Student Study Project

On

VECTOR SPACE

Submitted by

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Under the Guidance of

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Submitted to

DEPARTMENT OF MATHEMATICS

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CERTIFICATE

This is to certify that the student study project work entitled VECTOR SPACE is a bonafide work done by the students of III MPC II MPCs A. Naga Rani, B. Rajavardhan, A. Pranitha , Nishad , S. Karthikeya under my supervision for the award of student study project work in Mathematics, Department of Mathematics, Dr. BRR Government Degree College, Jadcherla.

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DECLARATION

We hereby declare that student study project work entitled VECTOR SPACE is a genuine work done by us under the supervision of S.Madhavi Latha, Lecurter in Mathematics, Department of Mathematics, Dr.BRR Government Degree College, Jadcherla.

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AIM: TO STUDY VECTOR SPACES

Introduction:

A Vector Space is a set of objects called vectors which it is possible to add and to multiply by scalars. Vector Spaces occur in numerous branches of mathematics as well as in many applications. The linearity of vector Spaces has made these abstract objects important in diverse areas such as statistics, physics and economics where the vectors may indicate probabilities, forces.

OBJECTIVES:

1. **DEFINE VECTOR SPACE**
2. **DIFFERENT EXAMPLES OF VECTOR SPACE**
3. **PROPERTIES OF VECTOR SPACE**
4. **DEFINE SUBSPACE**
5. **EXAMPLES OF SUBSPACE**

METHODOLOGY:

STUDY THE CONCEPT OF ALGEBRAIC

DEFINITION OF GROUP

RING AND FIELD

VECTOR SPACES

Definition : A vector space "v" is a non-empty set containing vectors on which are defined two operators called Addition and Multiplication by scalars that satisfies the following conditions for all vectors $\bar{u}$, $\bar{v}$, $\bar{w}$ in v,& c, d in R

i) $\bar{u} + \bar{v}$ € V $\bar{u}$,$\bar{v}$ € V (Closure property)

ii) $\bar{u} + \bar{v} = \bar{v} + \bar{u}$ $\bar{u}$,$\bar{v}$ € V (Commutative property)

iii) $(\bar{u}+\bar{v})+\bar{w} = \bar{u}+(\bar{v}+\bar{w})$,$\bar{u}$,$\bar{v}$,$\bar{w}$ €V(Associative property)

iv) There exist $\bar{o}$€V ;$\bar{u}+\bar{o} = \bar{u}$ ∀ $\bar{u}$ €V(Identity property)

v) There exist u € V, $\bar{u} + (-\bar{u}) = \bar{o}$ ∀ $\bar{u}$ € V(Inverse property)

vi) There scalar multiple of $C\bar{u}$ by C denoted by $C\bar{u}$ € V ,$\bar{u}$ €V, c €R

vii) $C(\bar{u}+\bar{v}) = c\bar{u} + c\,\bar{v}$;∀ $\bar{u}$€ V R€R

viii) $(c + d)\,\bar{u} = c\bar{u} + d\bar{u}$; ∀ $\bar{u}$ €V ,c ,d € R

ix) $C(d\bar{u}) = c\,d(\bar{u})$;∀ $\bar{u}$ € V , c ,d € R

x) $I.\bar{u} = \bar{u}$; ∀ $\bar{u}$ €V

(R, +,.) is a field (v ,+) is order triple group R ,∀ x_i ,y_i € R c € R defined

i) $x_1 , x_2 ,x_3) +(y_1 ,y_2 ,y_3) =(x_1 +y_1 ,x_2 +y_2 ,x_3+y_3)$

ii) $C(x_1, x_2, x_3) = (cx_1, cx_2, cx_3)$ then show that v(R) is a vector space

