x_{11}=(x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9+x_{10}) \, mod \, 9$

If the Entered incorrect number is $x_1x_2x_3x_4x_5x_6x_7x_8x_9y_{10}$ error would go undetected if

$(x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9+x_{10}) mod \, 9=(x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9+y_{10}) \, mod \, 9$

$\Rightarrow x_{10}\ mod\ 9 = y_{10}\ mod\ 9$ in general $x_i\ mod\ 9 = y_i\ mod\ 9$

It detects the error only if $y_i\ mod\ 9 \neq x_i\ mod\ 9$

If the number 0254375059 were incorrectly entered into a computer as say 0524375059 the computer would calculate the check as 4 only since

$$0 + 5 + 2 + 4 + 3 + 7 + 5 + 0 + 5 + 9 = 40\ mod\ 9 = 4$$

In this case computer cannot detect this error because addition is commutative. in general It detects errorif only

$$(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10})\ mod\ 9$$
$$\neq (x_2 + x_1 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10})\ mod\ 9$$

AIR TICKET

Some Airline companies use Modulo 7 to assign check digit of identification of their tickets. For example following Airline Ticket identification number **2 001 6027808715 1**has the check digit **1** appended to it

Because $20016027808715\ mod\ 7 = 1$ since $\mathbf{20016027808715 = 2859432544102} \times 7 + \mathbf{1}$

If the number 2 001 6027808715 were incorrectly entered into a computer as say 2001**5**027808715

the computer would calculate the check as **2** since $2001\mathbf{5}027808715 = 2859289686959 \times 7 + 2$

Where as the check digit would be 3. Thus the error would be detected

The formula of last digit is$\boldsymbol{x_{15} = (x_1x_2x_3x_4x_5x_6x_7x_8x_9x_{10}x_{11}x_{12}x_{13}x_{14})\ mod\ 7}$

If the number 2 001 6027808715 were incorrectly entered into a computer as say 20016097808715

the computer would calculate the check as **1** since $20016097808715 = 2859442544102 \times 7 + 1$

In this case we cannot detect the error if

$$(x_1x_2x_3x_4x_5x_6x_7x_8x_9x_{10}x_{11}x_{12}x_{13}x_{14})\ mod\ 7\ =\ (x_1x_2x_3x_4x_5x_6y_7x_8x_9x_{10}x_{11}x_{12}x_{13}x_{14})\ mod\ 7$$

$\Rightarrow x_7\ mod\ 9 = y_7\ mod\ 7$

In general single digit errors cannot be detected if $x_i\ mod\ 7 = y_i\ mod\ 7$

here9 $mod\ 7 = 2\ mod\ 7$hence unable to detect the error

If the Ticket identification number **2 001 6297808715 5** has the check digit **1** appended to it

Because $20016297808715\ mod\ 7 = 5$Since $\mathbf{2001629808715} = \mathbf{2859471115530} \times 7 + \mathbf{5}$

If the number 2 001 6297808715 were incorrectly entered into a computer as say 20016**92**7808715

the computer would calculate the check as 5 since $20016297808715 = 2859561115530 \times 7 + 5$

It is clear that check digit is same sytem cannot detect this transition error in general transposition errorcannot be detected if $x_i\ mod\ 7 = x_{i-1} mod\ 7$

UPC (UNIVERSAL PRODUCT CODE)

Universal Product Code is printed on package items it has 12 digits (Shown below). The first sixdigits identify the manufacture; the next five identify the product and last is a check

9 87654 32109 8

The check digit is calculated by following formula

$(9,8,7,6,5,4,3,2,1,0,9,8).(3,1,3,1,3,1,3,1,3,1,3,1)\ mod\ 10 = 0$

$L.H.S = 9.3 + 8.1 + 7.3 + 6.1 + 5.3 + 4.1 + 3.3 + 2.1 + 1.3 + 0.1 + 9.3 + 8.1$

$= 27 + 8 + 21 + 6 + 15 + 4 + 9 + 2 + 3 + 27 + 8$

$= 130$

$130\ mod\ 10 = 0$

UPC should satisfy the following formula

$$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}).(3,1,3,1,3,1,3,1,3,1,3,1)\ mod\ 10 = 0$$

The fixed 12-tuple used in the calculation of check digits is called the weighting vector

Suppose a single error is made in entering the number into computer. Say for instance that $98765\mathbf{3}321098$ is entered. Then computer (Which is programmed to calculate check) calculates

$(9,8,7,6,5,4,3,2,1,0,9,8).(3,1,3,1,3,1,3,1,3,1,3,1)\ mod\ 10$

$= 9.3 + 8.1 + 7.3 + 6.1 + 5.3 + 4.1 + 3.3 + 2.1 + 1.3 + 0.1 + 9.3 + 8.1$

$= 27 + 8 + 21 + 6 + 15 + 3 + 9 + 2 + 3 + 27 + 8$

$= 129$

But $129\ mod\ 10 = 9 \neq 0$

Therefore entered number cannot be correct

In general any single error will result in a sum modulo 10 not equal to zero

Suppose a transposition error is made in entering the number into computer. Say for instance that 987653321089 is entered. Then computer calculates

