

STUDENT STUDY PROJECT 2020-21

History of Zero



Supervised by

Dr.V.Yadaiah

Lecturer in Mathematics

1. 18044028468181	B.Sc(MPCs) III	P.Dharani
2. 18044028468004	B.Sc(MPCs) III	A.Soumya Sree
3. 18044028468193	B.Sc(MPCs) III	P.Krishna Sree
4. 18044028442022	B.Sc(MPG) III	M.Amulya
5. 18044028468072	B.Sc(MPCs) III	G.Harika

NAGARJUNA GOVERNMENT COLLEGE (A), NALGONDA

DEPARTMENT OF MATHEMATICS

2020-21

Nagajuna Govt College

Maths- Project

Topic :- History of Zero

zero was invented independently by the Babylonians, Mayans and Indians

Although some researchers say the Indian number system was influenced by the Babylonians

The Babylonians got their number system from the Sumerians. The first people in the world to develop a counting system

In algebra the zero-product property states that the product of the two non-zero element is non-zero. In other words, it is the following assertion

If $ab=0$ then $a=0$ (or) $b=0$

One of the commonest questions which the readers of this archive ask is who discovered zero?

Why then have we not written an article on zero as one of the first in the archive? The reason is basically because of the difficulty of answering the question in a satisfactory form. If someone had come up with the concept of zero which everyone then saw as a brilliant innovation to enter mathematics from that time on, the question would have a satisfactory answer even if we did not know which genius invented it. The historical record, however, shows quite a different path towards the concept. Zero makes shadowy appearances only to vanish again almost as if mathematicians were searching for it yet did not recognize its fundamental significance even when they saw it.

The first thing to say about zero is that there are two uses of zero which are both extremely important but are somewhat different. One use is an empty place indicator in our place-value number system. Hence in a number like 2016 the zero is used so that the positions of the 2 and 1 are correct. Clearly 216 means something quite different. The second use

of zero is as a no of itself in the form we use it as 0. There are also different aspects of zero within these two uses, namely the concept, the notation, and the name. Our name "zero" derives ultimately from the Arabic سيف which also gives us the word "cipher")

Neither of the above uses has an easily described history. It just did not happen that someone invented them, and then everyone started to use them. Also it is fair to say that the number zero is far from an intuitive concept. Mathematical problems started as 'real' problems rather than abstract problems. Number in early historical times were thought of much more concretely than the abstract concepts which are our numbers today. These are giant mental leaps from 5 horses to 5 "things" and then to the abstract idea of "five". If ancient peoples solved a problem about how many horses a farmer needed then the problem was not going to have 0 (or) 23 as an answer.

One might think that once a place-value number system came into existence then the 0 as an empty place indicator is a necessary idea, yet the

yet the Babylonians had a place-value number system without this feature for over 1000 years. Moreover there is absolutely no evidence that the Babylonians felt that there was any problem with the ambiguity which existed. Remarkably original texts survive from the era of Babylonian mathematics. The Babylonians wrote on tablets of unbaked clay, using cuneiform writing. The symbols were pressed into soft clay tablets with the slanted edge of a stylus and so had a wedge-shaped appearance (and hence the name cuneiform) many tablets from around 1700 BC survive and we can read the original texts. Of course their notation they would not distinguish between 200 and 216 (the context would have to show which was intended) it was not until around 1400 BC that the Babylonians put two wedge symbols into the place where we would put zero to indicate which was meant was in 216 or 21["]6.

The two wedge were not the only notation used, however, and on a tablet found at Kish, an ancient Mesopotamian city located east of Babylon in what is today south-central Iraq, a different notation is used

we have an inscription on a stone tablet which contains a date which translates to 876. The inscription concerns the town of Gwalior, 400 km south of Delhi, where they planted a garden 187 by 270 hastas which would produce enough flowers to allow 50 garlands per day to be given to the local temple. Both of the numbers 270 and 50 are denoted almost as they appear today although the 0 is smaller and slightly raised.

