GOVERNMENT DEGREE COLLEGE (W), NALGONDA

DEPARTMENT OF MATHEMATICS

ACADEMIC YEAR

2020-2021

STUDENT STUDY PROJECT

ON

“Extension of Corollary of Euler’s theorem for functions of three variables”

Submitted

By

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Problem:

In this study Project we shall Extend Euler's theorem for Homogeneous function of three variables and we state some equations containing third order partial derivatives using Euler's theorem for function of two variables.We shall verify above statements by taking examples.

1. Introduction: The Euler's theorem on Homogeneous function is a part of Differential Calculus which is a course in mathematics for undergraduate students of semester-I. If Z is a Homogeneous function of x and y of degree'n', then the theorem is useful for finding the values of expression of type $xZ_x + yZ_y$, $x^2Z_{xx} + 2xyZ_{xy} + y^2Z_{yy}$ etc.

In this study project we shall extend this theorem to a Homogeneous function of x, y & z variables of degree 'n'. I.e we shall extend Euler's theorem to a Homogeneous function of three variables.

2. Euler's theorem on Homogeneous function of two variables:

Definition: a function f(x, y) is said to be a Homogeneous function of degree 'n' if

$f(kx, ky) = k^n f(x, y)$ For $k > 0$

2.1 Euler's theorem: If Z is a Homogeneous function of x, y of degree 'n'and first order partial derivatives of Z are exit then $xZ_x + yZ_y = nZ$.

Corollary 2.2: If Z is a Homogeneous function of x, y of degree 'n'and first and second order partial derivatives of Z are exit and are continuous then $x^2Z_{xx} + 2xyZ_{xy} + y^2Z_{yy} = n(n-1)Z$.

Corollary 2.3: If f is a Homogeneous function of x, y of degree 'n'and first, second and third order partial derivatives of f are exit and are continuous then

$x^3f_{xxx} + 3xy^2f_{xyy} + 3x^2yf_{xxy} + y^3f_{yyy} = n(n-1)(n-2)f$.

Proof:

Given that f is a Homogeneous function of x, y of degree 'n'and first, second and third order partial derivatives of f are exit and are continuous.

By 2.2 Corollary we have $x^2f_{xx} + 2xyf_{xy} + y^2f_{yy} = n(n-1)f$ .... (1)

Differentiate equation (1) partially with respect to 'x' on both sides then we get

$$\frac{\partial\left(x^2f_{xx}+2xyf_{xy}+y^2f_{yy}\right)}{\partial x} = \frac{\partial(n(n-1)f}{\partial x}$$

$$x^2f_{xxx} + 2xf_{xx} + 2y\left(xf_{xxy} + f_{xy}\right) + y^2f_{xyy} = n(n-1)\frac{\partial f}{\partial x} \quad \text{...............} (2)$$

Differentiate equation (1) partially with respect to 'y' on both sides then we get

$$\frac{\partial\left(x^2 f_{xx}+2xyf_{xy}+y^2 f_{yy}\right)}{\partial y} = \frac{\partial(n(n-1)f}{\partial y}$$

$$x^2 f_{xxy} + 2x\left(yf_{xyy} + f_{xy}\right) + y^2 f_{yyy} + 2yf_{yy} = n(n - 1)\frac{\partial f}{\partial y} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

Multiply equation (2) with 'x' and multiply equation (3) with 'y' then we get

$$x^3 f_{xxx} + 2x^2 f_{xx} + 2xy\left(xf_{xxy} + f_{xy}\right) + xy^2 f_{xyy} = n(n - 1)x\frac{\partial f}{\partial x} \quad \ldots\ldots\ldots. (4)$$

$$x^2 yf_{xxy} + 2xy\left(yf_{xyy} + f_{xy}\right) + y^3 f_{yyy} + 2y^2 f_{yy} = n(n - 1)y\frac{\partial f}{\partial y} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

Adding equation (4) & (5) then we get

$$x^3 f_{xxx} + 3xy^2 f_{xyy} + 3x^2 yf_{xxy} + y^3 f_{yyy} + 2\left(x^2 f_{xx} + 2xyf_{xy} + 2xyf_{xy} + y^2 f_{yy}\right)$$

$$= n(n - 1)\left(x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}\right)$$

$$\Rightarrow x^3 f_{xxx} + 3xy^2 f_{xyy} + 3x^2 yf_{xxy} + y^3 f_{yyy} + 2n(n - 1)f = n(n - 1)(nf)$$

$$\Rightarrow x^3 f_{xxx} + 3xy^2 f_{xyy} + 3x^2 yf_{xxy} + y^3 f_{yyy} = n^2(n - 1)f - 2n(n - 1)f$$

$$\Rightarrow x^3 f_{xxx} + 3xy^2 f_{xyy} + 3x^2 yf_{xxy} + y^3 f_{yyy} = n(n - 1)(n - 2)f$$

Hence the theorem.

Now we apply this theorem to Homogeneous function of three variables.

3. Euler's theorem on Homogeneous function of three variables:

Definition: a function $f(x,\ y, z)$ is said to be a Homogeneous function of degree 'n' if

$$f(kx, ky, kz) = k^n f(x, y, z) \text{ For } k > 0$$

Euler's theorem 3.1: If f is a Homogeneous function of $x,\ y$ & z of degree 'n'and first order partial derivatives of f are exit then $xf_x + yf_y + zf_z = nf$.

Proof:

Given that f is a Homogeneous function of $x,\ y$ & z of degree 'n and first order partial derivatives of f are exit then $f(x, y, z) = x^n f\left(\frac{y}{x}, \frac{z}{x}\right)$

$\Rightarrow f(x, y, z) = x^n f(u, v)$ ……….. (1) Where $u = \frac{y}{x},\ v = \frac{z}{x}$

Differentiate equation (1) partially with respect to 'x' on both sides then we get

$$\frac{\partial f}{\partial x} = \frac{\partial\left(x^n f(u,v)\right)}{\partial x}$$

$$\Rightarrow \frac{\partial f}{\partial x} = x^n \ \frac{\partial(f(u,v))}{\partial x} + \ f(u,v)\frac{\partial x^n}{\partial x}$$

$$\Rightarrow \frac{\partial f}{\partial x} = x^n\left(\frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x}\right) + \ f(u,v)nx^{n-1}$$ $(since \ \ u \ = \frac{y}{x} \ then \ \frac{\partial u}{\partial x} =- \frac{y}{x^2} ,$

$v \ = \frac{z}{x} \ \ then \ \frac{\partial v}{\partial x} = \ \ \ - \frac{z}{x^2} \ )$

$$\Rightarrow \frac{\partial f}{\partial x} = x^n\left(\frac{\partial f}{\partial u}\left(- \frac{y}{x^2}\right) \ + \frac{\partial f}{\partial v}\left(- \frac{z}{x^2}\right)\right) + \ f(u,v)nx^{n-1}$$

$$\Rightarrow \frac{\partial f}{\partial x} =- \ x^{n-2}y\frac{\partial f}{\partial u} - \ x^{n-2}z\frac{\partial f}{\partial v} + \ f(u,v)nx^{n-1}$$ ………… (2)

Differentiate equation (1) partially with respect to ‘y’ on both sides then we get

$$\frac{\partial f}{\partial y} = \frac{\partial\left(x^n f(u,v)\right)}{\partial y}$$

$$\Rightarrow \frac{\partial f}{\partial y} = x^n \ \frac{\partial(f(u,v))}{\partial y}$$

$$\Rightarrow \frac{\partial f}{\partial y} = x^n\left(\frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y}\right)$$ $(since \ \ u \ = \frac{y}{x} \ then \ \frac{\partial u}{\partial y} = \frac{1}{x}$

$v \ = \frac{z}{x} \ \ then \ \frac{\partial v}{\partial y} = \ \ 0 \ )$

$$\Rightarrow \frac{\partial f}{\partial y} = x^n \frac{\partial f}{\partial u}\left(\frac{1}{x}\right) + \ 0$$

$$\Rightarrow \frac{\partial f}{\partial y} = x^{n-1}\frac{\partial f}{\partial u}$$ ……….. (3)

Differentiate equation (1) partially with respect to ‘z’ on both sides then we get

$$\frac{\partial f}{\partial z} = \frac{\partial\left(x^n f(u,v)\right)}{\partial z}$$

$$\Rightarrow \frac{\partial f}{\partial z} = x^n \ \frac{\partial(f(u,v))}{\partial z}$$

$$\Rightarrow \frac{\partial f}{\partial z} = x^n \left(\frac{\partial f}{\partial u}\frac{\partial u}{\partial z} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial z}\right)$$ $(since \ \ u \ = \frac{y}{x} \ then \ \frac{\partial u}{\partial z} = \ 0$

$v \ = \frac{z}{x} \ \ then \ \frac{\partial v}{\partial z} = \ \ \frac{1}{x} \ )$

$$\Rightarrow \frac{\partial f}{\partial z} = x^n \frac{\partial f}{\partial u}\left(\frac{1}{x}\right) + \ 0$$

$\Rightarrow \frac{\partial f}{\partial z} = x^{n-1}\frac{\partial f}{\partial u}$ ……….. (4)

Now

$$xf_x + yf_y + zf_z = x\left(-x^{n-2}y\frac{\partial f}{\partial u} - x^{n-2}z\frac{\partial f}{\partial v} + f(u,v)nx^{n-1}\right) + yx^{n-1}\frac{\partial f}{\partial u} + zx^{n-1}\frac{\partial f}{\partial u}$$

$$\Rightarrow xf_x + yf_y + zf_z = -yx^{n-1}\frac{\partial f}{\partial u} - zx^{n-1}\frac{\partial f}{\partial u} + x^n f(u,v) + yx^{n-1}\frac{\partial f}{\partial u} + zx^{n-1}\frac{\partial f}{\partial u}$$

$$\Rightarrow xf_x + yf_y + zf_z = x^n f(u,v)$$

$$\Rightarrow xf_x + yf_y + zf_z = nf$$

Hence proved.

Corollary 3.2: If f is a Homogeneous function of $x,\ y$ & z of degree 'n'and first and second order partial derivatives of f are exist and are continuous then

$$x^2f_{xx} + y^2f_{yy} + z^2f_{zz} + 2xyf_{xy} + 2yzf_{yz} + 2xzf_{xz} = n(n-1)f.$$

Proof:

Given that f is a Homogeneous function of $x,\ y$ & z of degree 'n and first and second order partial derivatives of f are exit and are continuous.

By 3.1 Euler's theorem we have $xf_x + yf_y + zf_z = nf$ ………………. (1)

Differentiate equation (1) partially with respect to 'x' on both sides then we get

$$\frac{\partial(xf_x+yf_y+zf_z)}{\partial x} = \frac{\partial(nf)}{\partial x}$$

$\Rightarrow xf_{xx} + f_x + yf_{xy} + zf_{xz} = nf_x$ ……….. (2)

Differentiate equation (1) partially with respect to 'y' on both sides then we get

$$\frac{\partial(xf_x+yf_y+zf_z)}{\partial y} = \frac{\partial(nf)}{\partial y}$$

$\Rightarrow xf_{xy} + f_y + yf_{yy} + zf_{yz} = nf_y$ ……….. (3)

Differentiate equation (1) partially with respect to 'z' on both sides then we get

$$\frac{\partial(xf_x+yf_y+zf_z)}{\partial z} = \frac{\partial(nf)}{\partial z}$$

$\Rightarrow xf_{xz} + yf_{yz} + zf_{zz} + f_z = nf_z$ ……….. (4)

Multiply Equation (2) with 'x', (3) with 'y' and (4) with 'z' then we get

$x^2f_{xx} + xf_x + xyf_{xy} + xzf_{xz} = nxf_x$

$xyf_{xy} + yf_y + y^2f_{yy} + yzf_{yz} = nyf_y$

$xzf_{xz} + yzf_{yz} + z^2f_{zz} + zf_z = nzf_z$

Adding above three equations then we get

$$x^2f_{xx} + y^2f_{yy} + z^2f_{zz} + 2xyf_{xy} + 2yzf_{yz} + 2xzf_{xz} + xf_x + yf_y + zf_z$$

$$= nxf_x + nyf_y + nzf_z$$

$\Rightarrow x^2f_{xx} + y^2f_{yy} + z^2f_{zz} + 2xyf_{xy} + 2yzf_{yz} + 2xzf_{xz} + nf = n(xf_x + yf_y + zf_z)$

$\Rightarrow x^2f_{xx} + y^2f_{yy} + z^2f_{zz} + 2xyf_{xy} + 2yzf_{yz} + 2xzf_{xz} = n^2f - nf$

$\Rightarrow x^2f_{xx} + y^2f_{yy} + z^2f_{zz} + 2xyf_{xy} + 2yzf_{yz} + 2xzf_{xz} = n(n - 1)f$

Hence proved.

Corollary 3.3: If f is a Homogeneous function of x, y & z of degree 'n'and first, second and third order partial derivatives of f are exit and are continuous then

$x^3f_{xxx} + y^3f_{yyy} + z^3f_{zzz} + 3xy^2f_{xyy} + 3x^2yf_{xxy} + 3xz^2f_{xzz} + 3x^2zf_{xxz} + 3y^2zf_{yyz} + 3yz^2f_{yzz} + 6xyzf_{x}$

.