$(9,8,7,6,5,3,3,2,1,8,9).(3,1,3,1,3,1,3,1,3,1,3,1)\ mod\ 10$

$= 9.3 + 8.1 + 7.3 + 6.1 + 5.3 + 3.1 + 3.3 + 2.1 + 1.3 + 0.1 + 8.3 + 9.1$

$= 27 + 8 + 21 + 6 + 15 + 3 + 9 + 2 + 3 + 24 + 9$

$= 135$

But $135\ mod\ 10 = 5 \neq 0$

Therefore entered number cannot be correct

If the following UPC is wrongly entered in the computer as 892680501003.Then computer calculates

$(8,9,2,6,8,0,5,0,1,0,0,3).(3,1,3,1,3,1,3,1,3,1,3,1)\ mod\ 10$

$= 8.3 + 9.1 + 2.3 + 6.1 + 8.3 + 0.1 + 5.3 + 0.1 + 1.3 + 0.1 + 0.3 + 3$

$= 24 + 9 + 6 + 6 + 24 + 0 + 15 + 0 + 3 + 0 + 0 + 3 = 90$

Hence $90\ mod\ 10 = 0$

8 92685 00100 3

To verify this, we observe that a transposition error of the form

$x_1x_2 \ldots . x_5x_6 \ldots . . x_{12} - -\rightarrow x_1x_2 \ldots . x_6x_5 \ldots \ldots . x_{12}$ is undetected if

$$(x_1, x_2 \ldots . x_6x_5 \ldots . . x_{12}). (3,1,3,1,3,1,3,1,3,1,3,1) \; mod \; 10$$

$$= 0(x_1, x_2 \ldots . x_5x_6 \ldots . . x_{12}). (3,1,3,1,3,1,3,1,3,1,3,1) \; mod \; 10$$

$$= (x_1, x_2 \ldots . x_6x_5 \ldots . . x_{12}). (3,1,3,1,3,1,3,1,3,1,3,1) \; mod \; 10$$

$\Rightarrow (3x_5 + x_6) \; 10 = (3x_6 + x_5) \; mod \; 10$

$\Rightarrow (2x_5 - 2x_6) \; 10 = 0$

$\Rightarrow x_5 - x_6 = 5$

In general this scheme cannot detect transposition error if $|x_i - x_{i-1}| = 5$

Advantage of UPC Method

Detects nearly all errors involving the transposition of two adjacent digits as well as errors involving onedigit

ISBN

Stands for International Standard Book number identifier. It has 10 digits First two digits represents Group , Second four digits indicates Publisher ,third three digits denotes title and last is check

ISBN 1-55615-678-2

9 781556 156786

The check digit is calculated by the following formula

$$(x_1.10 + x_2.9 + x_3.8 + x_4.7 + x_5.6 + x_6.5 + x_7.4 + x_8.3 + x_9.2 + x_{10}.1)\ mod\ 11 = 0$$

That is $(1, 5, 5, 6, 1, 5, 6, 7, 8, 2).(10, 9, 8, 7, 6, 5, 4, 3, 2, 1) mod\ 11 = 0$

$L.H.S = 10.1 + 9.5 + 8.5 + 7.6 + 6.1 + 5.5 + 4.6 + 3.7 + 2.8 + 1.2$

$= 10 + 45 + 40 + 42 + 06 + 25 + 24 + 21 + 16 + 2$

$= 231$

$231\ mod\ 11 = 0$

Here weighing vector is $(10,9,8,7,6,5,4,3,2,1)$

Suppose a single error is made in entering the number into computer.

Say for instance that 1554156782

is entered. Then computer calculates

$(1,5,5,4,1,5,6,7,8,2).(10,9,8,7,6,5,4,3,2,1) mod\ 11$

$= 10.1 + 9.5 + 8.5 + 7.4 + 6.1 + 5.5 + 4.6 + 3.7 + 2.8 + 1.2$

$= 10 + 45 + 40 + 28 + 06 + 25 + 24 + 21 + 16 + 2$

$= 217$

$217\ mod\ 11 = 8$

Therefore entered number cannot be correct We observe that a single digit error of the form $x_1x_2 \ldots.x_5x_6 \ldots..x_{10} --\rightarrow x_1x_2 \ldots.y_5x_6 \ldots\ldots.x_{10}$ is undetected if

$$(10x_1 + 9x_2 + 8x_3 + 7x_4 + 6x_5 + 5x_6 + 4x_7 + 3x_8 + 2x_9 + x_{10}) mod\ 11$$
$$= (10x_1 + 9x_2 + 8x_3 + 7x_4 + 6y_5 + 5x_6 + 4x_7 + 3x_8 + 2x_9 + x_{10})\ 11$$

