

# **Student Study Project**

**On**

## **Magic Squares and its Applications**



Submitted by

B. Rajesh	B.Sc MPC III Yr
T. Shireesha	B.Sc MPC III Yr
U. Anitha	B.Sc MPC III Yr
N. Prashanth Reddy	B.Sc MPC III Yr
E. Lalitha	B.Sc MPC III Yr

Under the Guidance of

**K. Sindhu**  
**Lecturer in Mathematics**

Submitted to

**DEPARTMENT OF MATHEMATICS**

**Dr.BRR GOVERNMENT DEGREE COLLEGE, JADCHERLA**

**MAHABUBNAGAR (DIST), TELANGANA**

# CERTIFICATE

This is to certify that the student study project work entitled "Magic Squares and its applications" is a bonafide work done by the students of III MPC B. Rajesh, T. Shireesha, U. Anitha, N. Prashanth Reddy E. Lalitha under my supervision for the award of student study project work in Mathematics, Department of Mathematics, Dr.BRR Government Degree College, Jadcherla.



Signature of Supervisor



Signature of Principal  
Dr.B.R.R. Degree College  
Jadcherla-509 301  
Dist.Mahabubnagar (TS)

## DECLARATION

We hereby declare that student study project work entitled Magic Squares and its Applications is a genuine work done by us under the supervision of K.Sindhu, Department of Mathematics, Dr.BRR Government Degree College, Jadcherla.

Name of the Student	Class	Hall Ticket Number	Signature
B. Rajesh	III MPC	19033006441008	B. Rajesh
T. Shireesha	III MPC	19033006441529	T. shireesha
U. Anitha	III MPC	19033006468028	U. Anitha
N.Prashanth Reddy	III MPC	19033006468022	prashanth
E. Lalitha	III MPCs	19033006441012	E. Lalitha

## Objectives

What is magic square?

How many types of magic squares?

How many methods to construct magic squares?

What are the applications in day to day life?

## Introduction

Magic squares are one of the beautiful examples of the mathematical objects with several

Real-world applications. They have always had a great influence upon mankind's attitude. Although a definitive judgment of early history of magic squares is not available, it has been suggested that magic squares probably date back to pre-Islamic Persian origins. The study of magic squares in medieval Islam in Persia was came after the introduction of Chess in Persia .In the tenth Century, the Persian mathematician Buzjani has left a manuscript in which there is a magic Square, which are filled by numbers in arithmetic progression.

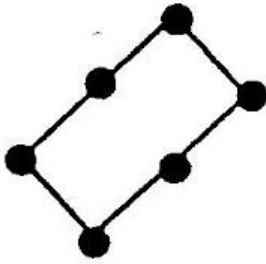
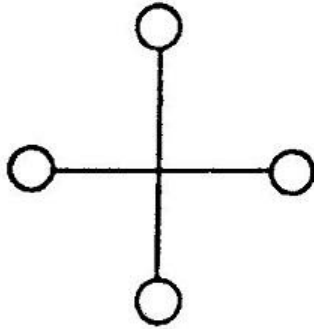
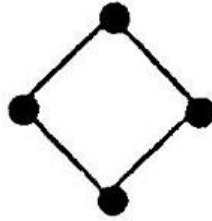
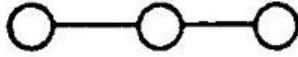
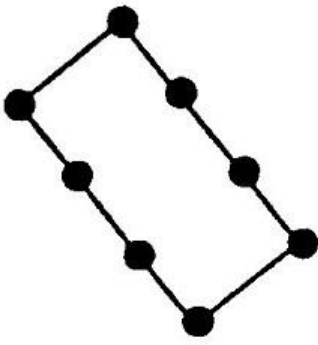
In this paper, initially some types of magic geometrical shapes such as square, rectangle,

Cube, tesseract, hypercube, circle, sphere, triangle, star and hexagon are briefly introduced. Then, the most noticeable types of magic squares and their features are expressed. After that, construction methods of natural magic squares are described.

## **HISTORY OF MAGIC SQUARE**

Magic squares have a rich history. The magic square is said to have been discovered in the third millenium B.C. by the Chinese Emperor Yu. According to tradition, the Emperor, while walking on the river bank, found a turtle with an odd diagram on its shell (see below left).

The Emperor saw in the unusual pattern a numerical sequence. He called this pattern the "Lo Shu." His discovery was a magic square of the third order, and it is symbolically the same as the first magic square in this article (below right):



The object of a magic square is to arrange numbers within the cells of the square such that the horizontal rows, the vertical columns, and the two diagonals each add up to the same number called the magic number. In the magic square below, the numbers 1 through 9 have been arranged such that the magic number 15 results.