Proof:

Given that f is a Homogeneous function of x, y & z of degree 'n and first, second and third order partial derivatives of f are exit and are continuous.

3.2 Corollary we have $x^2f_{xx} + y^2f_{yy} + z^2f_{zz} + 2xyf_{xy} + 2yzf_{yz} + 2xzf_{xz} = n(n - 1)f$ … (1)

Differentiate equation (1) partially with respect to 'x' on both sides then we get

$x^2f_{xxx} + 2xf_{xx} + y^2f_{xyy} + z^2f_{xzz} + 2y(xf_{xxy} + f_{xy}) + 2yzf_{xyz} + 2z(xf_{xxz} + f_{xz})$

$$= (n-1)f_x \text{ .... (2)}$$

Differentiate equation (1) partially with respect to 'y' on both sides then we get

$$x^2 f_{xxy} + 2yf_{yy} + y^2 f_{yyy} + z^2 f_{yzz} + 2x\left(yf_{xyy} + f_{xy}\right) + 2z(yf_{yyz} + f_{yz}) + 2xzf_{xyz}$$

$$= (n-1)f_y \text{ .... (3)}$$

Differentiate equation (1) partially with respect to 'z' on bothsides then we get

$$x^2 f_{xxz} + y^2 f_{yyz} + z^2 f_{zzz} + 2zf_{zz} + 2xyf_{xyz} + 2y(zf_{yzz} + f_{yz}) + 2x\left(zf_{xzz} + f_{xz}\right)$$

$$= (n-1)f_z \text{ .... (4)}$$

Multiply Equation (2) with 'x', (3) with 'y' and (4) with 'z' then we get

$$x^3 f_{xxx} + 2x^2 f_{xx} + xy^2 f_{xyy} + xz^2 f_{xzz} + 2xy\left(xf_{xxy} + f_{xy}\right) + 2xyzf_{xyz} + 2xz\left(xf_{xxz} + f_{xz}\right)$$

$$= (n-1)xf_x \text{ .... (5)}$$

$$yx^2 f_{xxy} + 2y^2 f_{yy} + y^3 f_{yyy} + yz^2 f_{yzz} + 2xy\left(yf_{xyy} + f_{xy}\right) + 2yz(yf_{yyz} + f_{yz}) + 2xyzf_{xyz}$$

$$= (n-1)yf_y \text{ .... (6)}$$

$$zx^2 f_{xxz} + zy^2 f_{yyz} + z^3 f_{zzz} + 2z^2 f_{zz} + 2xyzf_{xyz} + 2yz(zf_{yzz} + f_{yz}) + 2xz\left(zf_{xzz} + f_{xz}\right)$$

$$= (n-1)zf_z \text{ .... (7)}$$

Adding equations (5), (6) & (7) then we get

$$x^3 f_{xxx} + y^3 f_{yyy} + z^3 f_{zzz} + 3xy^2 f_{xyy} + 3x^2 yf_{xxy} + 3xz^2 f_{xzz} + 3x^2 zf_{xxz} + 3y^2 zf_{yyz} + 3yz^2 f_{yzz} + 2(x^2 f_{xx}$$

$$= n(n-1)\left(xf_x + yf_y + zf_z\right)$$

$$\Rightarrow x^3 f_{xxx} + y^3 f_{yyy} + z^3 f_{zzz} + 3xy^2 f_{xyy} + 3x^2 yf_{xxy} + 3xz^2 f_{xzz} + 3x^2 zf_{xxz} + 3y^2 zf_{yyz} + 3yz^2 f_{yzz} + 2n$$

$$\Rightarrow x^3 f_{xxx} + y^3 f_{yyy} + z^3 f_{zzz} + 3xy^2 f_{xyy} + 3x^2 yf_{xxy} + 3xz^2 f_{xzz} + 3x^2 zf_{xxz} + 3y^2 zf_{yyz} + 3yz^2 f_{yzz} = n($$

$\Rightarrow x^3 f_{xxx} + y^3 f_{yyy} + z^3 f_{zzz} + 3xy^2 f_{xyy} + 3x^2 y f_{xxy} + 3xz^2 f_{xzz} + 3x^2 z f_{xxz} + \quad 3y^2 z f_{yyz} + 3yz^2 f_{yzz} + 6x$

Hence proved.

Now we shall verify above corollary for one Homogeneous function of three variables.

Problem (1): verify corollary **3.3** for the function $f(x, y, z) = x^3 + y^3 + z^3$

Sol:

Given that $f(x, y, z) = x^3 + y^3 + z^3$ ………… (8) Is a Homogeneous function of degree '3' then

By corollary 3.3 we have

$x^3 f_{xxx} + y^3 f_{yyy} + z^3 f_{zzz} + 3xy^2 f_{xyy} + 3x^2 y f_{xxy} + 3xz^2 f_{xzz} + 3x^2 z f_{xxz} + \quad 3y^2 z f_{yyz} + 3yz^2 f_{yyz} + 6xyz$

$\Rightarrow x^3 f_{xxx} + y^3 f_{yyy} + z^3 f_{zzz} + 3xy^2 f_{xyy} + 3x^2 y f_{xxy} + 3xz^2 f_{xzz} + 3x^2 z f_{xxz} + \quad 3y^2 z f_{yyz} + 3yz^2 f_{yyz} + 6x$

$\Rightarrow x^3 f_{xxx} + y^3 f_{yyy} + z^3 f_{zzz} + 3xy^2 f_{xyy} + 3x^2 y f_{xxy} + 3xz^2 f_{xzz} + 3x^2 z f_{xxz} + \quad 3y^2 z f_{yyz} + 3yz^2 f_{yzz} + 6x$
…….. (9)

Differentiate equation (1) partially with respect to 'x','y' & 'z' on both sides then we get

$f_x = 3x^2$ ….. (10)

$f_y = 3y^2$ …….. (11)

$f_z = 3z^2$ …….. (12)

Differentiate equation (10) partially with respect to 'x' on both sides then we get

$f_{xx} = 6x$ ….. (13) Similarly $f_{xxx} = 6, f_{xxy} = 0, f_{xyy} = 0, f_{xzz} = 0, f_{xxz} = 0, f_{xyz} = 0,$

Differentiate equation (11) partially with respect to 'y' on both sides then we get

$f_{yy} = 6y$ …….. (14) Similarly $f_{yyy} = 6, f_{yyz} = 0, f_{yzz} = 0,$

Differentiate equation (12) partially with respect to 'z' on both sides then we get

$f_{zz} = 6z$ …….. (15) Similarly $f_{zzz} = 6$

Now

$x^3 f_{xxx} + y^3 f_{yyy} + z^3 f_{zzz} + 3xy^2 f_{xyy} + 3x^2 y f_{xxy} + 3xz^2 f_{xzz} + 3x^2 z f_{xxz} + 3y^2 z f_{yyz} + 3yz^2 f_{yzz} + 6xyzf_{x}$

$x^3 f_{xxx} + y^3 f_{yyy} + z^3 f_{zzz} + 3xy^2 f_{xyy} + 3x^2 y f_{xxy} + 3xz^2 f_{xzz} + 3x^2 z f_{xxz} + 3y^2 z f_{yyz} + 3yz^2 f_{yzz} + 6xyzf_{x}$
…….. (16)

From equations (9), (16) we say that **corollary 3.3** verified

Problem (2): verify corollary **2.3** for the function $f(x, y, z) = x^3 + y^3$

Sol:

Given that $f(x, y, z) = x^3 + y^3$ ……….. (17) Is a Homogeneous function of degree '3' then

By corollary 2.3 we have

$$x^3 f_{xxx} + 3xy^2 f_{xyy} + 3x^2 y f_{xxy} + y^3 f_{yyy} = n(n-1)(n-2)f$$

$$\Rightarrow x^3 f_{xxx} + 3xy^2 f_{xyy} + 3x^2 y f_{xxy} + y^3 f_{yyy} = 3(3-1)(3-2)f$$

$$\Rightarrow x^3 f_{xxx} + 3xy^2 f_{xyy} + 3x^2 y f_{xxy} + y^3 f_{yyy} = 6f \quad \text{……… (19)}$$

Differentiate equation (1) partially with respect to 'x','y' & 'z' on both sides then we get

$f_x = 3x^2$ ….. (20)

$f_y = 3y^2$ …….. (21)

$f_z = 3z^2$ …….. (22)

Differentiate equation (10) partially with respect to 'x' on both sides then we get

$f_{xx} = 6x$ ….. (23) Similarly $f_{xxx} = 6$, $f_{xxy} = 0, f_{xyy} = 0$

Differentiate equation (11) partially with respect to 'y' on both sides then we get

$f_{yy} = 6y$ …….. (24) Similarly $f_{yyy} = 6$

Now

$$x^3 f_{xxx} + 3xy^2 f_{xyy} + 3x^2 y f_{xxy} + y^3 f_{yyy} = 6x^3 + 0 + 0 + 6y^3$$

$$x^3 f_{xxx} + 3xy^2 f_{xyy} + 3x^2 y f_{xxy} + y^3 f_{yyy} = 6f \quad \text{......... (25)}$$

From equations (19), (25) we say that **corollary 2.3** verified

References:

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GOVERNMENT DEGREE COLLEGE (W), NALGONDA

DEPARTMENT OF MATHEMATICS

ACADEMIC YEAR 2019-20

STUDENT STUDY PROJECT

ON

"FROM KNOWN TO UNKNOWN"

-An application of curve fitting

Submitted

By

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Problem:

In this study project we have obtained data of worldwide usage of electric vehicles for the last 8 years from 2012 to 2019 from Statista website.We shall find which curve is best to represent this data and using this function we shall estimate future usage of electric vehicles.

Our motto is to find a suitable model for the following data and to estimate the future usage electric vehicles

Worldwide number of battery electric vehicles in use from 2012-2019

Year	2012	2013	2014	2015	2016	2017	2018	2019
In millions	0.11	0.22	0.4	0.72	1.18	1.93	3.27	4.7

Table1

Source:Statista

Table1 can be represented as discrete points on the graph.

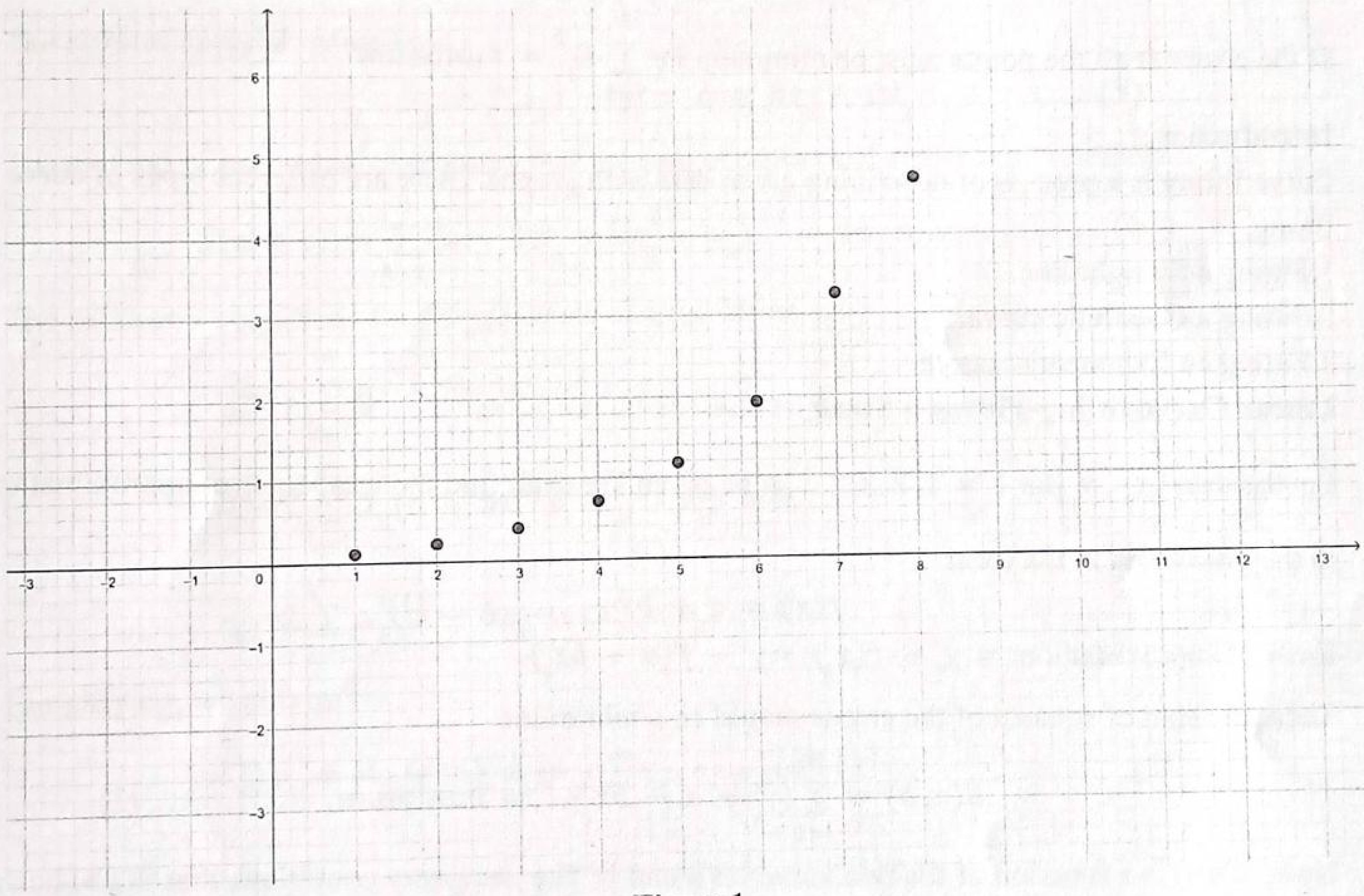

Figure1

Objectives:

1.To understand the Least squares method.