$\Rightarrow 6x_5\ mod\ 11 = 6y_5\ mod\ 11$

$\Rightarrow 6(x_5 - y_5)\ 11 = 0$

$\Rightarrow x_5 - y_5 = 11$

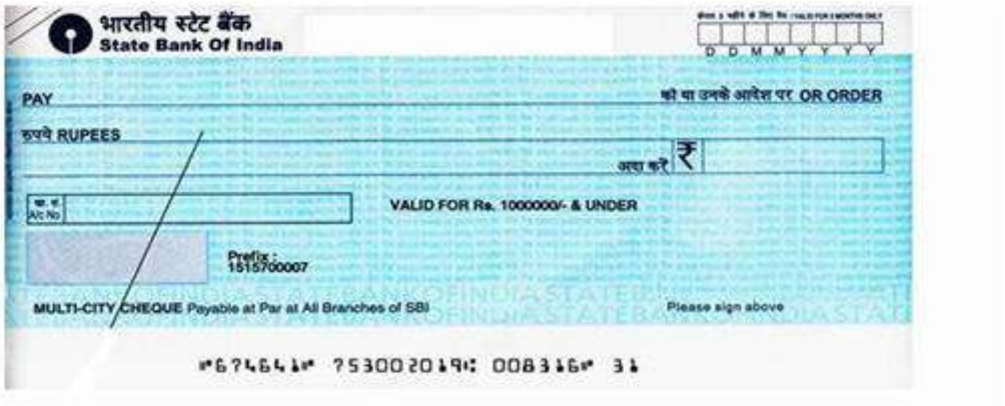
As it is not possible because all the digits are less than 10 so it detects all the single digit errors always.Similarly we can show all transposition errors can be detected.

Advantage

ISBN method is capable of detecting all single digit errors and all transposition errors involving adjacentdigits.

BANK CHEQUE NUMBER

Bank cheque is identified by a number printed between two colons consists of an eight digit number anda check digit.



The check digit is calculated by the following formula

$(x_1.7 + x_2.3 + x_3.9 + x_4.7 + x_5.3 + x_6.9 + x_7.7 + x_8.3 + x_9.9)\ mod\ 10 = 0$

That is

$(7,5,3,0,0,2,0,1,9).(7,3,9,7,3,9,7,3,9)\ mod\ 10 = 0$

$L.H.S = 7.7 + 5.3 + 3.9 + 0.7 + 0.3 + 2.9 + 0.7 + 1.3 + 9.9$

$= 49 + 15 + 24 + 18 + 3 + 81$

$= 190$

$190\ mod\ 10 = 0$

Here weighing vector is (7, 3, 9, 7, 3, 9, 7, 3, 9)

BANKNOTES

Now we see a check digit system based on Dihedral groupD_5.It is a very sophisticated scheme to append check digit to identification number among these schemes only the ISBN method is capable of detecting all single digit errors and all transposition errors involving adjacent digits. It was found by J.Verhoeff .D_5 consists of 10 elements five elements from rotations and five elements from reflexions namely 0,1,2,3,4,5,6,7,8,9 this group is represented by following composition table

O	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	0	6	7	8	9	5
2	2	3	4	0	1	7	8	9	5	6
3	3	4	0	1	2	8	9	5	6	7
4	4	0	1	2	3	9	5	6	7	8
5	5	9	8	7	1	0	4	3	2	1
6	6	5	9	8	7	1	0	4	3	2
7	7	6	5	9	8	2	1	0	4	3
8	8	7	6	5	9	3	2	1	0	4
9	9	8	7	6	5	4	3	2	1	0

Check digit x_n is appended to $x_1x_2 \ldots. x_{n-1}$ such that

Similarly we can write other exponents of σ

$\sigma(x_1)O\sigma^2(x_2)O \dots \dots O\sigma^{n-1}(x_{n-1})Ox_n = 0$

Here $\sigma = (01589427)(36) = \begin{pmatrix} 0 & 1 & 23 & 4 & 56 & 7 & 89 \\ 1 & 5 & 76 & 2 & 83 & 0 & 94 \end{pmatrix}$

And $\sigma^i(a) \neq \sigma^i(b) if\ a \neq b$

$aO\sigma(b) \neq bO\sigma(a)\ if\ a \neq b$

$\sigma^2 = \sigma O\sigma = \begin{pmatrix} 0 & 1 & 23 & 4 & 56 & 7 & 89 \\ 1 & 5 & 76 & 2 & 83 & 0 & 94 \end{pmatrix} O \begin{pmatrix} 0 & 1 & 23 & 4 & 56 & 7 & 89 \\ 1 & 5 & 76 & 2 & 83 & 0 & 94 \end{pmatrix}$

$= \begin{pmatrix} 0 & 1 & 23 & 4 & 56 & 7 & 89 \\ 5 & 8 & 03 & 7 & 96 & 1 & 42 \end{pmatrix}$