Let $\bar{u} = (x_1, x_2, x_3)$

$\bar{v} = (y_1, y_2, y_3)$ C, d €R

$\bar{w} = (z_1, z_2, z_3)$ €V

i)To prove $\bar{u} + \bar{v}$ €V

$\overline{u}+\overline{v} = (x_1, x_2, x_3) + (y_1, y_2, y_3)$

$= (x_1+y_1)\ (x_2+y_2)\ (x_3+y_3)$

$\in v$

∵ $\overline{u}+\overline{v} \in v, \forall\ \overline{u}, \overline{v}, \in V$

ii)To prove $\overline{u}+\overline{v} = \overline{v}+\overline{u}$

$\overline{u}+\overline{v} = (x_1, x_2, x_3) + (y_1, y_2, y_3)$

$=(x_1+y_1, x_2, y_2, x_3+y_3)$

$= (y_1+x_1, y_2+x_2, y_3+x_3)$

$= (y_1, y_2, y_3) + (x_1, x_2, x_3)$

$= \overline{v} + \overline{u}$

iii) To prove $(\overline{u}+\overline{v}) + \overline{w} = \overline{u} + (\overline{v}+\overline{w})$

$(\overline{u}+\overline{v})+\overline{w}=(x_1+y_1, x_2+y_2, x_3+y_3) + (z_1, z_2, z_3)$

$= [(x_1+y_1) +z_1, (x_2+y_2) + z_2(x_3+y_3) + z_3)]$

$=[x_1 + (y_1+z_1), x_2+ (y_2+z_2), x_3(y_3+z_3)$

$= [(x_1, x_2, x_3) + (y_1+z_1, y_2+z_2, y_3+z_3)]$

$=\overline{u} + [\overline{v}+\overline{w}]$

∵ $(\overline{u}+\overline{v}) + \overline{w} = \overline{u} + (\overline{v}+\overline{w}), \forall\ \overline{u}, \overline{v}\ \overline{w} \in R$

iv)To prove $\overline{u}+\overline{o}= \overline{u}$

Let $\overline{o} = (0, 0, 0) \in V$

$\overline{u}+\overline{o}=(x_1, x_2, x_3) + (0, 0, 0)$

$=(x_1+0, x_2+0, x_3+0)$

$=(x_1, x_2, x_3)$

$=\overline{u}$

$\overline{u}+\overline{o} = \overline{u}, \forall\ \overline{u} \in v$

v)To prove $\bar{u}+(-\bar{u})=\bar{o}$

Let $(-\bar{u}) = (-x_1\, x_2, x)$ €V

$\bar{u} + (-\bar{u}) = (x_1, x_2, x_3) + (-x_1, x_2\, x_3)$

$=(x_1-x_1, x_2-x_2, x_3-x_3)$

$= (0, 0, 0) = 0$

$\bar{u} + (-\bar{u}) = 0\ \forall\ \bar{u}, -\bar{u}$ €V

vi)To prove $C\bar{u}+v$

$C\bar{u} = C(x_1, x_2, x_3)$

$= cx_1\, cx_2, cx_3$

€V

$\forall\ c\bar{u}$ € V c € R

vii)To prove $C(\bar{u}+\bar{v}) = c\bar{u}+c\bar{v}$

$C(\bar{u}+\bar{v}) = c(x_1+y_1, x_2, + y_2, x_3+y_3)$

$= [cx_1+ cy_1, cx_2+ cy_2, cx_3+cy_3]$

$= c(x_1, x_2, x_3) + (y_1, y_2, y_3)$

$= c\bar{u}+c\bar{v}$

$\forall\ \bar{u}, \bar{v}$ € V €R

Viii) To prove $(c+d)\bar{u} = (\bar{u}+d\bar{u}$

$(c+d)\bar{u} = (c+d)(x_1,x_2,x_3)$

$=(c+d)\, x_1, (c+d)\, x_2, (c+d)\, x_3$

$=cx_1+dx_1, cx_2+dx_2, cx_3+dx_3$

$= (cx_1, cx_2, cx_3) + (dx_1, dx_2, dx_3)$

$=c(x_1, x_2, x_3) + d(x_1, x_2, x_3)$

$=c\bar{u}+d\bar{u}$

∵ $(c+d)\bar{u}=c\bar{u}+d\bar{u}\ \forall\ \bar{u} \in V\ \ c, d \in R$

Properties of vector spaces

Theorem: V is a vector space then

i) r is unique
ii) if u€ V then $-\bar{u}$ unique in "v"

Proof: V is a vector space

i) To prove $\bar{o}$ is unique

Since V is vector space

∵ $\bar{o} \in V$ such that $\bar{u}+\bar{o}=\bar{u} \rightarrow (1)\ \forall\ \bar{u} \in \bar{v}$