2.To learn about linear curve fitting.

3.To learn about quadratic curve fitting.

4.To learn about exponential curve fitting.

5.To learn application of curve fitting.

Methodology:

In Science , Engineering and Social sciences numerical data is obtained through experimentation or survey.Obtained data can be represented by two variables $x$and y.Suppose these data is denoted by $(x_1, y_1), (x_2, y_2),...., (x_n, y_n)$ points. First question comes into our minds that which continuous function best suits these n points.One solution is least squares method.Let $y = f(x)$ be continuous curve fitting the given data $(x_1, y_1), (x_2, y_2),...., (x_n, y_n)$.The error of the approximation is $E_i = y_i - f(x_i)$.Then the least squares method requires that the sum of squares of the errors at all the points must be minimum i.e $\sum_{i=1}^{n} E_i^2 = minimum$

Introduction:

Curve fitting is a process of describing given data with graphs.There are different types of curve fitting.

1.Fitting a Straight line.

2.Fitting a Quadratic curve.

3.Fitting an Exponential curve.

Linear Curve fitting-Fitting a Straight line

Let the data $\left(x_i , y_i\right)$for $i = 1, 2, 3,........, n$ be given.We shall find the least-squares approximation to the given data in the form

$$f(x) = a + bx \quad .... (1)$$

Error of approximation $= y_i - f(x_i) = y_i - f(a + bx_i)$.

Then, the sum of squares of the errors should be a minimum.

$$S(a, b) = \sum_{i=1}^{n} (y_i - a - bx_i)^2 = minimum. \quad .... (2)$$

Now, S(a,b) is a function of the two variables a and b. The necessary conditions that S(a,b) has a minimum are

$$\frac{\partial s}{\partial a} = \frac{\partial s}{\partial b} = 0........... (3)$$

Therefore,

$$\frac{\partial s}{\partial a} = - 2\sum_{i=1}^{n} (y_i - a - bx_i) = 0....... (4)$$

and $$\frac{\partial s}{\partial b} = - 2\sum_{i=1}^{n} x_i(y_i - a - bx_i) = 0.........(5)$$

Simplifying these equations, we get

$$\sum_{i=1}^{n} y_i = na + b\sum_{i=1}^{n} x_i \quad ....(6)$$

$$\sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 \text{............} (7)$$

Since $\sum_{i=1}^{n} = n$.From the given data, we compute all the sums. We solve these equations for a and b. The required approximation is $f(x) = a + bx$.Equations (6) and (7) are called "the normal equations" for fitting a straight line to a given data.

Linear Curve fitting-Fitting a Quadratic curve

In this session, we present a method to fit a parabola to a given data of n points $\left(x_i, y_i\right)$, $i = 1, 2, 3,........, n$ in xy-plane. A parabola is a second degree polynomial and hence is a nonlinear curve.

We write the quadratic approximation in the form

$$f(x) = a + bx + cx^2 \text{.............} (8)$$

The error of approximation is

$$E_k = f(x_k) - y_k = a + bx_k + cx_k^2 - y_k \text{...........} (9)$$

The method of least squares requires that the sum of squares of errors is a minimum.

$$S(a, b, c) = \sum_{k=1}^{n} (a + bx_k + cx_k^2 - y_k)^2 = minimum. \text{............} (10)$$

The necessary conditions for the existence of a minimum are

$$\frac{\partial s}{\partial a} = 2\sum_{k=1}^{n} (a + bx_k + cx_k^2 - y_k) = 0,$$

$$\frac{\partial s}{\partial b} = 2\sum_{k=1}^{n} x_k(a + bx_k + cx_k^2 - y_k) = 0,$$

$$\frac{\partial s}{\partial c} = 2\sum_{k=1}^{n} x_k^2(a + bx_k + cx_k^2 - y_k) = 0.$$

Simplifying, we obtain

$$\sum y_k = na + b\sum x_k + c\sum x_k^2, \text{...........} (11)$$

$$\sum x_k y_k = a\sum x_k + b\sum x_k^2 + c\sum x_k^3, \text{..........} (12)$$

$$\sum x_k^2 y_k = a\sum x_k^2 + b\sum x_k^3 + c\sum x_k^4, \text{..........} (13)$$

Since $\sum_{i=1}^{n} = n$.These equations are called "the normal equations" for fitting a parabola. From the given data, we compute all the sums. We solve these equations for a, b and c. The required approximation is $f(x) = a + bx + cx^2$.

Fitting an exponential curve

Many practical problems in Science and Engineering are based on decaying processes or growth processes of exponential nature. The approximation to the functions representing such data should reflect the same, that is. They should show exponential decay/growth. Therefore. It is important to consider fitting of an exponential curve to a data by least squares method.

Now, let us fit a curve of the form

$$y = f(x) = a_0 e^{a_1 x} \quad \text{..........} \quad (14)$$

to a given data at n points (x_i, y_i), $i = 1, 2, 3, \ldots\ldots, n$. Taking logarithms on both sides, we have

$$\ln f(x) = \ln a_0 + a_1 x \quad \ldots\ldots\ldots\ldots \quad (15)$$

Where the natural logarithm is taken. This equation (15) can be written as

$$Y = A + BX \quad \ldots\ldots\ldots. \quad (16)$$

Where $Y = \ln f(x)$, $A = \ln a_0$ and $B = a_1$. This equation is a straight line in terms of the variables x, Y. Therefore, we fit a least squares straight line to the new data (x_i, Y_i). We determine A and B, and then obtain $a_0 = e^A$ and $a_1 = b$. Using these values of a_0 and a_1, we obtain the exponential curve fit (14)

We first fit straight line to the data and see whether it is a appropriate fit or not

Let the straight line fitting be $p(x) = A + Bx$--------[0]

Number of given data values are $N = 8$

Normal equations are given by

$$8A + B\sum_{i=1}^{8} x_i = \sum_{i=1}^{8} y_i \text{------[1]}$$

$$A\sum_{i=1}^{8} x_i + B\sum_{i=1}^{8} (x_i)^2 = \sum_{i=1}^{8} x_i y_i \text{------[2]}$$

x_i	y_i	$(x_i)^2$	$x_i y_i$
1	0.11	1	0.11
2	0.22	4	0.44
3	0.4	9	1.2
4	0.72	16	2.88
5	1.18	25	5.9

6	1.93	36	11.58
7	3.27	49	22.89
8	4.7	64	37.6
$\sum_{i=1}^{8} x_i = 3$	$\sum_{i=1}^{8} y_i = 12.$	$\sum_{i=1}^{8} (x_i)^2 = 204$	$\sum_{i=1}^{8} x_i y_i = 82.6$

Substituting summation values in equation [1] and [2] we get

$8A + B36 = 12.53$------[3]

$A36 + B204 = 82.6$------[4]

Solving [3] and [4] we get

$A = -1.2425, B = 0.624167$

Keeping these values in equation [0] we $p(x) = -1.2425 + 0.624167x$

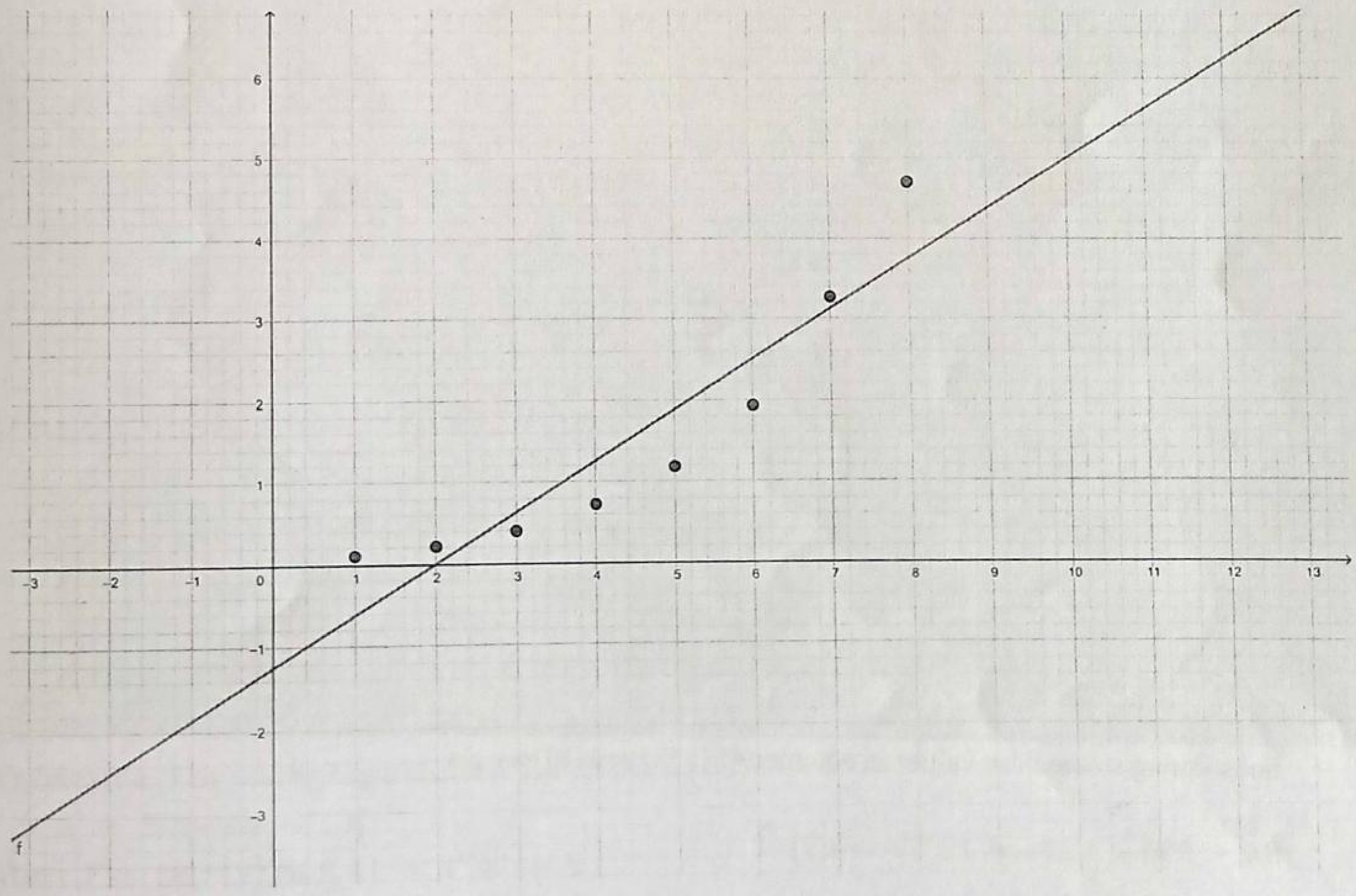

Figure 2

Analysis:

Figure 2 is representing data points and straight line fit $p(x)$.It is clearly noted that $p(x)$ is near to only two data points.This fitting is not appropriate to represent the table1.

Now we fit the quadratic function to table1.