$\sigma^{11} = \sigma^{10}O\sigma = \begin{pmatrix} 0 & 1 & 23 & 4 & 56 & 7 & 89 \\ 5 & 8 & 03 & 7 & 96 & 1 & 42 \end{pmatrix} O \begin{pmatrix} 0 & 1 & 23 & 4 & 56 & 7 & 89 \\ 1 & 5 & 76 & 2 & 83 & 0 & 94 \end{pmatrix}$

$= \begin{pmatrix} 0 & 1 & 23 & 4 & 56 & 7 & 89 \\ 8 & 9 & 16 & 0 & 43 & 5 & 27 \end{pmatrix}$

$\sigma^{12} = \sigma^{11}O\sigma = \begin{pmatrix} 0 & 1 & 23 & 4 & 56 & 7 & 89 \\ 8 & 9 & 16 & 0 & 43 & 5 & 27 \end{pmatrix} O \begin{pmatrix} 0 & 1 & 23 & 4 & 56 & 7 & 89 \\ 1 & 5 & 76 & 2 & 83 & 0 & 94 \end{pmatrix}$

$= \begin{pmatrix} 0 & 1 & 23 & 4 & 56 & 7 & 89 \\ 9 & 4 & 53 & 1 & 26 & 8 & 70 \end{pmatrix}$

Similarly we can write other exponents of σ

The following table gives the values of the functions $\sigma, \sigma^2, \dots, \sigma^{10}$needed for computations

O	0	1	2	3	4	5	6	7	8	9
σ	1	5	7	6	2	8	3	0	9	4
σ^2	5	8	0	3	7	9	6	1	4	2
σ^3	8	9	1	6	0	4	3	5	2	7
σ^4	9	4	5	3	1	2	6	8	7	0
σ^5	4	2	8	6	5	7	3	9	0	1
σ^6	2	7	9	3	8	0	6	4	1	5
σ^7	7	0	4	6	9	1	3	2	5	8
σ^8	0	1	2	3	4	5	6	7	8	9
σ^9	1	5	7	6	2	8	3	0	9	4
σ^{10}	5	8	0	3	7	9	6	1	4	2
σ^{11}	8	9	1	6	0	4	3	5	2	7
σ^{12}	9	4	5	3	1	2	6	8	7	0

To any strings of digits $x_1 x_2 \dots x_{10}$ check digit x_{11} is chosen so that

$$(x_1) O \sigma^2(x_2) O \ \dots\ O \sigma^{10}(x_{10}) O x_{11} = 0$$

Let us verify above formula with following bank note bearing the number **DL2496197N7**

Let x be required check digit satisfying the following expression

$\sigma(D)O\sigma^2(L)O\sigma^3(2)O\sigma^4(4)O\sigma^5(9)O\sigma^6(6)O\sigma^7(1)O\sigma^8(9)O\sigma^9(7)O\sigma^{10}(5)Ox = 0$

$5O7O1O1O1O6O0O9O0O0O9Ox = 0$ [using above table]

$7Ox = 0$ [from the above table]

$x = 7$

So 7 is check digit hence verified

Here serial numbers on the bank notes are alphanumeric hence it is necessary to assign numericalvalues to alphabets to compute check digit this assignment is shown in the following table

A	D	G	K	L	N	S	U	Y	Z
0	1	2	3	4	5	6	7	8	9

Suppose a single error is made in entering the number into computer. Say for instance that $DL2496198N7$ is entered. The scheme calculates

$\sigma(D)O\sigma^2(L)O\sigma^3(2)O\sigma^4(4)O\sigma^5(9)O\sigma^6(6)O\sigma^7(1)O\sigma^8(9)O\sigma^9(8)O\sigma^{10}(5)O7$ *shoul be zero*

$5O7O1O1O1O6O0O0O9O0O9O9O7 = 7 \neq 0$ [Using above table]

Hence error can be found

Draw backs

- This scheme does not distinguish between a letter and its assigned value Thus a substitution of 2for G or G for 2 becomes an error
- Does not detect all transpositions of adjacent character involving check digit itself.

Both theses defects can be over come by verhoeff method with D_{18} which is dihedral group of order 36 using this scheme all single position errors and transposition errors involving adjacent digits can be detected

Now we verify check digits of some PMOs using the scheme discussed above

Problems on PMO method

PMO

1. Find check digit of PMO 0421300001

$0+4+2+1+3+0+0+0+0+1$

$= 11 \; mod \; 9$

$= 2$

$\therefore \boldsymbol{Check\ digit = 2}$

2. Find check digit of PMO 0771330536

$0+7+7+1+3+3+0+5+3+6$

$= 17 \; mod \; 9$

$= 8$

$\therefore \boldsymbol{Check\ digit = 8}$

3. Find check digit of PMO 7404348478

$7+4+0+4+3+4+8+4+7+8 = 31 \; mod \; 9 = 4$

$Check\ digit = 4$

4. Find check digit of PMO 0254375059

$0+2+5+4+3+7+5+0+5+9$

$= 40 \; mod \; 9$

$= 4$

$\therefore \boldsymbol{Check\ digit = 4}$

PMO

NUMBER	**CHECK**
0421300001	2
0771330536	8
7404348478	4
0254375059	4

Now we verify check digits of some AIRTICKETSs using the scheme discussed above

Formula:

$$x_{15} = (x_1x_2x_3x_4x_5x_6x_7x_8x_9x_{10}x_{11}x_{12}x_{13}x_{14})\ \boldsymbol{mod}\ \mathbf{7}$$