If possible $\bar{w}$ is another a vector in v such that $\bar{u}+\bar{w}=\bar{u}$, $\forall\ \bar{u} \in V$

For $\bar{o} \in v$; $\bar{o}+\bar{w}=\bar{o} \rightarrow (2)$ but

$\bar{o}+w=w \rightarrow (3)$ [property]

from (2) & (3)

∵ $\bar{o}=\bar{o}+\bar{w}=\bar{w}$

$\rightarrow \bar{w}=\bar{o}$

∵ $\bar{o}$ vector is unique in

ii)To prove $\bar{u} \in v$ then $-\bar{u}$ is negative if $\bar{u} \in v$

then definition of vector space

$\bar{u}+(-\bar{u})=\bar{o} \rightarrow (4)$ [∵ v^{th} property]

Suppose that let $\bar{w} \in V$ be another vector such that

$\bar{u}+\bar{w}=\bar{o} \rightarrow (5)$

Add both sides with (- is)

$(-\bar{u})+(\bar{u}+\bar{w}=(-\bar{u})+v$

$(-\bar{u})+(\bar{u})+\bar{w}=-\bar{u}$

$\bar{o}+\bar{w} = -\bar{u}$

$\bar{w} = -\bar{u}$

∴ $-\bar{u}$ is unique V

Theorem: Let v be a vector space then

i) $o\bar{u}=o$, $\bar{u}$ € V

ii) $C\bar{o} = \bar{o}$; ∀ scalar

iii) if $\bar{u}$ €V then $(-1)\bar{u} =\bar{u}$

Proof: V is a vector space

i) To prove $o\bar{u} = \bar{o}$, $\bar{u}$ €V

$o\bar{u} = (0+0)\ \bar{u}$

$O\bar{U} = O\bar{U} +O\bar{U}$

Add both sides $-O\bar{U}$

$-O\bar{U}+O\bar{U} = -\ O\bar{U} + (O\bar{U}+o\bar{u})$

$\bar{o}= o + o\bar{u}$

∴ $\bar{o} = o\bar{u}$, ∀ $\bar{u}$ € V

ii) To prove $c\bar{o} = \bar{o}$, ∀ scalar

$c\bar{o} = c\ (\bar{o}+\bar{o})$

$c\bar{o} = c\bar{o} +c\bar{o}$

Add - co both sides

$-c\bar{o} + c\bar{o} = -c\bar{o} + (c\bar{o}+c\bar{o})$

$\bar{o} = (-c\bar{o}+c\bar{o}) +c\bar{o}$

$\bar{o} = o+c\bar{o}$

$\bar{o} = c\bar{o}$ ∀ scalar

iii) To prove $\bar{u}$€V then $(-1)\ \bar{u} =-\bar{u}$

$\bar{u} + (-1)\bar{u} = \bar{u} + (-\bar{u})$

$= \bar{o}$

$(-1)\bar{u} = -\bar{u}$

(OR)

$\bar{u} + (-1)\bar{u} = \bar{o}$

Add $(-\bar{u})$ both side

$-\bar{u} + (\bar{u} + (-1)\bar{u}) = -\bar{u} + \bar{o}$

$(-\bar{u} + \bar{u}) + (-1)(\bar{u}) = -\bar{u}$

$O + (-1)\bar{u} = -\bar{u}$

$(-1)(\bar{u}) = -\bar{u}$

SUB SPACE

- H is a subset of a vector space "v' if H satisfies following conditions then ,H is called Subspace of V

i) Zero vector ($\bar{o}$) in V is also in H

i.e., $\bar{o}$ € H

ii) H is closure with respect to addition

i.e., $\bar{u}+\bar{v}$ €H, ∀ $\bar{u}$, $\bar{v}$ € H

iii) H is closure with respect to scalar Multiplication

i.e., $c\bar{u}$ €H, $\bar{u}$ € H $\bar{c}$ is scalar

Theorem: V is a vector space W is a non –empty subset of 'v' w is subspace of v

↔$c\bar{u}+d\bar{v}$ € w, $\bar{u}$, $\bar{v}$ € w c, d are scalar

Proof: V is a vector space

W is a non-empty subset of V

Necessary condition: W is a subspace of v now, we prove that $c\bar{u}+d\bar{v}$ €w; $\bar{u},\bar{v}$ € w c, d are scalar from scalar Multiplication