Let it be $q(x) = A + Bx + Cx^2$------[5]

Normal equations are

$$8A + B\sum_{i=1}^{8} x_i + C\sum_{i=1}^{8} (x_i)^2 = \sum_{i=1}^{8} y_i \text{------[6]}$$

$$A\sum_{i=1}^{8} x_i + B\sum_{i=1}^{8} (x_i)^2 + C\sum_{i=1}^{8} (x_i)^3 = \sum_{i=1}^{8} x_i y_i \text{------[7]}$$

$$A\sum_{i=1}^{8} (x_i)^2 + B\sum_{i=1}^{8} (x_i)^3 + C\sum_{i=1}^{8} (x_i)^4 = \sum_{i=1}^{8} (x_i)^2 y_i \text{-------[8]}$$

x_i	y_i	$x_i y_i$	$(x_i)^2$	$(x_i)^2 y_i$	$(x_i)^3$	$(x_i)^4$
1	0.11	0.11	1	0.11	1	1
2	0.22	0.44	4	0.88	8	16
3	0.4	1.2	9	3.6	27	81
4	0.72	2.88	16	11.52	64	256
5	1.18	5.9	25	29.5	125	625
6	1.93	11.58	36	69.48	216	1296
7	3.27	22.89	49	160.23	343	2401
8	4.7	37.6	64	300.8	512	4096
$\sum_{i=1}^{8} x_i = 36$	$\sum_{i=1}^{8} y_i = 12.53$	$\sum_{i=1}^{8} x_i y_i = 82.$	$\sum_{i=1}^{8} (x_i)^2 = 20$	$\sum_{i=1}^{8} (x_i)^2 y_i = 576.$	$\sum_{i=1}^{8} (x_i)^3 = 129$	$\sum_{i=1}^{8} (x_i)^4 = 87$

Substituting summation values in equation·[6] , [7] and [8] we get

$8A + B36 + C204 = 12.53$------[9]

$A36 + B204 + C1296 = 82.6$------[10]

$A204 + B1296 + C8772 = 576.12$-------[11]

Solving [9] ,[10] and [11] we get

$A = 0.603036, B = -0.483155, C = 0.123036$

Keeping these values in equation [5] we get $q(x) = 0.63036 - 0.483155x + 0.123036x^2$

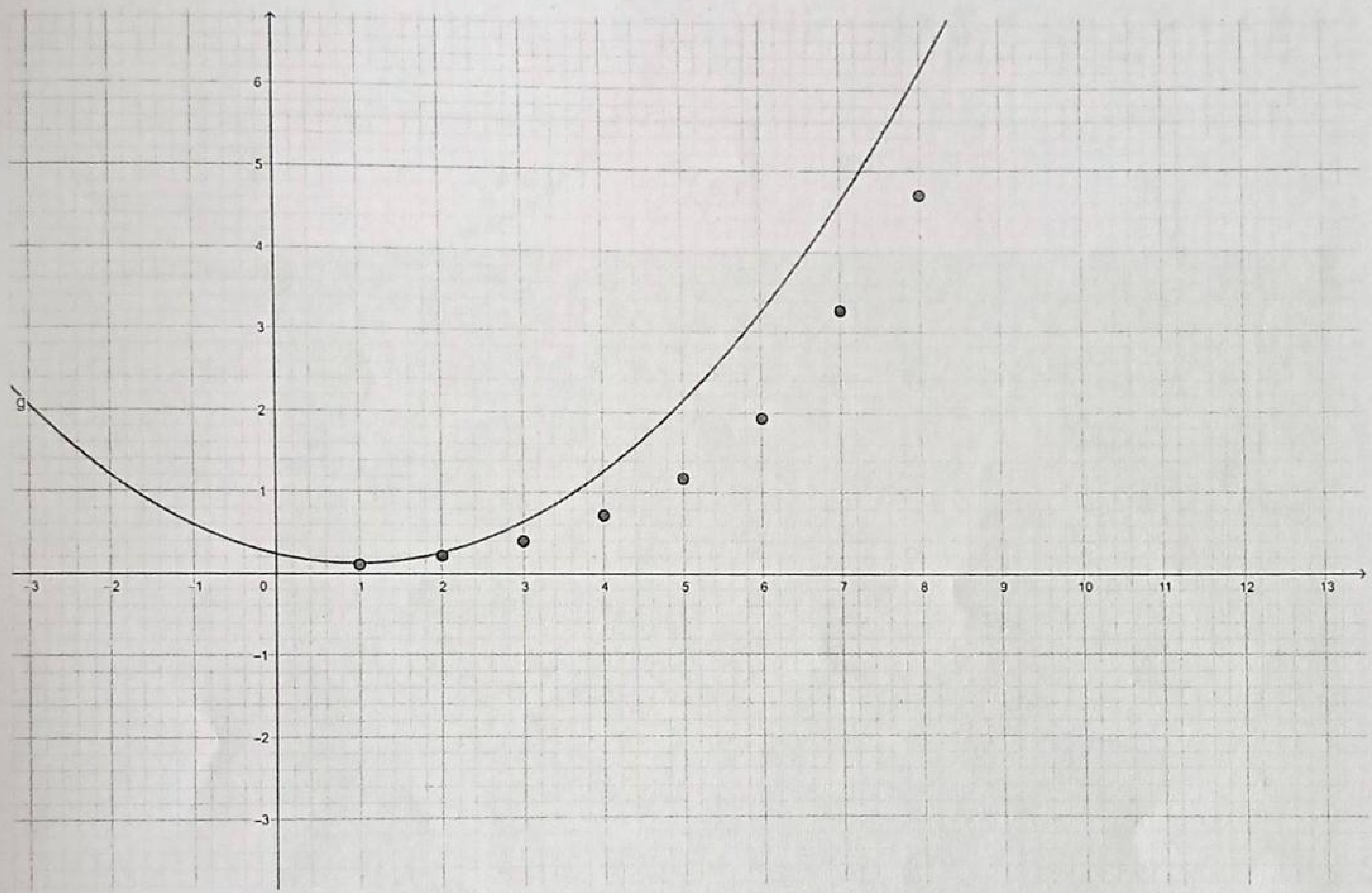

Figure3

Analysis:
Figure 3 is representing data points and quadratic fit $q(x)$.It is observed that $q(x)$ is passing through only two data points.This fitting is not appropriate to represent the table1.However we can say it is a better approximation than $p(x)$.

Now we fit the exponential function to table1.
Let it be $r(x) = Ae^{Bx}$------[12]
We linearise it by taking logarithm on either sides
$Y = A' + B'x$----------[13]
Where $Y = ln(r(x)), ln(A) = A', B' = B$

The equation is a straight line in terms of variables x, Y.we fit a straight line to the new data (x_i, Y_i).we find A', B' then $A = e^{A'}, B' = B$

Normal equations are

$$8A' + B' \sum_{i=1}^{8} x_i = \sum_{i=1}^{8} Y_i \text{------[14]}$$

$$A' \sum_{i=1}^{8} x_i + B' \sum_{i=1}^{8} (x_i)^2 = \sum_{i=1}^{8} x_i Y_i \text{------[15]}$$

x_i	y_i	$Y_i = lny_i$	$(x_i)^2$	$x_i Y_i$
1	0.11	-2.21	1	-2.21
2	0.22	-1.51	4	-3.02
3	0.4	-0.92	9	-2.76
4	0.72	-0.33	16	-1.32
5	1.18	0.17	25	0.85
6	1.93	0.66	36	3.96
7	3.27	1.18	49	8.26
8	4.7	1.55	64	12.4
$\sum_{i=1}^{8} x_i = 3$	$\sum_{i=1}^{8} y_i = 12.$	$\sum_{i=1}^{8} Y_i =- 1.4$	$\sum_{i=1}^{8} (x_i)^2 = 2($	$\sum_{i=1}^{8} x_i Y_i = 16.16$

Substituting above summations in [14] and [15]

$8A' + B'36 =- 1.41$------[14]

$A'36 + B'204 = 16.16$------[15]

Solving [14] and [15] we get

$A' =- 2.59, B' = 0.54$

$A = e^{A'} = 0.08, B' = 0.54$

$r(x) = 0.08e^{054x}$

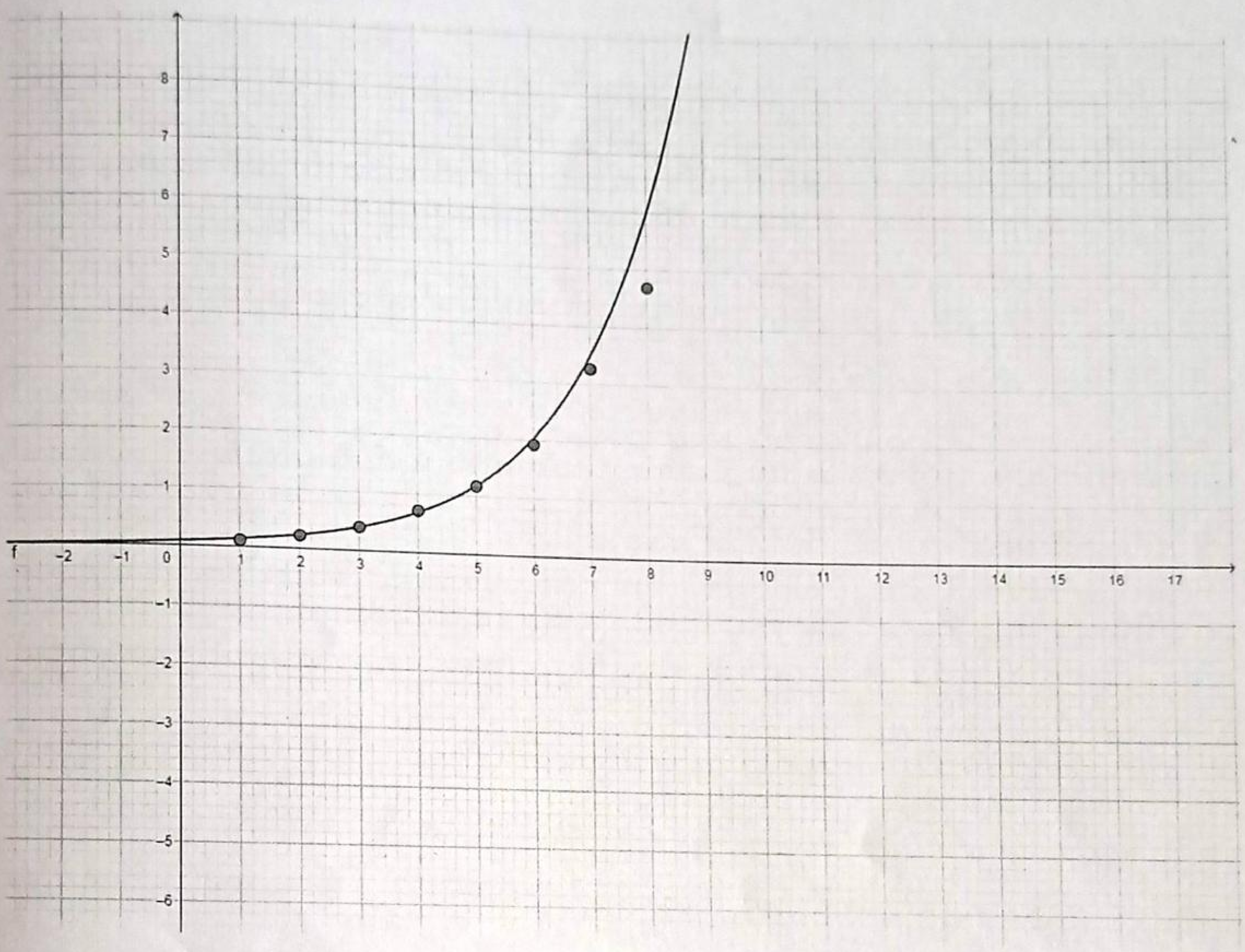

Figure4

Analysis:

Figure 4 is representing data points and exponential fit $r(x)$.It is observed that $r(x)$ is passing through only six data points.This fitting is most appropriate to represent the table1.We can say it is a better approximation than $p(x), q(x)$.Hence It is a curve of best fit for the table 1.

References:

1. Statista website
2. Telugu Academy

Softwares:

1. Geogebra

GOVERNMENT DEGREE COLLEGE (W), NALGONDA

DEPARTMENT OF MATHEMATICS

ACADEMIC YEAR 2019-20

STUDENT STUDY PROJECT

ON

"EXCEL PROGRAM TO SOLVE SIMULTANEOUS EQUATIONS"

Submitted

By

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Problem:

In this project we write an excel program to solve system of 10 linear equations with 10 unknowns. This system can be expressed in the following way.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + a_{16}x_6 + a_{17}x_7 + a_{18}x_8 + a_{19}x_9 + a_{110}x_{10} = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + a_{26}x_6 + a_{27}x_7 + a_{28}x_8 + a_{29}x_9 + a_{210}x_{10} = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{44}x_4 + a_{45}x_5 + a_{46}x_6 + a_{47}x_7 + a_{48}x_8 + a_{49}x_9 + a_{410}x_{10} = b_3$$
$$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 + a_{46}x_6 + a_{47}x_7 + a_{48}x_8 + a_{49}x_9 + a_{410}x_{10} = b_4$$
$$a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 + a_{56}x_6 + a_{57}x_7 + a_{58}x_8 + a_{59}x_9 + a_{610}x_{10} = b_5$$
$$a_{61}x_1 + a_{62}x_2 + a_{63}x_3 + a_{64}x_4 + a_{65}x_5 + a_{66}x_6 + a_{67}x_7 + a_{68}x_8 + a_{69}x_9 + a_{610}x_{10} = b_6$$
$$a_{71}x_1 + a_{72}x_2 + a_{73}x_3 + a_{74}x_4 + a_{75}x_5 + a_{76}x_6 + a_{77}x_7 + a_{78}x_8 + a_{79}x_9 + a_{710}x_{10} = b_7$$
$$a_{81}x_1 + a_{82}x_2 + a_{83}x_3 + a_{84}x_4 + a_{85}x_5 + a_{86}x_6 + a_{87}x_7 + a_{88}x_8 + a_{89}x_9 + a_{810}x_{10} = b_8$$
$$a_{91}x_1 + a_{92}x_2 + a_{93}x_3 + a_{94}x_4 + a_{95}x_5 + a_{96}x_6 + a_{97}x_7 + a_{98}x_8 + a_{99}x_9 + a_{910}x_{10} = b_9$$
$$a_{011}x_1 + a_{102}x_2 + a_{103}x_3 + a_{104}x_4 + a_{105}x_5 + a_{106}x_6 + a_{107}x_7 + a_{108}x_8 + a_{109}x_9 + a_{1010}x_{10} = b_{10}$$

This program would prompt us to enter coefficients of variables $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$ and values of $b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}$ in the entry panel.