1. Find check digit of 00121373147367

$00121373147367\ mod\ 7$

$= 3$

$\therefore \boldsymbol{Check\ digit = 3}$

2. Find check digit of 10745778782465

$10745778782465\ mod\ 7$

$= 6$

$\therefore \boldsymbol{Check\ digit = 6}$

3. Find check digit of 10017048327873

$10017048327873\ mod\ 7$

$= 6$

$\therefore \boldsymbol{Check\ digit = 6}$

4. Find check digit of 12174851913640

$12174851913640\ mod\ 7$

$= 3$

$\therefore \boldsymbol{Check\ digit = 3}$

5. Find check digit of 10010424308333

$10010424308333\ mod\ 7$

$= 1$

$\therefore \boldsymbol{Check\ digit = 1}$

AIR TICKET

NUMBER	**CHECK**
00121373147367	3
10745778782465	6
10017048327873	6
12174851913640	3
10010424308333	1

Now we verify check digits of some UPCs using the scheme discussed above

Formula:

$$(x_1.10 + x_2.9 + x_3.8 + x_4.7 + x_5.6 + x_6.5 + x_7.4 + x_8.3 + x_9.2 + x_{10}.1)\ mod\ 11 = 0$$

1. Find check digit of 812190306

$(8,1,2,1,9,0,3,0,6,x).(10,9,8,7,6,5,4,3,2,1)\ mod\ 11 = 0$

$\Rightarrow (8 \times 10 + 1 \times 9 + 2 \times 8 + 1 \times 7 + 9 \times 6 + 0 \times 5 + 3 \times 4 + 0 \times 3 + 6 \times 2 + x) mod\ 11 = 0$

$\Rightarrow (80 + 9 + 16 + 7 + 54 + 0 + 12 + 0 + 12 + x) mod\ 11 = 0$

$\Rightarrow (80 + 9 + 16 + 7 + 54 + 0 + 12 + 0 + 12 + x) mod\ 11 = 0$

$\Rightarrow (80 + 9 + 16 + 7 + 54 + 0 + 12 + 0 + 12 + x) mod\ 11 = 0$

$\Rightarrow (190 + x) mod\ 11 = 0$

$\Rightarrow x = 8$

$\therefore \boldsymbol{Check\ digit = 7}$

2. Find check digit of 817525766

$(8,1,7,5,2,5,7,6,6,x).(10,9,8,7,6,5,4,3,2,1)\ mod\ 11 = 0$

$\Rightarrow (8 \times 10 + 1 \times 9 + 7 \times 8 + 5 \times 7 + 2 \times 6 + 5 \times 5 + 7 \times 4 + 6 \times 3 + 6 \times 2 + x) mod\ 11 = 0$

$\Rightarrow (80 + 9 + 56 + 35 + 12 + 25 + 28 + 18 + 12 + x) mod\ 11 = 0$

$\Rightarrow (275 + x) mod\ 11 = 0$

$\Rightarrow x = 0$

$\therefore \boldsymbol{Check\ digit = 0}$

3. Find check digit of 155615678

$(1,5,5,6,1,5,6,7,8,x).(10,9,8,7,6,5,4,3,2,1)\ mod\ 11 = 0$

$\Rightarrow (1 \times 10 + 5 \times 9 + 5 \times 8 + 6 \times 7 + 1 \times 6 + 5 \times 5 + 5 \times 4 + 7 \times 3 + 8 \times 2 + x) mod\ 11 = 0$

$\Rightarrow (10 + 45 + 40 + 42 + 6 + 25 + 20 + 21 + 16 + x) mod\ 11 = 0$

$\Rightarrow (225 + x) mod\ 11 = 0$

$\Rightarrow x = 6$

$\therefore \boldsymbol{Check\ digit = 6}$

4. Find check digit of 812192661

$(8,1,2,1,9,2,6,6,1)(10,9,8,7,6,5,4,3,2,1)\ mod\ 11 = 0$

$\Rightarrow (8 \times 10 + 1 \times 9 + 2 \times 8 + 1 \times 7 + 9 \times 6 + 2 \times 5 + 6 \times 4 + 6 \times 3 + 1 \times 2 + x) mod\ 11 = 0$