C is a scalar $\bar{u}$ € w →$c\bar{u}$ € w

D is a scalar $\bar{v}$ € w→ $d\bar{v}$ € w

$c\bar{u}$, $d\bar{v}$ € w from addition closure

∴ $c\bar{u}+d\bar{v}$ € w, ∀ $\bar{u}$, $\bar{v}$ € w, c, d are scalar

Sufficient condition: Let $c\bar{u}+d\bar{v}$ € w $\bar{u}$, $\bar{v}$ € w, c, d are scalars

Now, we will subspace of v

i) $c\bar{u}+d\bar{v}$ = w→①

let c=o, d=o

Put c=o, d=o in equation ①

①→o ($\bar{u}$) + o ($\bar{v}$) € w

O+O € w

ii) Put c=1, d=1, in equation①

1($\bar{u}$) + 1 ($\bar{v}$) € w

$\bar{u}+\bar{v}$ € w, ∀ $\bar{u}$, $\bar{v}$ € w

iii) Put d=o in equation ①

($\bar{u}$ + o ($\bar{v}$) €w

$c\bar{u}$ € w ∀ $\bar{u}$ € w, c is a scalar

W is a subspace of v

- Consider the vector space R^2 with vector addition? scalar Multiplication i.e., R^2={(x, y)/X, y € R } Show that the subset H={(x_1-x)/x €R} is a subspace of R^2

R^2 = {(x, y) /x, y € R}

$H = \{(x_1 - x) / x \in R\}$

i) To prove $\bar{o} \in H$

$(0, 0) \in R^2$

$(0, 0) \in = (0,-0) \in H, o \in R$

ii) To prove $\bar{u}+\bar{v} \in w - v\ \bar{u}, \bar{v} \in H$

Let $\bar{u} = (x_1-x), \bar{v} = (y_1-y) \in +1\ \forall\ x_1\ y \in R$

$\bar{u}+\bar{v} = (x_1-x) + (y_1-y)$

$= (x + y, -(x-y))$

$\in H, \forall\ \bar{u}, \bar{v} \in H$

iii) To prove $c\bar{u} \in H, \forall\ \bar{u} \in H$

Let $\bar{u} = (x_1-x) \in H$

$c\bar{u} = c\ (+\ x_1-x)$

$= (cx_1-cx)$

$\in H$

Consider a vector space R3 with vector addition and vector

Multiplication i.e., $R^3 \begin{cases} x1 \\ x1 \\ x1 \end{cases} / x_1, x_2, x_3 \in R \}$ show that $H = \left\{ \begin{matrix} x \\ x^2 \\ 0 \end{matrix} / x_1, x_2 \in R \right\}$ is a sequences

$R^3 = \{[x_1\ x_2 x_3]\} / x_1, x_2, x_3 \in R\}$

$H = \{[x_1\ x_2\ 0]\} / x_1\ x_2 \in R\}$

i) To prove $\bar{o} \in H$

$\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \in R^3\ o\ \in R$

$\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$ € H o € R

ō € H, ∀ € R

ii) To prove ū+v̄ € H → ū, v̄ € H

Let ū = $\begin{matrix} x1 \\ x2 \\ 0 \end{matrix}$ v̄ = $\begin{matrix} y \\ y_2 \\ 0 \end{matrix}$ ∀ x_1, x_2, y_1, y_2 € R

ū+v̄ = $\begin{matrix} x \\ x_2 \\ 0 \end{matrix}$ + $\begin{matrix} y \\ y_2 \\ 0 \end{matrix}$

ū +v̄ = $\begin{matrix} x & y \\ x_2 & +y_2 \\ 0 & 0 \end{matrix}$

€ H, ∀ ū, v̄ €H

iii) To prove cū €H ∀ ū € H

ū = $\begin{matrix} x \\ x_2 \\ 0 \end{matrix}$

cū = c $\begin{matrix} x \\ x_2 \\ 0 \end{matrix}$

ū = $\begin{matrix} cx \\ cx_2 \\ 0 \end{matrix}$

€ H, ∀ ū € H c is a scalar

Theorem: V is a vector space H &K are two subspaces in v then H∩K is a subspaces of v