We now come to considering the first appearance of zero as a number. Let us first note that it is not in any sense anatural candidate for a number. From early times numbers are words which refer to collections of objects. Of course the problems which arises when one tries to consider zero and negatives as numbers is how they interact in regard to operations of arithmetic, addition, subtraction, multiplication and division. In 3 important books the Indian mathematicians Brahmagupta, Mahavira and Bhaskar tried to answer these questions.

Brahmagupta attempted to give the rules for arithmetic involving zero and negative numbers in the seventh century. He explained that given a number then if you subtract it from itself you obtain zero. He gave the following rule

for addition which involve zero:-

The sum of zero and a negative number is negative

The sum of a positive number and zero is positive

The sum of zero and zero and zero is zero

Subtraction is a little harder :-

a negative number subtracted from zero is positive

a positive number subtracted from zero is negative

zero subtracted from a negative is negative, zero

subtracted from a positive number is positive, zero

subtracted from zero is zero

Brahmagupta then says that any number
when multiplied by zero but struggles when it
comes divisions:-

a positive (or) negative number when divided by zero
is a fractions with the zero as denominator - zero

divided by a negative (or) positive number is either
zero (or) is expressed as a fraction with zero as numerator
and the finite quantity as denominator. zero divided

by zero is zero

Really Brahmagupta is saying very little when he suggests that n divided by zero is n/o. clearly he is a struggling here. He is certainly wrong when he then claims that zero divided by zero is zero however it is a brilliant attempt from the first person that we know who tried to extend arithmetic to negative numbers and zero.

In 830, around 200 years after Brahmagupta wrote his masterpiece mahavira wrote his masterpiece, Mahavira. wrote Ganita Sara Samgraha which was designed as an updating of Brahmagupta's book, he correctly states that

a number multiplied by zero is zero and a number remains the same when zero is subtracted from it

however his attempts to improve on Brahmagupta's statements on dividing by zero seem to lead him into error. He writes.

* A number remains unchanged when divided by zero

since this is clearly incorrect my use of the words Despite the passage of time he is still struggling

Bhaskara wrote over 500 years after Brahma. Despite the passage of time he is still struggling to explain division by zero. He writes:-

A quantity divided by zero becomes a fraction the denominator of which is zero. This fraction is termed an infinite quantity. In this quantity consisting of that which has zero for its divisors there is no alteration, though many may be inserted or extracted; as no change takes place in the infinite and immutable God when worlds are created or destroyed, though numerous orders of beings are absorbed (or) put forth.

So Bhaskara tried to solve the problem by writing $n/0 = \infty$. At first sight we might be tempted to believe that Bhaskara has it correct but of course he does not. If this were true then 0 times ∞ must be equal to every number. Then all numbers are equal. The Indian mathematicians could not bring themselves to the point of admitting that one could not divide by zero. Bhaskara did correctly state other properties of zero, however, such as $0^2 = 0$, and $\sqrt{0} = 0$.

perhaps we should note at this point that there was another civilisation which developed a place value number system with a zero. This was the may people who lived in Central America occupying the area which today is Southern Mexico, Guatemala, and northern Belize. This was an ~~occupying~~ ancient civilisation but flourished particularly b/w 250 and 900 A.D. by 665 they used a place value number system to base 20 with a symbol for zero. However their use of zero goes back further than this and was in use before they introduced the place-value number system. This is a remarkable achievement but sadly did not influence other peoples.

The brilliant work of the Indian mathematicians was transmitted to the Islamic and Arabic mathematicians further west.

It came at an early stage for al-Khwarizmi wrote Al-Khwarizmi on the Hindu Art of Reckoning which describes the Indian place-value system of numerals based on 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0. This book was the first in what is now Iraq to use zero as a place holder in positional base

0 is the integer immediately preceding 1. Zero is an even number because it is divisible by 2. 0 is neither positive nor negative. By most definitions 0 is a natural number, and then the only natural number not to be positive. Zero is a negative thing, (or) quantities less than zero, was accepted.