8	3	4
1	5	9
6	7	2

### What is a magic square?

An  $n \times n$  matrix whose entries consists of the integers 1 through  $n^2$  is called a magic square if all row sums, column sums and diagonal sums are equal and they add up to the number  $n(n^2+1)$

2. This number is called the magic constant.



# Types of Magic squares

## 1. Magic square

A natural magic square of order  $n$  is a square array of numbers consisting of the distinct

Positive integers  $1, 2, \dots, n^2$  arranged such that the sum of the numbers in any horizontal,

Vertical, or main diagonal lines is always the same number, known as the magic constant.

One can verify that the magic constant is equal to

$$M_n = \frac{n}{2}(n^2 + 1).$$

8	1	6
3	5	7
4	9	2

Natural magic square of order 3.

## 2. Magic rectangle

A magic  $(m,n)$ -rectangle  $R$  is an  $m \times n$  array in which the first  $mn$  positive integers are

placed so that the sum over each row of  $R$  is constant and the sum over each column of  $R$

is another (different if  $m \neq n$ ) constant. For  $m, n > 1$ , there is a magic  $(m,n)$ -rectangle  $R$  if

and only if  $m \equiv n \pmod{2}$  and  $(m,n) \neq (2,2)$ . For centrally symmetric rectangles,

the row sums are equal to

$$M_{n,m} = \frac{n}{2}(mn + 1).$$

and the column sums are equal to

$$N_{n,m} = \frac{m}{2}(mn + 1).$$

14	10	4	5	7
1	3	8	13	15
9	11	12	6	2

Centrally symmetric rectangle of order  $(3,5)$

### 3. Magic cube

A magic cube is an  $n \times n \times n$  version of a magic square in which each of the  $n^2$  rows,  $n^2$

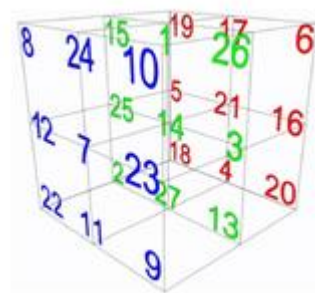
Columns,  $n^2$  pillars, and four space diagonals sum to a single number  $M_3$  ( $n$ ) known as the

Cube's magic constant. Magic cubes are most commonly assumed to be "normal", i.e. to

have elements that are the consecutive integers  $1, 2, \dots, n^3$ . If it exists, a normal magic cube

has magic constant equal to

$$M_n = \frac{n}{2}(n^3 + 1).$$



Natural Magic cube of order 3

## Methods for constructing Magic Squares

- De La Loubere's method.
- Pheru's method.
- Pyramid method.
- Method for even magic square divisible by 4.

## • De La Loubere's Method

Was the French ambassador to Siam (now Thailand) at the end of the seventeenth century. On his return to France he brought with him a method for constructing magic squares with an odd number of rows and columns, otherwise Known as squares of odd order. Begin by finding the middle cell in the top row of the magic square, and write the number 1 in it. Continue writing the numbers 2, 3, 4, and so on, each in the diagonally adjacent cell north-east of the previously filled one. When you reach the edge of the Square, continue from the opposite edge, as if opposite edges were glued together. If you encounter a cell that is already filled, move to the cell immediately below the cell you have just filled, and continue as before. When all the cells are filled, the two main

diagonals and every row and column should add up to the same number, as if by magic.

8	1	6
3	5	7
4	9	2

- **Pheru's method**

The first known mathematical use of magic squares in India was by Thakkura Pheru in his work Ganitasara (ca. 1315 A.D.). Pheru provided a method for constructing odd magic squares, that is, squares in which  $n$  is an odd integer. Start by placing the number 1 in the bottom cell of the central column.

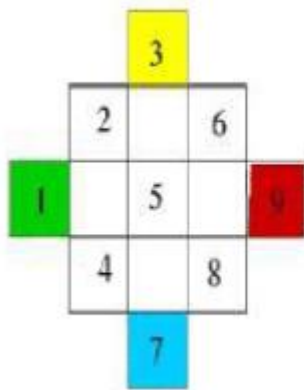
To obtain the next cell above it, add  $n+1$ , getting  $n+2$ . To obtain the next cell above  $n+2$ , add  $n+1$  again, getting  $2n+3$ . Continuing to add in this way to obtain the cell values in the central column results in an arithmetic progression with a common difference of  $n+1$ . Continue adding  $n+1$  until reaching the central column's top cell, which has a value of  $n^2$ .