Proof: V is a vector space H & k, are two subspace in v

i) To prove Ō € H, Ō € K

Ō € H∩K

ii) To prove $\bar{u}+\bar{v}$ € H∩K

Since H&K are two subspaces in v

↔ $\bar{u}+\bar{v}$ €H, ∀ $\bar{u}$, $\bar{v}$ € H → ①

↔ $\bar{u}+\bar{v}$ €K, ∀ $\bar{u}$, $\bar{v}$ €K→ ②

$\bar{u}+\bar{v}$ € H∩K, ∀ $\bar{u}$, $\bar{v}$€H∩K

iii) To prove c$\bar{u}$ €H∩K

Let $\bar{u}$ € H∩K c is a scalar

$\bar{u}$ € H∩K

c$\bar{u}$ €H, c$\bar{u}$ €K, ∀ $\bar{u}$ € H∩K, [H&K, are subspace]

H∩K is a subspace of v

Definition: H&K, are two subset of a v we define the sum H+K as H+K = {$\bar{w}$/w = $\bar{u}+\bar{v}$} $\bar{u}$€K, $\bar{v}$€H}

Theorem: H, K are two subspaces in a vector space v then H+K, is also subspace in v

Proof: V is a vector space

H&K, are two subspace in a, v

Now, we will prove that H&K, is also subspace u in v

H+K, = {$\bar{w}$/w = $\bar{u}+\bar{v}$, $\bar{u}$ €H, $\bar{v}$ €K}

i) H is a subspace of v

$\bar{o}$ €H

K is subspace of v, $\bar{o}$ €K

$\bar{o}$€H, $\bar{o}$€K

$\bar{o}+\bar{o}$ € H+K

$\bar{o}$€H +K

ii) Let $\overline{w}_1 = \overline{u}_1+\overline{v}_1$, $\overline{u}_1$ €H_1 , $\overline{v}_1$ €K

$\overline{w}_2 = \overline{u}_2+\overline{v}_2$, $\overline{u}_2$ €H_2, $\overline{v}_2$€K

$\overline{w}_1+\overline{w}_2 = (\overline{v}_1+\overline{v}_2) + (\overline{u}_1+\overline{u}_2)$

$= (\overline{u}_1+\overline{u}_2) + (\overline{v}_1+\overline{v}_2)$

$\overline{u}_1$, $\overline{u}_2$, €H → $\overline{u}_1+\overline{u}_2$ €K

$\overline{v}_1$, $\overline{v}_2$, €H → $\overline{v}_1+\overline{v}_2$ €K

∴ $\overline{w}_1+\overline{w}_2$ €H+K

iii) Let c be a scalar

$\overline{u}_1$€H, $\overline{v}_1$€K

H&K are vector space in v

$c\overline{u}_1$ €H, ∀ $\overline{u}_1$ €H

$c\overline{v}_1$ €H, ∀ $\overline{v}_1$ €K

→ $c\overline{u}_1+c\overline{v}_1$ €H+K

→ C $(\overline{u}_1 +\overline{v}_1)$ €H+K ∀ $\overline{u}_1 +\overline{v}_1$ € H+K

H+K is a subspace of v

Theorem: V is a vector space H and K, are two subspace in v then H∩K is not a sub-space of v

Proof: V is a vector space

H&K are subspace in v

We will prove that H∩K is not a subspace of v

Ex: Let v is a vector space in R^2

H = {(x_1-x)/x€R} ꞓ k = {(x1, 2x)/x €R} are two subspace of R^2

H = {(x_1-x),(x, 2x)/x €R}

Let x = 2

H = {(2,-2)}, k= {(2, 4)}

HUK = {(2,-2), (2, 4)}

V_1+v_2 = {(2,-2), + (2, 4)}

= (2+2, -2+4)

= (4, 2) +HUK

HUK is not subspace in v_1

CONCLUSION:

WE KNOW

1. **DEFINATION OF VECTOR SPACE**
2. **DIFFERENT EXAMPLES OF VECTOR SPACE**
3. **PROPERTIES OF VECTOR SPACE**
4. **DEFINATION OF SUBSPACE**
5. **EXAMPLES OF SUBSPACE**