Methodology:

We would convert the entered coefficient matrix to row echelon form by applying series of row transformations. To do this we write excel coding for each set of row transformations and output matrix will be displayed after such transformations. We have written coding for each iteration. Coding enables us find the solution of the system without actually solving the system manually. In this project we are showing only coding for first Iteration.

Complete coding has been given in excel file. Which can be accessed with the following link

Link:

Input:

C	D	E	F	G	H	I	J	K	L	M
1	2	1	1	2	1	1	2	3	2	16
1	3	1	2	9	2	1	1	2	3	25
3	1	3	8	2	3	3	1	1	2	27
2	3	4	2	3	1	10	8	4	1	38
3	7	2	6	4	1	1	8	2	6	40
5	4	2	1	3	6	9	1	3	7	41
1	2	5	7	6	8	4	6	3	1	43
2	6	3	4	8	7	4	5	1	2	42
7	5	6	1	2	6	4	8	2	6	47
8	4	5	6	2	1	1	4	6	3	40

First Iteration Coding:

Column C:

```
IF($C$6<>0,C6,C15)
IF($C$6<>0,$C$6*C7-$C$7*C6,$C$18*C7-$C$7*C18)
IF($C$6<>0,$C$6*C8-$C$8*C6,$C$18*C8-$C$8*C18)
IF($C$6<>0,$C$6*C9-$C$9*C6,$C$18*C9-$C$9*C18)
IF($C$6<>0,$C$6*C10-$C$10*C6,$C$18*C10-$C$10*C18)
IF($C$6<>0,$C$6*C11-$C$11*C6,$C$18*C11-$C$11*C18)
IF($C$6<>0,$C$6*C12-$C$12*C6,$C$18*C12-$C$12*C18)
```

```
IF($C$6<>0,$C$6*C15-$C$15*C6,
```

Column D:

```
IF($C$6<>0,D6,D15)
IF($C$6<>0,$C$6*D7-$C$7*D6,$C$18*D7-$C$7*D18)
IF($C$6<>0,$C$6*D8-$C$8*D6,$C$18*D8-$C$8*D18)
IF($C$6<>0,$C$6*D9-$C$9*D6,$C$18*D9-$C$9*D18)
IF($C$6<>0,$C$6*D10-$C$10*D6,$C$18*D10-$C$10*D18)
IF($C$6<>0,$C$6*D11-$C$11*D6,$C$18*D11-C$11*D18)
IF($C$6<>0,$C$6*D12-$C$12*D6,$C$18*D12-C$12*D18)
IF($C$6<>0,$C$6*D13-$C$13*D6,$C$18*D13-C$13*D18)
IF($C$6<>0,$C$6*D14-$C$14*D6,$C$18*D14-C$14*D18)
IF($C$6<>0,$C$6*D15-$C$15*D6,D6)
```

Column E:

```
IF($C$6<>0,E6,E15)
IF($C$6<>0,$C$6*E7-$C$7*E6,$C$18*E7-$C$7*E18)
IF($C$6<>0,$C$6*E8-$C$8*E6,$C$18*E8-$C$8*E18)
IF($C$6<>0,$C$6*E9-$C$9*E6,$C$18*E9-$C$9*E18)
IF($C$6<>0,$C$6*E10-$C$10*E6,$C$18*E10-C$10*E18)
IF($C$6<>0,$C$6*E11-$C$11*E6,$C$18*E11-$C$11*E18)))
IF($C$6<>0,$C$6*E12-$C$12*E6,$C$18*E12-$C$12*E18)
IF($C$6<>0,$C$6*E13-$C$13*E6,$C$18*E13-$C$13*E18)
IF($C$6<>0,$C$6*E14-$C$14*E6,$C$18*E14-$C$14*E18)
IF($C$6<>0,$C$6*E15-$C$15*E6,E6)
```

Column F:

```
IF($C$6<>0,F6,F15)
IF($C$6<>0,$C$6*F7-$C$7*F6,$C$18*F7-$C$7*F18)
IF($C$6<>0,$C$6*F8-$C$8*F6,$C$18*F8-$C$8*F18)
IF($C$6<>0,$C$6*F9-$C$9*F6,$C$18*F9-$C$9*F18)
IF($C$6<>0,$C$6*F10-$C$10*F6,$C$18*F10-$C$10*F18)
IF($C$6<>0,$C$6*F11-$C$11*F6,$C$18*F11-$C$11*F18)
IF($C$6<>0,$C$6*F12-$C$12*F6,$C$18*F12-$C$12*F18)
IF($C$6<>0,$C$6*F13-$C$13*F6,$C$18*F13-$C$13*F18)
IF($C$6<>0,$C$6*F14-$C$14*F6,$C$18*F14-$C$14*F18)
IF($C$6<>0,$C$6*F15-$C$15*F6,F6)
```

Column G:

```
IF($C$6<>0,G6,G15)
IF($C$6<>0,$C$6*G7-$C$7*G6,$C$18*G7-$C$7*G18)
IF($C$6<>0,$C$6*G8-$C$8*G6,$C$18*G8-$C$8*G18)
IF($C$6<>0,$C$6*G9-$C$9*G6,$C$18*G9-$C$9*G18)
IF($C$6<>0,$C$6*G10-$C$10*G6,$C$18*G10-$C$10*G18)
IF($C$6<>0,$C$6*G11-$C$11*G6,$C$18*G11-$C$11*G18)
IF($C$6<>0,$C$6*G12-$C$12*G6,$C$18*G12-$C$12*G18)
IF($C$6<>0,$C$6*G13-$C$13*G6,$C$18*G13-$C$13*G18)
IF($C$6<>0,$C$6*G14-$C$14*G6,$C$18*G14-$C$14*G18)
IF($C$6<>0,$C$6*G15-$C$15*G6,G6)
```

Column H :

```
IF($C$6<>0,H6,H15)
IF($C$6<>0,$C$6*H7-$C$7*H6,$C$18*H7-$C$7*H18)
IF($C$6<>0,$C$6*H8-$C$8*H6,$C$18*H8-$C$8*H18)
IF($C$6<>0,$C$6*H9-$C$9*H6,$C$18*H9-$C$9*H18)
```

```
IF($C$6<>0,$C$6*H10-$C$10*H6,$C$18*H10-$C$10*H18)
IF($C$6<>0,$C$6*H11-$C$11*H6,$C$18*H11-$C$11*H18)
IF($C$6<>0,$C$6*H12-$C$12*H6,$C$18*H12-$C$12*H18)
IF($C$6<>0,$C$6*H13-$C$13*H6,$C$18*H13-$C$13*H18)
IF($C$6<>0,$C$6*H14-$C$14*H6,$C$18*H14-$C$14*H18)
IF($C$6<>0,$C$6*H15-$C$15*H6,H6)
```

Column I:

```
IF($C$6<>0,I6,I15)
IF($C$6<>0,$C$6*I7-$C$7*I6,$C$18*I7-$C$7*I18)
IF($C$6<>0,$C$6*I8-$C$8*I6,$C$18*I8-$C$8*I18)
IF($C$6<>0,$C$6*I9-$C$9*I6,$C$18*I9-$C$9*I18)
IF($C$6<>0,$C$6*I10-$C$10*I6,$C$18*I10-$C$10*I18)
IF($C$6<>0,$C$6*I11-$C$11*I6,$C$18*I11-$C$11*I18)
IF($C$6<>0,$C$6*I12-$C$12*I6,$C$18*I12-$C$12*I18)
IF($C$6<>0,$C$6*I13-$C$13*I6,$C$18*I13-$C$13*I18)
IF($C$6<>0,$C$6*I14-$C$14*I6,$C$18*I14-$C$14*I18)
IF($C$6<>0,$C$6*I15-$C$15*I6,I6)
```

Column J:

```
IF($C$6<>0,J6,J15)
IF($C$6<>0,$C$6*J7-$C$7*J6,$C$18*J7-$C$7*J18)
IF($C$6<>0,$C$6*J8-$C$8*J6,$C$18*J8-$C$8*J18)
IF($C$6<>0,$C$6*J9-$C$9*J6,$C$18*J9-$C$9*J18)
IF($C$6<>0,$C$6*J10-$C$10*J6,$C$18*J10-C$10*J18)
IF($C$6<>0,$C$6*J11-$C$11*J6,$C$18*J11-$C$11*J18)
IF($C$6<>0,$C$6*J12-$C$12*J6,$C$18*J12-$C$12*J18)
IF($C$6<>0,$C$6*J13-$C$13*J6,$C$18*J13-$C$13*J18)
IF($C$6<>0,$C$6*J14-$C$14*J6,$C$18*J14-$C$14*J18)
IF($C$6<>0,$C$6*J15-$C$15*J6,J6)
```

Column K:

```
IF($C$6<>0,K6,K15)
IF($C$6<>0,$C$6*K7-$C$7*K6,$C$18*K7-$C$7*K18)
IF($C$6<>0,$C$6*K8-$C$8*K6,$C$18*K8-$C$8*K18)
IF($C$6<>0,$C$6*K9-$C$9*K6,$C$18*K9-$C$9*K18)
IF($C$6<>0,$C$6*K10-$C$10*K6,$C$18*K10-$C$10*K18)
IF($C$6<>0,$C$6*K11-$C$11*K6,$C$18*K11-$C$11*K18)
IF($C$6<>0,$C$6*K12-$C$12*K6,$C$18*K12-$C$12*K18)
IF($C$6<>0,$C$6*K13-$C$13*K6,$C$18*K13-$C$13*K18)
IF($C$6<>0,$C$6*K14-$C$14*K6,$C$18*K14-$C$14*K18)
IF($C$6<>0,$C$6*K15-$C$15*K6,K6)
```

Column L:

```
IF($C$6<>0,L6,L15)
IF($C$6<>0,$C$6*L7-$C$7*L6,$C$18*L7-$C$7*L18)
IF($C$6<>0,$C$6*L8-$C$8*L6,$C$18*L8-$C$8*L18)
IF($C$6<>0,$C$6*L9-$C$9*L6,$C$18*L9-$C$9*L18)
IF($C$6<>0,$C$6*L10-$C$10*L6,$C$18*L10-$C$10*L18)
IF($C$6<>0,$C$6*L11-$C$11*L6,$C$18*L11-$C$11*L18)
IF($C$6<>0,$C$6*L12-$C$12*L6,$C$18*L12-$C$12*L18)
IF($C$6<>0,$C$6*L13-$C$13*L6,$C$18*L13-$C$13*L18)
IF($C$6<>0,$C$6*L14-$C$14*L6,$C$18*L14-$C$14*L18)
IF($C$6<>0,$C$6*L15-$C$15*L6,L6)
```

Column M:

```
IF($C$6<>0,M6,M15)
IF($C$6<>0,$C$6*M7-$C$7*M6,$C$18*M7-$C$7*M18)
IF($C$6<>0,$C$6*M8-$C$8*M6,$C$18*M8-$C$8*M18)
IF($C$6<>0,$C$6*M9-$C$9*M6,$C$18*M9-$C$9*M18)
IF($C$6<>0,$C$6*M10-$C$10*M6,$C$18*M10-$C$10*M18)
IF($C$6<>0,$C$6*M11-$C$11*M6,$C$18*M11-$C$11*M18)
IF($C$6<>0,$C$6*M12-$C$12*M6,$C$18*M12-$C$12*M18)
IF($C$6<>0,$C$6*M13-$C$13*M6,$C$18*M13-$C$13*M18)
IF($C$6<>0,$C$6*M14-$C$14*M6,$C$18*M14-$C$14*M18)
IF($C$6<>0,$C$6*M15-$C$15*M6,M6)
```

Output:

C	D	E	F	G	H	I	J	K	L	M
1	2	1	1	2	1	1	2	3	2	16
0	1	0	1	7	1	0	-1	-1	1	9
0	-5	0	5	-4	0	0	-5	-8	-4	-21
0	-1	2	0	-1	-1	8	4	-2	-3	6
0	1	-1	3	-2	-2	-2	2	-7	0	-8
0	-6	-3	-4	-7	1	4	-9	-12	-3	-39
0	0	4	6	4	7	3	4	0	-1	27
0	2	1	2	4	5	2	1	-5	-2	10
0	-9	-1	-6	-12	-1	-3	-6	-19	-8	-65
0	-12	-3	-2	-14	-7	-7	-12	-18	-13	-88

Final output:

Q
Unique solution
1
1
1
1
1
1
1
1
1
1

```
IF($C$6<>0,$C$6*C13-$C$13*C6,$C$18*C13-$C$13*C18)
IF($C$6<>0,$C$6*C14-$C$14*C6,$C$18*C14-$C$14*C18)
```

GOVERNMENT DEGREE COLLEGE (W), NALGONDA

DEPARTMENT OF MATHEMATICS

ACADEMIC YEAR 2018-19

STUDENT STUDY PROJECT

ON

"SOME PROBLEMS IN INTERPOLATION"

Submitted

By

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INTERPOLATION

When the function $y = f(x)$ is known explicitly, it is easy to find the values of $f(x)$ for different values of x .Interpolation is art of reconstructing $f(x)$ when $f(x)$ is not given explicitly but values of $f(x)$ is available at various x values. Through the process of interpolation the function $f(x)$ can be approximated with much simple function like polynomial.