$\Rightarrow (80 + 9 + 16 + 7 + 54 + 10 + 24 + 18 + 2 + x) mod\ 11 = 0$

$\Rightarrow (445 + x) mod\ 11 = 0$

$\Rightarrow x = 6$

$\therefore \boldsymbol{Check\ digit = 6}$

5. Find check digit of 007048298

$(0,0,7,0,4,8,2,9,8).(10,9,8,7,6,5,4,3,2,1)\ mod\ 11 = 0$

$\Rightarrow (0 \times 10 + 0 \times 9 + 7 \times 8 + 0 \times 7 + 4 \times 6 + 8 \times 5 + 2 \times 4 + 9 \times 3 + 8 \times 2 + x) mod\ 11 = 0$

$\Rightarrow (0 + 0 + 56 + 0 + 24 + 40 + 8 + 27 + 16 + x) mod\ 11 = 0$

$\Rightarrow (171 + x) mod\ 11 = 0$

$\Rightarrow x = 5$

$\therefore \boldsymbol{Check\ digit = 5}$

ISBN

NUMBER	**CHECK**
812190306	8
817525766	0
155615678	2
812192661	0
007048298	5

Now we verify check digits of some CHEQUESs using the scheme discussed above

Formula:

$$(x_1.7 + x_2.3 + x_3.9 + x_4.7 + x_5.3 + x_6.9 + x_7.7 + x_8.3 + x_9.9)\ mod\ 10 = 0$$

1. Find check digit of 09190204

$(0,9,1,9,0,2,0,4,x).(7,3,9,7,3,9,7,3,9)\ mod\ 10 = 0$

$\Rightarrow (0 \times 7 + 9 \times 3 + 1 \times 9 + 9 \times 7 + 0 \times 3 + 2 \times 9 + 0 \times 7 + 4 \times 3 + x \times 9) mod\ 10 = 0$

$\Rightarrow (0 + 27 + 9 + 63 + 0 + 18 + 0 + 12 + 9x) mod\ 10 = 0$

$\Rightarrow (129 + 9x) mod\ 10 = 0$

$\Rightarrow x = 9$

$\therefore \boldsymbol{Check\ digit = 9}$

2. Find check digit of 82700200

$(8,2,7,0,0,2,0,0,x).(7,3,9,7,3,9,7,3,9)\ mod\ 10 = 0$

$\Rightarrow (8 \times 7 + 2 \times 3 + 7 \times 9 + 0 \times 7 + 0 \times 3 + 2 \times 9 + 0 \times 7 + 0 \times 3 + x \times 9) mod\ 10 = 0$

$\Rightarrow (56 + 6 + 63 + 0 + 0 + 18 + 0 + 0 + 9x) mod\ 10 = 0$

$\Rightarrow (143 + 9x) mod\ 10 = 0$

$\Rightarrow x = 3$

$\therefore \boldsymbol{Check\ digit = 3}$

3. Find check digit of 40024000

$(4,0,0,2,4,0,0,0,x).(7,3,9,7,3,9,7,3,9)\ mod\ 10 = 0$

$\Rightarrow (4 \times 7 + 0 \times 3 + 0 \times 9 + 2 \times 7 + 4 \times 3 + 0 \times 9 + 0 \times 7 + 0 \times 3 + x \times 9) mod\ 10 = 0$

$\Rightarrow (28 + 0 + 0 + 14 + 12 + 0 + 0 + 0 + 9x) mod\ 10 = 0$

$\Rightarrow (54 + 9x) mod\ 10 = 0$

$\Rightarrow x = 4$

$\therefore \boldsymbol{Check\ digit = 4}$

CHEQUES

NUMBER	**CHECK**
09190204	9
82700200	3
40024000	4

Now we verify check digits of some UPCs using the scheme discussed above

Formula:

$$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}).(3,1,3,1,3,1,3,1,3,1,3,1)\ mod\ 10 = 0$$

1. Find check digit of 02035712268

$(0, 2,0,3,5,7,1,2,2,6,8, x).(3,1,3,1,3,1,3,1,3,1,3,1)\ mod\ 10 = 0$

$(0 \times 3 + 2 \times 1 + 0 \times 3 + 3 \times 1 + 5 \times 3 + 7 \times 1 + 1 \times 3 + 2 \times 1 + 2 \times 3 + 6 \times 1 + 8 \times 3 + x) mod\ 10 = 0$

$\Rightarrow (0 + 2 + 0 + 3 + 15 + 7 + 3 + 2 + 6 + 6 + 24 + x) mod\ 10 = 0$

$\Rightarrow x = 2$

$\therefore \boldsymbol{Check\ digit = 2}$

2. Find check digit of 02233454545

$(0,2,2,3,3,4,5,4,5,4,5, x).(3,1,3,1,3,1,3,1,3,1,3,1)\ mod\ 10 = 0$

$(0 \times 3 + 2 \times 1 + 2 \times 3 + 3 \times 1 + 3 \times 3 + 4 \times 1 + 5 \times 3 + 4 \times 1 + 5 \times 3 + 4 \times 1 + 5 \times 3 + x) mod\ 10 = 0$