	46	57	68	79	90		22	33	44	
	45	47	58	69	80		12	23	34	
	35	37	48	59	70	81	2	13	24	35
	25	36	38	40	60	71	73	3	14	25
	15	26	28	39	50	61	72	74	4	15
	86	16	27	29	40	51	62	64	75	5
	76	6	17	19	30	41	52	63	65	26
	66	77	7	18	20	31	42	53	55	66
	56	67	78	8	10	21	32	43	54	56
		57	68	70	9	11	22	33	44	46
		47	58	60	80	1	12	23	34	45

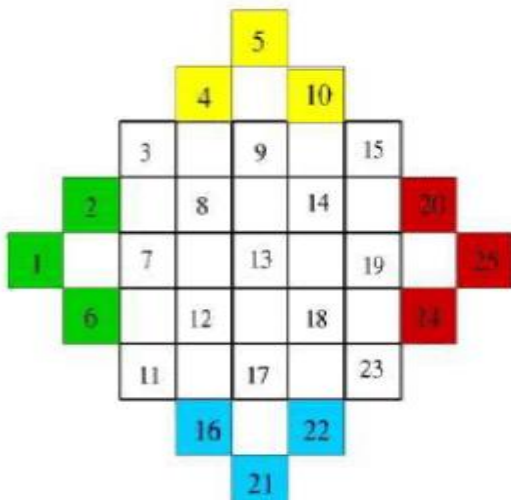
## • Pyramid method

This method consists of three steps:

1. Draw a pyramid on each side of the magic square. The pyramid should have two less squares on its base than the number of squares on the side of the magic square. This creates a square standing on a vertex.
2. Sequentially place the numbers 1 to  $n^2$  of the  $n \times n$  magic square in the diagonals as shown in Figures 1 and 2.
3. Relocate any number not in the  $n \times n$  square to the opposite hole inside the square (shaded)



The same Pyramid method can be used for any odd order magic square as shown below for the 5×5square



3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23



- The method for even magic square divisible by 4

### A construction of a magic square of order 4

Go left to right through the square filling counting and filling in on the diagonals only. Then continue by going left to right from the top left of the table and fill in counting down from 16 to 1, as shown in the figure,

64	63	3	4	5	6	58	57
56	55	11	12	13	14	50	49
17	18	46	45	44	43	23	24
25	26	38	37	36	35	31	32
33	34	30	29	28	27	39	40
41	42	22	21	20	19	47	48
16	15	51	52	53	54	10	9
8	7	59	60	61	62	2	1

## Applications of Magic Squares

### 1. Music

The main area of the application of magic squares to music is in rhythm, rather than notes. Indian musicians seem to have applied them to their music and they seem to be useful in time cycles and additive rhythm. In this case it is not the usual magic properties of a square that are important, but the relationship of the central number to the total sum of all the numbers in the magic square. This is because for rhythm, consecutive numbers 1 to  $n$  are not used to fill the cells of the magic square.

This relationship is:

The total sum of the magic square  $\hat{=}$  as numbers = central number  $\times$  9. This is important to music as it shows the size of the magic square, which is how many pulses or sub-divisions there are in the sequence, this will indicate how and where to apply it.

## 2.Sudoku

Sudoku was first introduced in 1979 and became popular in Japan during the 1980's. It has recently become a very popular puzzle in Europe, but it is actually a form of Latin square. A Sudoku square is a 9x9 grid, split into 9 3x3 sub-squares. Each sub-square is filled in with the numbers 1 to n where

$n=9$ , so that the 9x9 grid becomes a Latin square. This means each row and column contain the numbers 1 to 9 only once. Therefore each row, column and sub-square will sum to the same amount.

	3					9		
		6						
			2	4	1			3
			9			7		
					2			4
	8			7				2
8	5							
	9		7		4			
					6			1

1	3	2	5	6	7	9	4	8
5	4	6	3	8	9	2	1	7
9	7	8	2	4	1	6	3	5
2	6	4	9	1	8	7	5	3
7	1	5	6	3	2	8	9	4
3	8	9	4	7	5	1	2	6
8	5	7	1	2	3	4	6	9
6	9	1	7	5	4	3	8	2
4	2	3	8	9	6	5	7	1

## Conclusion

Mathematicians today do not need to speculate and attach meaning to magic squares to make them important, as has been done in the past with Chinese and other myths. The squares were thought to be mysterious and magic, although now it is clear that they are just ways of arranging numbers and symbols using certain rules. They can be applied to music and Sudoku as has been discussed but are mainly of interest in Mathematics for their “magic” properties rather than their practical applications.

## References

1. Adler, A. (1996). What is a magic square? Available from <http://mathforum.org/alejandre/magic.square/adler/adler.whatsquare.html>
2. Anderson, D. L. (2001). Magic Squares . Available from <http://illuminations.nctm.org/LessonDetail.aspx?id=L263>
3. Farrar, M. S. (1997). History of magic squares . Available from <http://www.markfarrar.co.uk/msqhst01.htm>
4. [www.google.com](http://www.google.com)