Suppose the following table of x and y are available

x	x_0	x_1	x_2	$\cdots$	$\cdots$	$\cdots$	x_n
$y = f(x)$	y_0	y_1	y_2	$\cdots$	....	$\cdots$	y_n

Interpolation is the method by which one can find the value of y for non tabulated values x between the range $[x_0 , x_n]$ or to find a simple function like polynomial say $\varphi(x)$ which does satisfy the above table. Evaluating the value of y outside the interval $[x_0 , x_n]$ is called extrapolation. In the area of Numerical analysis, Interpolation is a technique to construct new data points within the range of a discrete set of known data points. In science, number of data points is obtained by sampling or experimentation, which represents the values of a function for a limited number of values of a independent variable. It is often required to find the value of the function which is not available for value of the independent variable.

Objectives

1. Understanding what is interpolation.
2. Knowing practical applications of interpolation.
3. Improving the numerical calculations.
4. Learning new software like mathtype, geogebra etc.
5. How to use Newtons, Gauss interpolation formulae.

Methodology

In the first problem, we have verified interpolated data with the original data. These results were interpreted in the **graph1.1** graphs were drawn by using geogebra software. In this case approximant values have errors by range 0.2%-1.2%. In **Graph1.1** the exact data points are

$$(H,K,L,M,Z,O,P,Q,R,S,T,U,V,W,N,A1,B_1,C_1,D_1,E_1,F_1,G_1,H_1,I_1,J_1,K_1,L_1,M_1,N_1,O_1,P_1)$$

and approximant data points are (H,J,I).

In the second problem we have taken a class room example. We have collected marks of second year students. This is represented by (**Table2.1**) from this we have constructed a cumulative frequency table (**Table2.1**) then we interpolated a data and results were interpreted in a graph **graph2.1.** In this case approximant values have errors by range 1.3%-9.2%. In **graph2.1.** exact data points are (K,L) and approximant data points are (I,J).

In the third problem we have collected share values of a company over 30 days from these we have taken a few share values of the company at a length of four days. By interpolation we have found the share values of some missing days and compare with the exact data. We got the results which were error by range 0.5%-1%.These were interpreted in the **graph3.1** exact data points are

$$(A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P,Q,R,S,T,U,V,W)$$

And approximant data points are (Z,B_1,A_1).

PROBLEM 1

This problem is dealing about time and temperature. In this problem we have collected data points of time and temperature from observations made in the lab. First oil was heated upto $80^{0}C$ then allowed to cool. While oil was cooling; we have noted a temperature in degree Celsius for every minute till the 30th minute. These were tabulated in(**Table1.1**) which is showing the temperatures against time. From this table we have taken sub table. **(Table1.2)** which consists of temperatures of cooling oil at regular intervals of time of length 5 minute. Now our aim was to obtain the temperatures of oil at $4^{th}, 26^{th}, \& 16^{th}$ minutes respectively which are not available in the **(Table1.2)**.To do this we need technique of interpolation. After finding the values our aim was to compare the interpolated result with the experimental result and analyse the errors which were shown in a graphs.

Time (minutes)	Temperature T(^{0}c)
0	80
1	78
2	76
3	75
4	73
5	71.8
6	70
7	69.3
8	68
9	66.5
10	65
11	64.8
12	63.5
13	62.5
14	61.8
15	60.9
16	60
17	59.8
18	58.8
19	58
20	57
21	56.5
22	55.9
23	55.2
24	54.9
25	54
26	53.9
27	53
28	52.3
29	52
30	51.5

(Table1.1)

Time (minutes) (x)	Temperature(T) ^{0}c (y)
0	80
5	71.8
10	65
15	60.9
20	57
25	54
30	51.5

(Table1.2)

Now the aim is to interpolate the data at the specific times i.e. to evaluate the temperature of the oil at non tabulated time and to find out the error. To do this we do need to construct the difference table(**Table1.3**).

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0	80						
		-8.2					
5	71.8		1.4				
		-6.8		1.3			
10	65		2.7		-3.8		
		-4.1		-2.5		7	
15	60.9		0.2		3.2		-11.3
		-3.9		0.7		-4.3	
20	57		0.9		-1.1		
		-3		-0.4			
25	54		0.5				
		-2.5					
30	51.5						

(Table1.3)

Evaluating temperature of oil at time 4^{th} minute i.e. when $x = 4$

As this time is at the beginning of the difference table **(Table1.3)** now we do need Newton's forward interpolation formula.

Newtons forward interpolation

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \frac{p(p-1)(p-2)(p-3)(p-4)}{5!}\Delta^5 y_0 + \frac{p(p-1)(p-2)(p-3)(p-4)(p-5)}{6!}\Delta^6 y_0$$

$$x = 4, x_0 = 0, h = 5$$

$$p = \frac{x - x_0}{h} = \frac{4-0}{5} = \frac{4}{5} = 0.8$$

$$y = 80 + (0.8)(-8.2) + \frac{(0.8)(-0.2)}{2}(1.4) + \frac{(0.8)(-0.2)(-1.2)}{6}(1.3) + \frac{(0.8)(-0.2)(-1.2)(-2.2)}{24}(-3.8) + \frac{(0.8)(-0.2)(-1.2)(-2.2)(-3.2)}{120}(7) + \frac{(0.8)(-0.2)(-1.2)(-2.2)(-3.2)(-4.2)}{720}(-11.3)$$

$$y = 80 - 6.56 - 0.112 + 0.0416 + 0.0669 + 0.0788 + 0.0891$$

$$y = 73.6044$$

Therefore temperature of the oil at 4^{th} minute i.e. $y(4) = 73.6044^0 C$.

Error in the approximation is.

$$E_A = |True\,value - approximate\,value|$$

$$E_A = |73 - 73.6044|$$

$$= 0.6044^0 C$$

Error analysis

Numerical error of the interpolated data is $0.6044^0 C$. The Point H is obtained by interpolation and the pont Z is true data point. The deviation is evident in the **graph1.1.**The approximate value is error by 0.82% i.e. 1%

Evaluating temperature of the oil at 26^{th} minute i.e. when $x = 26$

As this time is at the end of the difference table **(Table1.3)** now we do need Newton's backward interpolation formula.

Newtons backward interpolation

$$y = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)}{5!}\nabla^5 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)(p+5)}{6!}\nabla^6 y_n$$

$x = 26, x_n = 30$

$$p = \frac{x - x_n}{h} = \frac{26-30}{5} = \frac{-4}{5} = -0.8$$

$$y = 51.5 + (-0.8)(-2.5) + \frac{(-0.8)(0.2)}{2}(0.5) + \frac{(-0.8)(0.2)(1.2)}{6}(-0.4) + \frac{(-0.8)(0.2)(1.2)(2.2)}{24}(-1.1) + \frac{(-0.8)(0.2)(1.2)(2.2)(3.2)}{120}(-4.3) + \frac{(-0.8)(0.2)(1.2)(2.2)(3.2)(4.2)}{720}(-11.3)$$

$$y = 51.5 + 2 - 0.04 + 0.0128 + 0.0195 + 0.0484 + 0.0891$$

$$y = 53.6298$$

Therefore temperature of the oil at 26 minutes i.e. $x = 26$ is $y\ (26)\ - 53.62900C$.

Error in the approximation is

$$E_A = |True value - approximate value|$$

$$E_A = |53.9 - 53.6298|$$

$$= 0.6298^0 C$$

Error analysis

Numerical error of the interpolated data is $0.6298^0\,C$. The Point I is obtained by interpolation and the pont L_1 is true data point. The deviation is evident in the **graph1.1.**The approximate value is error by 1.2%

Evaluating temperature of the oil at 16^{th} minute i.e. when $x = 16$

As this time is in the middle of the difference table (**Table1.3**) we do need gauss central difference formula.

Gauss central difference interpolation

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{p(p-1)(p+1)}{3!}\Delta^3 y_{-1} + \frac{p(p^2-1)(p-2)}{4!}\Delta^4 y_{-2} + \frac{p(p^2-1)(p^2-4)}{5!}\Delta^5 y_{-2} + \frac{p(p^2-1)(p^2-4)(p-3)}{6!}\Delta^6 y_{-3}$$

$$x = 16, x_0 = 15, h = 5$$

$$p = \frac{x - x_0}{h} = \frac{16-15}{5} = \frac{1}{5} = 0.2$$

$$y = 60.9 + (0.2)(-3.9) + \frac{(0.2)(-0.8)}{2}(0.2) + \frac{(0.2)(-0.96)}{6}(0.7) + \frac{(0.2)(-0.96)(-1.8)}{24}(3.2) + \frac{(0.2)(-0.96)(-3.96)}{120}(-4.3) + \frac{(0.2)(-0.96)(-3.96)(-2.8)}{720}(-11.3)$$

$$y = 60.9 - 0.78 - 0.016 - 0.0224 + 0.04608 - 0.02724 + 0.033411$$

$$y = 60.133851$$

Therefore temperature of the oil at 16 minutes i.e. $x = 16$ is $y\ (16) = 60.1338510C$.

Error in the approximation is

$$E_A = |Truevalue - approximatevalue|$$

$$E_A = |60 - 60.133851|$$

$$= 0.133851^0 C$$

Error analysis

Numerical error of the interpolated data is $0.133851^0 C$. The Point J is obtained by interpolation and the pont B_1 is true data point. The deviation is evident in the **graph1.1.**The approximate value is error by 0.2%

PROBLEM 2

In this problem we have collected Marks obtained in mathematics by 115 students of Class MPCS, MPE, and MPC II of a college for the academic year 2015-2016.From this data we have extracted grouped frequency distribution table (**Table 2.2**) and then cumulative frequency table (**Table2.3**).In this problem we are aiming to calculate number of students who obtained less than 15 marks, 75 marks respectively by technique of interpolation. Results were compared with the exact data.

34	34	18	54	0
43	21	9	0	39
0	54	45	8	0
22	14	9	21	23
0	22	17	45	20
15	35	35	37	41
18	80	35	10	39
0	48	16	6	41
10	35	8	34	37
34	10	64	13	0
0	20	15	0	36
0	8	44	47	21
42	34	37	47	35
72	79	34	70	0
66	54	56	62	54
0	54	41	41	36
11	39	52	57	36
34	22	21	34	12
34	34	22	49	52
23	34	9	0	36
22	26	39	7	45
35	10	40	34	79
62	12	66	64	45

(Table 2.1)

(Grouped frequency distribution table) **(Table 2.2)**

S.No.	Marks obtained	No. Of students
1	0-10	21
2	10-20	15
3	20-30	14
4	30-40	29
5	40-50	16
6	50-60	09

7	60-70	06
8	70-80	05

Cumulative frequency table

S.No.	Marks less than $< x$	No. Of students with $< x$ marks $y = f(x)$
1	10	21
2	20	36
3	30	50
4	40	79
5	50	95
6	60	104
7	70	110
8	80	115

(Table 2.3)

To interpolate data we need to construct the difference table(**Table 2.4**)

X	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$
10	**21**							
		15						
20	36		-1					
		14		16				
30	50		15		-44			
		29		-28		78		
40	79		-13		34		-114	
		16		6		-38		150
50	95		-7		-2		36	
		9		4		0		
60	104		-3		-2			
		6		2				
70	110		-1					
		5						
80	115							

(Table 2.4)

Finding the number of students with less than 15 marks. As this mark is at the beginning of the (**Table 2.4**) hence we do need Newton's forward interpolation formula

Newtons forward interpolation

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \frac{p(p-1)(p-2)(p-3)(p-4)}{5!}\Delta^5 y_0 + \frac{p(p-1)(p-2)(p-3)(p-4)(p-5)}{6!}\Delta^6 y_0 + \frac{p(p-1)(p-2)(p-3)(p-4)(p-5)(p-6)}{7!}\Delta^7 y_0$$

$$x = 15, x_0 = 10, h = 10$$

$$p = \frac{x - x_0}{h} = \frac{15-10}{10} = \frac{5}{10} = 0.5$$

$$y = 21 + (0.5)15 + \frac{(0.5)(-0.5)}{2}(-1) + \frac{(0.5)(-0.5)(-1.5)}{6}16 + \frac{(0.5)(-0.5)(-1.5)(-2.5)}{24}(-44) + \frac{(0.5)(-0.5)(-1.5)(-2.5)(-3.5)}{120}78 + \frac{(0.5)(-0.5)(-1.5)(-2.5)(-3.5)(-4.5)}{720}(-114) + \frac{(0.5)(-0.5)(-1.5)(-2.5)(-3.5)(-4.5)(-5.5)}{5040}(150)$$

$$y = 21 + 7.5 + 0.125 + 1 + 1.71875 + 2.132820 + 2.33789 + 2.41699$$

$$y = 38.23145$$

Therefore number of students who secured marks less than 15 is equal to 38

The error in the approximation is

$$E_A = |True value - approximate value|$$

$$E_A = |35 - 38.23|$$

$$= 3.23 (Candidates)$$

Error analysis

Numerical error of the interpolated data is 3.23. The Point I is obtained by interpolation and the point K is true data point. The deviation is evident in the **graph2.1.**The approximate value is error by 9.2%

Finding the number of students with less than 75 marks. As this mark is at the end of the (**Table 2.4)** hence we do need Newton's backward interpolation formula.