$\Rightarrow (0 + 2 + 6 + 3 + 9 + 4 + 15 + 4 + 15 + 4 + 15 + x) mod\ 10 = 0$

$\Rightarrow (0 + 2 + 6 + 3 + 9 + 4 + 15 + 4 + 15 + 4 + 15 + x) mod\ 10 = 0$

$\Rightarrow (57 + x) mod\ 10 = 0$

$\Rightarrow x = 3$

$\therefore \boldsymbol{Check\ digit = 3}$

3. Find check digit of 12222233344

$(1,2,2,2,2,2,3,3,3,4,4\ , x).(3,1,3,1,3,1,3,1,3,1,3,1)\ mod\ 10 = 0$

$(1 \times 3 + 2 \times 1 + 2 \times 3 + 2 \times 1 + 2 \times 3 + 2 \times 1 + 3 \times 3 + 3 \times 1 + 3 \times 3 + 4 \times 1 + 4 \times 3 + x) mod\ 10 = 0$

$\Rightarrow (3 + 2 + 6 + 2 + 6 + 2 + 9 + 3 + 9 + 4 + 12 + x) mod\ 10 = 0$

$\Rightarrow (38 + x) mod\ 10 = 0$

$\Rightarrow x = 2$

$\therefore \boldsymbol{Check\ digit = 2}$

4. Find check digit of 89268500100

$(8,9,2,6,8,5,0,0,1,0,0,x).(3,1,3,1,3,1,3,1,3,1,3,1)\ mod\ 10 = 0$

$(8\times3+9\times1+2\times3+6\times1+8\times3+5\times1+0\times3+0\times1+1\times3+0\times1+0\times3+x)mod\ 10 = 0$

$\Rightarrow (24+9+6+6+24+5+0+0+3+0+0+x)mod\ 10 = 0$

$\Rightarrow (77+x)mod\ 10 = 0$

$\Rightarrow x = 3$

$\therefore \boldsymbol{Check\ digit = 3}$

5. Find check digit of 57241200001

$(5,7,2,4,1,2,0,0,0,0,1,x).(3,1,3,1,3,1,3,1,3,1,3,1)\ mod\ 10 = 0$

$(5\times3+7\times1+2\times3+4\times1+1\times3+2\times1+0\times3+0\times1+0\times3+0\times1+1\times3+x)mod\ 10 = 0$

$\Rightarrow (15+7+6+4+3+2+0+0+3+0+3+x)mod\ 10 = 0$

$\Rightarrow (3+x)mod\ 10 = 0$

$\Rightarrow x = 7$

$\therefore \boldsymbol{Check\ digit = 7}$

UPC

NUMBER	**CHECK**
02035712268	2
02233454545	3
12222233344	5
89268500100	3
57241200001	5

Now we verify check digits of some BANK NOTES using the scheme discussed above

Formula:

O	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	0	6	7	8	9	5
2	2	3	4	0	1	7	8	9	5	6
3	3	4	0	1	2	8	9	5	6	7
4	4	0	1	2	3	9	5	6	7	8
5	5	9	8	7	1	0	4	3	2	1
6	6	5	9	8	7	1	0	4	3	2
7	7	6	5	9	8	2	1	0	4	3
8	8	7	6	5	9	3	2	1	0	4
9	9	8	7	6	5	4	3	2	1	0

Check digit x_n is appended to $x_1x_2 \ldots . x_{n-1}$ such that

$\sigma(x_1)O\sigma^2(x_2)O \ldots \ldots O\sigma^{n-1}(x_{n-1})Ox_n = 0$

Here $\sigma = (01589427)(36) = \begin{pmatrix} 0 & 1 & 23 & 4 & 56 & 7 & 89 \\ 1 & 5 & 76 & 2 & 83 & 0 & 94 \end{pmatrix}$

O	0	1	2	3	4	5	6	7	8	9
σ	1	5	7	6	2	8	3	0	9	4
σ^2	5	8	0	3	7	9	6	1	4	2
σ^3	8	9	1	6	0	4	3	5	2	7
σ^4	9	4	5	3	1	2	6	8	7	0
σ^5	4	2	8	6	5	7	3	9	0	1
σ^6	2	7	9	3	8	0	6	4	1	5
σ^7	7	0	4	6	9	1	3	2	5	8
σ^8	0	1	2	3	4	5	6	7	8	9
σ^9	1	5	7	6	2	8	3	0	9	4
σ^{10}	5	8	0	3	7	9	6	1	4	2
σ^{11}	8	9	1	6	0	4	3	5	2	7
σ^{12}	9	4	5	3	1	2	6	8	7	0

To any strings of digits $x_1x_2 \ldots x_{10}$ check digit x_{11} is chosen so that

$$\sigma(x_1)O\sigma^2(x_2)O \ldots O\sigma^{10}(x_{10})Ox_{11} = 0$$

A	D	G	K	L	N	S	U	Y	Z
0	1	2	3	4	5	6	7	8	9

1. Find check digit of GA9556216U

$\sigma(G)o\sigma^2(A)o\sigma^3(9)o\sigma^4(5)o\sigma^5(5)o\sigma^6(6)o\sigma^7(2)o\sigma^8(1)\hat{}\sigma^9(6)o\sigma^{10}(U)ox = 0$