The value, (or) number, zero is not the same as the digit zero, used in numeral systems using positional notation successive positions of digits have higher weights, so inside a numeral the digit zero is used to skip a position and give appropriate weights to the preceding and following digits. A zero digit is not always necessary in a positional number system for example in the number 02. In some instance a leading zero may be used to distinguish a number.

Elementary ~ algebra

The number 0 is the smallest non-negative integer. The natural number following 0 is 1 and no natural number system, for example, in the number 02. In some instance a leading

Zero may be used to distinguish a number

The number 0 is neither positive nor negative and appears, in the middle of a number line, it is neither a prime number (or) a composite number. It can't be prime because it has an infinite number of factors and can't be composite because it can't be expressed by multiplying prime numbers (0 must always be one of the factors) Zero is however even

The following are some basic (elementary) rules for dealing with the number 0. These rules apply for any real or complex numbers, unless otherwise stated

- addition: $x+0=0+x=x$ That is, 0 is an identity element (or neutral element) w.r.t addition
- subtraction: $x-0=x$ and $0-x=-x$
- multiplication: $x \cdot 0 = 0 = 0 \cdot x = 0$
- Division: $0/x=0$ for all non-zero x But $x/0$ is undefined, because 0 has no multiplicative inverse (no real number multiplied by 0 produces 1) a consequence -

of the previous rule

* Exponentiation: $x^0 = x/x = 1$ except that the case $x=0$ may be left undefined in some contexts. For all positive real x , $0^x = 0$

The expression $\frac{0}{0}$ which may be obtained in an attempt to determine the limit of an expression of the form $f(x)/g(x)$ as a result of applying the \lim operator independently to both operands of the fraction, is also called "indeterminate form". That does not simply mean that the limit sought is necessarily undefined; rather, it means that the limit of $f(x)/g(x)$ if, for example, the sum of 0 numbers is 0, and the product of 0 numbers

such as from "Hopital's rule" the product of the sum of 0 numbers is 0, and the product of 0 numbers is 1. The factorial 0! evaluates to 1

Other branches of mathematics

* In set theory 0 is the cardinality of the empty set if one does not have any apples, then one has 0 apples. In fact, in certain axiomatic

developments of mathematics from set theory
0 is defined to be the empty set, when this is
done, the empty set is the von Neuman cardinal
assignment for a set with no elements, which is the
empty set. The cardinality function, applied to the

empty set as value, the cardinality function,
applied to the empty set, returns the empty set as a
value, thereby assigning it 0 elements.

* Also in set theory 0 is the lowest ordinal
number, corresponding to the empty set viewed
as a well-ordered set

* In propositional logic, 0 may be used to denote
the truth value false

* In abstract algebra, 0 is commonly used to
denote a zero element, which is a neutral element
for additions (if defined on the structure on the
under consideration) and an absorbing element
for multiplication (if defined)

* in lattice theory 0 may denote the bottom
element - or from the truth value false

* in abstract algebra, 0 is commonly used to denote
a zero element which is neutral element for +

* a zero of a function is a point x in the domain of the function such that $f(x)=0$, when there are finitely many zeros these are called the roots of the function

this is related to zeros of holomorphic function

* The zero function (or zero map) on a domain D is the constant function is the only possible output both even and odd. A particular zero function is a zero function is the only function that is both even and odd. A particular zero function is a zero morphism in category theory e.g. a zero map is the identity in the additive group of functions. The determinant on non-invertible square matrices is a zero map.

* Several branches of mathematics have zero elements which generalise either the property $0+x=x$ (or) the property $0 \times x=0$ (or) both.

The set theory 0 is the lowest ordinal number corresponding to the empty set viewed as a well-ordered set

* In propositional logic, 0 may be used to denote the truth value false

- * In abstract algebra 0 is commonly used to denote a zero element, which is a neutral element for addition (if defined on the structure under consideration) and an absorbing element for multiplication (if defined).
- * In lattice theory 0 may denote the bottom element of a bounded lattice.
- * In category theory, 0 is sometimes used to denote an initial object of a category.
- * In recursion theory, 0 can be used to denote the turning degree of the partial computable function.