Newtons backward interpolation

$$y = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)}{5!}\nabla^5 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)(p+5)}{6!}\nabla^6 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)(p+5)(p+6)}{7!}\nabla^7 y_n$$

$$x = 75, x_n = 80$$

$$p = \frac{x - x_n}{h} = \frac{75-80}{10} = \frac{-5}{10} = -0.5$$

$$y = 115 + (-0.5)5 + \frac{(-0.5)(0.5)}{2}(-1) + \frac{(-0.5)(0.5)(1.5)}{6}2 + \frac{(-0.5)(0.5)(1.5)(2.5)}{24}(-2) + \frac{(-0.5)(0.5)(1.5)(2.5)(3.5)(4.5)}{720}36 + \frac{(-0.5)(0.5)(1.5)(2.5)(3.5)(4.5)(5.5)}{5040}150$$

$$y = 115 - 2.5 + 1.25 - 0.125 + 0.07812 - 0.73828 - 2.41699$$

$$y = 110.54785$$

Therefore number of students who secured marks less than 75 is equal to 111

The error in the approximation is

$E_A = |True value - approximate value|$

$E_A = |112 - 110.54785|$

$= 1.45 (Candidates)$

Error analysis

Numerical error of the interpolated data is 1.45.The Point L is obtained by interpolation and the point J is true data point. The deviation is evident in the **graph2.1.**The approximate value is error by 1.3%

PROBLE 3

In this problem we have collected share values of WIPRO Company over 30 days from 1//11/16 to 30/11/16**(Table3.1).**From this we have considered a share values for every four days as shown in the table **(Table3.2).**Now our aim is to evaluate the share value of the company in intermediate days. We have interpolated company share values on day 7 , day 26 and day 15.Results were compared with the existing true value.

Date(x)	Shares value(y)
1/11/16	464.40
2/11/16	460.75
3/11/16	467.90
4/11/16	447.60
5/11/16	NA
6/11/16	NA
7/11/16	452.50
8/11/16	450.00
9/11/16	451.75
10/11/16	446.90
11/11/16	444.95
12/11/16	442.35

13/11/16	NA
14/11/16	NA
15/11/16	442.35
16/11/16	447.95
17/11/16	445.30
18/11/16	438.30
19/11/16	437.50
20/11/16	Holiday
21/11/16	437.15
22/11/16	441.80
23/11/16	450.40
24/11/16	448.90
25/11/16	450.75
26/11/16	464.75
27/11/16	NA
28/11/16	464.75
29/11/16	460.60
30/11/16	460.15

(Table3.1)Source: TOI newspaper

From the above table following table has drawn

Day(x)	Share value(y)
04	447.60
08	450
12	442.35
16	447.95

20	437.15
24	448.90
28	464.75

(Table3.2)

If we need to find the share value of a company on specific day which is non tabulated value, can be obtained by interpolating data at that date with suitable interpolation formula. To do this we do need construct the difference table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
04	447.60						
		2.4					
08	450		-10.5				
		-7.65		23.75			
12	442.35		13.25		-53.4		
		5.6		-29.65		122	
16	447.95		-16.4		68.6		-248
		-10.8		39.95		-126	
20	437.15		22.55		57.4		
		11.75		-18.45			
24	448.90		4.1				
		15.85					
28	464.75						

(Table3.3)

Finding the share value of a company on day 7 as this is at the beginning of the (**Table3.3**) we do require forward interpolation formula.

Newtons forward interpolation

$x = 7, x_0 = 4, h = 4$

$p = \frac{x - x_0}{h} = \frac{7-4}{4} = \frac{3}{4} = 0.75$

$$y = 447.60 + (0.75)(2.4) + \frac{(0.75)(-0.25)}{2}(-10.5) + \frac{(0.75)(-0.25)(-1.25)}{6}(23.75) + \frac{(0.75)(-0.25)(-1.25)(-2.25)}{24}(-53.4) + \frac{(0.75)(-0.25)(-1.25)(-2.25)(-3.25)(-4.25)}{720}(-248)$$

$$y = 447.60 + 1.8 + 0.984375 + 0.92773438 + 1.1733399 + 1.742431 + 2.508911133$$

$$y = 456.736792$$

Therefore share value of the company on 7 day $y(7) = 456.73672(Rs)$

The error in the approximation is

$$E_A = |True value - approximate value|$$
$$y = 447.60 + 1.8 + 0.984375 + 0.92773438 + 1.1733399 + 1.742431 + 2.508911133$$

$$= |456.736792 - 452.50|$$

$$= 4.236792(Rs)$$

Error analysis

Numerical error of the interpolated data is 4.23. The Point Z is obtained by interpolation and the pont E is true data point. The deviation is evident in the **graph3.1.** The approximate value is error by 1%

Finding the share value of a company on day 26 as this is at the end of the **(Table3.3)** we do require backward interpolation formula.

Newtons backward interpolation

$x = 26, x_n = 28, h = 4,$

$$p = \frac{x - x_n}{h} = \frac{26 - 28}{4} = \frac{-2}{4} = -0.5$$

$$y = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)}{5!}\nabla^5 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)(p+5)}{6!}\nabla^6 y_n$$

$$y = 464.75 + (-0.5)(15.85) + \frac{(-0.5)(0.5)(4.1)}{2} + \frac{(-0.5)(0.5)(1.5)}{6}(-18.45) + \frac{(-0.5)(0.5)(1.5)(2.5)}{24}(-57.4) + \frac{(-0.5)(0.5)(1.5)(2.5)(3.5)}{120}(-126) + \frac{(-0.5)(0.5)(1.5)(2.5)(3.5)(4.5)}{720}(-248)$$

$$y = 464.75 - 7.925 - 0.5125 + 1.153125 + 2.2421875 + 3.4453125 + 5.0859375$$

Therefore share value of the company on 26 day $y(26) = 468.239(Rs)$

The error in the approximation is

$$E_A = |Truevalue - approximatevalue|$$

$$= |468.2390625 - 464.75|$$

$$= 3.4890625(Rs)$$

Error analysis

Numerical error of the interpolated data is 3.489625. The Point B_1 is obtained by interpolation and the pont T is true data point. The deviation is evident in the **graph3.1.**The approximate value is error by 0.75%

Finding the share value of a company on day 15 as this is at the middle of the **(Table3.3)** we do require Gauss interpolation formula.

Gauss interpolation

$x = 15, x_0 = 12, h = 4$

$$p = \frac{x - x_0}{h} = \frac{15-12}{4} = \frac{3}{4} = 0.75$$

$$y = 442.35 + (0.75)(5.6) + \frac{(0.75)(-0.25)}{2}(13.25) + \frac{(0.75)(-0.25)(1.75)}{6}(-29.65) + \frac{(0.75)(-0.4375)(-1.25)}{24}(-53.4) + \frac{(0.75)(-0.4375)(-3.4375)}{120}(122) + \frac{(0.75)(-0.4375)(-3.4375)(-2.25)}{720}(-248)$$

$$\frac{p(p^2-1)(p^2-4)(p-3)}{6!}\Delta^6 y_{-3}$$

$$y = 442.35 + 4.2 - 1.2421875 - 1.621484375 - 0.9125976563 + 1.146728516 + 0.8741455078$$

$$y = 444.7946045$$

Therefore share value of the company on 15 day $y(15) = 444.794(Rs)$

The error in the approximation is

$$E_A = |Truevalue - approximatevalue|$$

$$= |442.35 - 444.794|$$

$$= 2.444(Rs)$$

Error analysis

Numerical error of the interpolated data is 2.444. The Point A_1 is obtained by interpolation and the pont K is true data point. The deviation is evident in the **graph3.1.**The approximate value is error by 0.5%.

References

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2. *Numerical analysis by S.ranganatham,Dr.M.V.S.S.N.Prasad, Dr.V.Ramesh babu-S.chand publications.*
3. *Numerical methods for scientific and engineering computations by M..K.Jain, S.R.K.Iyengar, R.K.Jain-New age publications.*
4. *Wikipedia.*
5. *Times of India news papers for data.*

Softwares:

1. ***Mathtype***
2. ***Geogebra for graphs.***

STUDENT STUDY PROJECT

TITLE: CONVERTING DATA INTO POLYNOMIALS USING NEWTON'S FORWARD INTERPOLATION FORMULA

Submitted to

THE DEPARTMENT OF MATHEMATICS

GOVERNMENT DEGREE COLLEGE FOR WOMEN, NALGONDA

NALGONDA DIST, TELANGANA

By

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Declaration

We hereby declare that the Study –Project entitled **"Converting data into Polynomials using newton's Forward interpolation formula"** is our own work, conducted under supervision of Sri B.S.S.P.Rajasekhar Asst.Prof. of Mathematics, Government Degree College for women Nalgonda, Nalgonda District is submitted by us in partial fulfilment of the requirement for the student –study project as a part of Study – Projects 2018-19.

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CERTIFICATE

This is to certify that the study project titled "**Converting data into Polynomials using newton's Forward interpolation formula**" is a bona fide work done by the following students in partial fulfilment of the requirement for the **student –study project 2018-19**

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Converting data into Polynomials using newton's Forward interpolation formula

1. Hypothesis

In engineering and science, one often has a number of data points, obtained by sampling or experimentation, which represent the values of a function for a limited number of values of the independent variable. It is often required to interpolate (i.e. estimate) the value of that function for an intermediate value of the independent variable. Newton's Forward Interpolation formula is one of the tools to interpolate the required value with a better approximation. But when there are many data points, it is difficult to use the Newton's Forward Interpolation formula manually due to number of calculations involved in it.

2. Aims & Objectives

The aims & objectives of this project are

1. To develop algorithms to the Newton's Forward Interpolation formula which can be executable in any spreadsheet/Ms- Excel which is readily available to the most of computer users.
2. To covert the data up to 10 points into a polynomial, so the we can draw graphs and use them to find the required data at any intermediate values.
3. To prepare a user friendly spread sheet program so that the user can interpolate the data at any intermediate value instantly.

3. Literary review

Numerical analysis is the study of algorithms that use numerical approximation for the problems of mathematical analysis.

One of the earliest mathematical writings is a Babylonian tablet from the Yale Babylonian Collection, which gives a sexagesimal numerical approximation of $\sqrt{2}$, the length of the diagonal in a unit square. Being able to compute the sides of a triangle is extremely important, for instance, in astronomy, carpentry and construction. Numerical analysis continues this long tradition of practical mathematical calculations. Much like the Babylonian approximation of $\sqrt{2}$, modern numerical analysis does not seek exact answers, because exact answers are often impossible to obtain in practice. Instead, much of numerical analysis is concerned with obtaining approximate solutions while maintaining reasonable bounds on errors.

Numerical analysis naturally finds applications in all fields of engineering and the physical sciences, but in the 21st century also the life sciences and even the arts have adopted elements of scientific computations. Ordinary differential equations appear in celestial mechanics (planets, stars and galaxies); numerical linear algebra is important for data analysis;

Interpolation

In numerical analysis, interpolation is a method of constructing new data points within the range of a discrete set of known data points.

In engineering and science, one often has a number of data points, obtained by sampling or experimentation, which represent the values of a function for a limited number of values of the independent variable. It is often required to interpolate (i.e. estimate) the value of that function for an intermediate value of the independent variable. Newton's Forward Interpolation formula is one of the tools to achieve this goal.

Newton's Forward Interpolation formula

$$y = f(x) = y_0 + p.\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \ldots\ldots + \frac{p(p-1)\ldots\ldots(p-n+1)}{n!}\Delta^n y_0$$

here $p = \frac{x-x_0}{h} > 0$, and x is the value at which we need to estimate the required data and x_0 is the initial value of the table and $x_1, x_2, x_3, \ldots\ldots, x_n$ are the subsequent values of x. Corresponding values of f(x) are denoted by $y_0, y_1, y_2, y_3, \ldots\ldots, y_n$.

$\Delta y_0, \Delta^2 y_0, \Delta^3 y_0, \ldots.. \Delta^n y_0$ can be obtained by the difference table using the following formulas.

$$\Delta y_0 = y_1 - y_0 = f(x+h) - f(x),$$

$$\Delta y_1 = y_2 - y_1 = f(x + 2h) - f(x + h),$$

$$\Delta y_2 = y_3 - y_2 = f(x + 3h) - f(x + 2h),$$

....

$$\Delta y_{n-1} = y_n - y_{n-1} = f(x + nh) - f(x + (n - 1)h)$$

And

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0,$$

$$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0,$$

....

$$\Delta^n y_0 = \Delta^{n-1} y_1 - \Delta^{n-1} y_0$$

Using the above Newton's Forward Interpolation formula one can convert the data obtained into a polynomial and interpolate the data at which sampling / experimentation was not done. This method gives better approximation when x is near to x_0.