$\Rightarrow \sigma(2)o\sigma^2(0)o\sigma^3(9)o\sigma^4(5)o\sigma^5(5)o\sigma^6(6)o\sigma^7(2)o\sigma^8(1)\hat{}\sigma^9(6)o\sigma^{10}(7)ox = 0$

$\Rightarrow 7o5o7o2o7o6o4o1o3o1ox = 0$

$\Rightarrow 2o7o2o7o6o4o1o3o1ox = 0$

$\Rightarrow 9o2o7o6o4o1o3o1ox = 0$

$\Rightarrow 7o7o6o4o1o3o1ox = 0$

$\Rightarrow 0o6o4o1o3o1ox = 0$

$\Rightarrow 7o1o3o1ox = 0$

$\Rightarrow 6o3o1ox = 0$

$\Rightarrow 8o1ox = 0$

$\Rightarrow 7ox = 0$

$\Rightarrow x = 7$

$\therefore \boldsymbol{Check\ digit = 7}$

2. Find check digit of $DL2496197N$

$\sigma(D)o\sigma^2(L)o\sigma^3(2)o\sigma^4(4)o\sigma^5(9)o\sigma^6(6)o\sigma^7(1)o\sigma^8(9)\hat{}\sigma^9(7)o\sigma^{10}(N)ox = 0$

$\Rightarrow \sigma(1)o\sigma^2(4)o\sigma^3(2)o\sigma^4(4)o\sigma^5(9)o\sigma^6(6)o\sigma^7(1)o\sigma^8(9)\hat{}\sigma^9(7)o\sigma^{10}(5)ox = 0$

$\Rightarrow 5o7o1o1o1o6o0o9o0o9ox = 0$

$\Rightarrow 3o1o1o1o6o0o9o0o9ox = 0$

$\Rightarrow 4o1o1o6o0o9o0o9ox = 0$

$\Rightarrow 0o1o6o0o9o0o9ox = 0$

$\Rightarrow 1o6o0o9o0o9ox = 0$

$\Rightarrow 7o0o9o0o9ox = 0$

$\Rightarrow 7o9o0o9ox = 0$

$\Rightarrow 3o0o9ox = 0$

$\Rightarrow 3o9ox = 0$

$\Rightarrow 7ox = 0 \Rightarrow$

$x = 7$

$\therefore \boldsymbol{Check\ digit = 7}$

BANK NOTES

NUMBER	**CHECK**
GA9556216U	7
DL2496197N	7

RESULTS AND DISCUSSIONS

1. Verified Formula of finding check digits for PMO and it found to be correct
2. Verified Formula of finding check digits for UPC of various products available in the market and they are found to be correct
3. Verified Formula of finding check digits for ISBN of various books available and they are found to be correct
4. Verified formula of finding check digits for Cheque numbers and they are found to be correct
5. Verified formula of verhoff scheme for German bank note serial number and they are found to be correct

CONCLUSIONS:

ITEM	NUMBER OF DIGITS	FORMULA OF CHECK	SINGLE DIGIT ERROR	TRANPOSITION ERROR
POSTAL MONEY ORDER	11	$x_{11} = (x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10})\ mod\ 9$	Except $x_i\ mod\ 9 = y_i\ mod\ 9$	Cannot detects
AIR TICKET	15	$x_{15} = (x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 x_{10} x_{11} x_{12} x_{13} x_{14})\ mod\ 7$	Except $x_i\ mo\quad 7 = y_i\ mod\ 7$	Except $x_i\ mod\ 7 = x_{i-1} mod\ 7$
UPC	12	$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12})$. (3,1,3,1,3,1,3,1,3,1,3 $= 0$	All can be detected	Except $\lvert x_i - x_{i-1} \rvert = 5$
ISBN	10	$(x_1.10 + x_2.9 + x_3.8 + x_4.7 + x_5.6 + x_6.5 + x_7.4 + x_8.3 + x_9.2 + x_{10}.1)\ mod\ 11 = 0$	All can be detected	All can be detected
CHEQUE	9	$(x_1.7 + x_2.3 + x_3.9 + x_4.7 + x_5.3 + x_6.9 + x_7.7 + x_8.3 + x_9.9)\ mod\ 10 = 0$	All can be detected	Except $\lvert x_i - x_{i-1} \rvert = 5$
BANK NOTE	11(not all) Contains alpha numeric	$\sigma(x_1) O \sigma^2(x_2) O \ldots O \sigma^{10}(x_{10}) O x_{11} = 0, \sigma = (01589427)(36)$	Cannot differentiate alphabets and numerals	Adjacent characters Involving thecheck digit itself

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