But, when there is more number of data points it is difficult to calculate the results manually. Even with the help of a calculator, it is not an easy task. Software's like MATLAB are required to convert the data into Polynomials, but they are costly to the common man.

Spreadsheet

A spreadsheet is essentially a matrix of rows and columns. Consider a sheet of paper on which horizontal and vertical lines are drawn to yield a rectangular grid. The grid namely a cell, is the result of the intersection of a row with a column. Such a structure is called a Spreadsheet.

A spreadsheet package contains electronic equivalent of a pen, an eraser and large sheet of paper with vertical and horizontal lines to give rows and columns. The cursor position uniquely shown in dark mode indicates where the pen is currently pointing. One can enter text or numbers at any position on the worksheet. One can enter a formula in a cell where he/she want to perform a calculation and results are to be displayed. A powerful recalculation facility jumps into action each time whenever the cell contents with new data were updated. MS-Excel is the most powerful spreadsheet package brought by Microsoft.

Microsoft Excel

Microsoft Excel is a commercial spreadsheet application, written and distributed by Microsoft for Microsoft Windows and Mac OS X. Microsoft Excel is a spreadsheet tool capable of performing calculations, analyzing data and integrating information from different programs. By default, documents saved in Excel 2010 are saved with the .xlsx extension whereas the file extension of the prior Excel versions are .xls.

MS- Excel is available as one of the application in the popular package MS- Office of Microsoft. Many of the computer users are familiar with MS-Excel , which is user friendly. Freeware spreadsheet programs like **Libreoffice Calc** are also available, in which most of the features of MS-Excel are available.

Keeping in view of the importance of the Newton Forward Interpolation formula, difficulty in calculating the same when there are more number of data points and features and advantages of MS-Excel, a program in Excel to estimate any intermediate value of a data is very useful to the society/needy.

4. Methodology

As there is need to create a program in Excel to estimate any intermediate value of a data practical method is used to create set of algorithms to compile the program. By default each Excel file contains three sheets and the same can be increased as per our need. More than one Billion cells are available in each sheet of Excel. Each cell can be converted into a calculator by typing "=" symbol. Hence, there are more than one billion calculators in a single Excel sheet. Further many powerful functions such as Paste link, CONCATENATE, VLOOKUP,HLOOKUP are available in Excel. Hence one can interlink these calculators to create powerful programs.

As the MS-Excel is user friendly and readily available to the most of the computer users the program prepared in Excel with algorithms will reach many people easily. Further similar algorithms will execute the program even in the non commercial, free softwares like Libreoffice Calc.

Practical Method is adopted to create the algorithms as detailed below.

Step:1 Creating Data Input Panel

i. Opened a new Excel file

ii. In the sheet opened, selected a range of cells to enter the data of the given problem and change the background of selected cells using cell styles under Home Tab to distinguish them from other cells. [ ] coloured cells are used to enter the input data. Hence the user has to fill [ ] coloured cells only.

As the Newton's forward interpolation formula is used to evenly spaced intervals, it is enough if first two "x" values are entered. The remaining "x" values can be auto filled by using the formulas shown below. The values of "h" and "p" which will be used in Newton's forward interpolation formula can also be Newton's forward interpolation formula.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Data Panel											
2		x			=D2+$I$5	=E2+$I$5	=F2+$I$5	=G2+$I$5	=H2+$I$5	=I2+$I$5	=J2+$I$5	=K2+$I$5	
3		y											
4													
5		x=						h=	=D2-C2		p=	=(C5-C2)/I5	
6													
7													

An empty Data Panel looks like as follows

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Data Panel											
2		x			0	0	0	0	0	0	0	0	
3		y											
4													
5		x=						h=	0		p=	#####	
6													

After entering the data the Data Panel looks like as follows

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Data Panel											
2		x	3	6	9	12	15	18	21	24	27	30	
3		y	10	9	-3	7	6	-5	8	1	-1	6	
4													
5		x=	4.2					h=	3		p=	0.4	
6													
7													

Step:2 Creating Difference Table

i. By using the formulas noted in each cell as shown below Difference Table can be obtained. For example enter " =C2 " in cell B14, enter " =G22-G20 " in cell H21.

ii. For "n" data points we have to get $\frac{n(n-1)}{2}$ first and higher order differences to complete the difference table. Hence for 10 data points we have to get 45 first and higher order differences to complete the difference table.

iii. "n" formulas are needed to auto fill "x" values and "n" more formulas are needed to auto fill "y" values. Thus totally "2n" formulas are needed to auto fill "x" and "y" values in the table. Hence 20 formulas are needed for 10 data points.

iv. Total formulas needed to auto fill the difference table for "n" data points =

$2n + \frac{n(n-1)}{2}$

$$= \frac{n^2 + 3n}{2} = \frac{n(n+3)}{2}$$

In the present case, where 10 data points are used $\frac{10(10+3)}{2} = 65$ formulas are used as shown in each cell.

Difference Table

	A	B	C	D	E	F	G	H	I	J	K	L	M
11													
12							Difference Table						
13		x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$	$\Delta^8 y$	$\Delta^9 y$	
14		=C2	=C3										
15				=C16-C14									
16		=D2	=D3		=D17-D15								
17				=C18-C16		=E18-E16							
18		=E2	=E3		=D19-D17		=F19-F17						
19				=C20-C18		=E20-E18		=G20-G18					
20		=F2	=F3		=D21-D19		=F21-F19		=H21-H19				
21				=C22-C20		=E22-E20		=G22-G20		=I22-I20			
22		=G2	=G3		=D23-D21		=F23-F21		=H23-H21		=J23-J21		
23				=C24-C22		=E24-E22		=G24-G22		=I24-I22		=K24-K22	
24		=H2	=H3		=D25-D23		=F25-F23		=H25-H23		=J25-J23		
25				=C26-C24		=E26-E24		=G26-G24		=I26-I24			
26		=I2	=I3		=D27-D25		=F27-F25		=H27-H25				
27				=C28-C26		=E28-E26		=G28-G26					
28		=J2	=J3		=D29-D27		=F29-F27						
29				=C30-C28		=E30-E28							
30		=K2	=K3		=D31-D29								
31				=C32-C30									
32		=L2	=L3										
33													

v. Using the formulas we can get the Difference Table as follows for the following data given at step-I.

Data Panel

x	3	6	9	12	15	18	21	24	27	30
y	10	9	-3	7	6	-5	8	1	-1	6

x= 4.2	h= 3	p= 0.4	

Difference Table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$	$\Delta^8 y$	$\Delta^9 y$
3	10									
		-1								
6	9		-11							
		-12		33						
9	-3		22		-66					
		10		-33		100				
12	7		-11		34		-101			
		-1		1		-1		-9		
15	6		-10		33		-110		377	
		-11		34		-111		368		-1240
18	-5		24		-78		258		-863	
		13		-44		147		-495		
21	8		-20		69		-237			
		-7		25		-90				
24	1		5		-21					
		-2		4						
27	-1		9							
		7								
30	6									

vi. If we change the data in "Data Panel", the difference table will be generated automatically using new values.

vii. We can extend the above difference table to any number of points using similar formulas.

viii. We can use copy, paste functions carefully to create large tables quickly.

Step-3 Finding the required value

i. Using Newton's Forward Interpolation formula

$$y = f(x) = y_0 + p.\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \ldots\ldots + \frac{p(p-1)\ldots\ldots(p-n+1)}{n!}\Delta^n y_0$$

and carefully linking the data we can get the required value.

ii. First prepare a table for factorials using the formulas as shown below.

	M	N	O	P
13				
14		N	FACTORIALS	
15		1	=FACT(N15)	
16		2	=FACT(N16)	
17		3	=FACT(N17)	
18		4	=FACT(N18)	
19		5	=FACT(N19)	
20		6	=FACT(N20)	
21		7	=FACT(N21)	
22		8	=FACT(N22)	
23		9	=FACT(N23)	
24		10	=FACT(N24)	
25				

We can get the display as follows

	M	N	O	P
13				
14		N	FACTORIALS	
15		1	1	
16		2	2	
17		3	6	
18		4	24	
19		5	120	
20		6	720	
21		7	5040	
22		8	40320	
23		9	362880	
24		10	3628800	
25				

iii. Using the following formulas in B37 and C37 ,

Formula in E5:

=CONCATENATE("f(",C5,")=")

Formula in F5:

=C14+L5*D15+L5*(L5-1)*E16/O16+L5*(L5-1)*(L5-2)*F17/O17+L5*(L5-1)*(L5-2)*(L5-3)*G18/O18+L5*(L5-1)*(L5-2)*(L5-3)*(L5-4)*H19/O19+L5*(L5-1)*(L5-2)*(L5-3)*(L5-4)*(L5-5)*I20/O20+L5*(L5-1)*(L5-2)*(L5-3)*(L5-4)*(L5-5)*(L5-6)*J21/O21+L5*(L5-1)*(L5-2)*(L5-3)*(L5-4)*(L5-5)*(L5-6)*(L5-7)*K22/O22+L5*(L5-1)*(L5-2)*(L5-3)*(L5-4)*(L5-5)*(L5-6)*(L5-7)*(L5-8)*L23/O23

We will get the final out put as

	A	B	C	D	E	F	G	H	I	J	K	L	M
1							Data						
2		x	3	6	9	12	15	18	21	24	27	30	
3		y	10	9	-3	7	6	-5	8	1	-1	6	
4													
5		x=	4.2		f(4.2)=	-0.65667		h=	3		p=	0.4	
6													

iv. While hiding the remaining part we can display the data panel and result only on the screen. Now the screen appears as shown below.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1							Data						
2		x	3	6	9	12	15	18	21	24	27	30	
3		y	10	9	-3	7	6	-5	8	1	-1	6	
4					Result								
5		x=	4.2		f(4.2)=	-0.65667							
6													

v. Thus one can get the result of intermediate values instantly, after entering the data and the value at which result is required.

Step-4 Finding the Polynomial Equation

i. To find out the polynomial equation to the given data first we prepared the following factor multiplication table.

	B	C	D	E	F	G	H	I
37								
38								
39								
40	x	c		c	x	x^2	x^3	x^4
41	1	=-C2		=C41	=B41			
42	1	=-D2		=E41*C42	=E41*B42+C42*F41	=F41*B42		
43	1	=-E2		=E42*C43	=F42*C43+E42*B43	=G42*C43+F42*B43	=G42*B43	
44	1	=-F2		=E43*C44	=F43*C44+E43*B44	=G43*C44+F43*B44	=H43*C44+G43*B44	=H43*B44
45	1	=-G2		=E44*C45	=F44*C45+E44*B45	=G44*C45+F44*B45	=H44*C45+G44*B45	=I44*C45+H44*B45
46	1	=-H2		=E45*C46	=F45*C46+E45*B46	=G45*C46+F45*B46	=H45*C46+G45*B46	=I45*C46+H45*B46
47	1	=-I2		=E46*C47	=F46*C47+E46*B47	=G46*C47+F46*B47	=H46*C47+G46*B47	=I46*C47+H46*B47
48	1	=-J2		=E47*C48	=F47*C48+E47*B48	=G47*C48+F47*B48	=H47*C48+G47*B48	=I47*C48+H47*B48
49	1	=-K2		=E48*C49	=F48*C49+E48*B49	=G48*C49+F48*B49	=H48*C49+G48*B49	=I48*C49+H48*B49
50				=SUM(E41:E49)	=SUM(F41:F49)	=SUM(G41:G49)	=SUM(H41:H49)	=SUM(I41:I49)

	J	K	L	M	N
37					
38					
39					
40	x^5	x^6	x^7	x^8	x^9
41					
42					
43					
44					
45	=I44*B45				
46	=J45*C46+I45*B46	=J45*B46			
47	=J46*C47+I46*B47	=K46*C47+J46*B47	=K46*B47		
48	=J47*C48+I47*B48	=K47*C48+J47*B48	=L47*C48+K47*B48	=L47*B48	
49	=J48*C49+I48*B49	=K48*C49+J48*B49	=L48*C49+K48*B49	=M48*C49+L48*B49	=M48*B49
50	=SUM(J41:J49)	=SUM(K41:K49)	=SUM(L41:L49)	=SUM(M41:M49)	=SUM(N41:N49)

The output of the above table will be as follows

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
38														
39														
40		x	c		c	x	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
41	1	1	-3		-3	1								
42	2	1	-6		18	-9	1							
43	3	1	-9		-162	99	-18	1						
44	4	1	-12		1944	-1350	315	-30	1					
45	5	1	-15		-29160	22194	-6075	765	-45	1				
46	6	1	-18		524880	-428652	131544	-19845	1575	-63	1			
47	7	1	-21		-11022480	9526572	-3191076	548289	-52920	2898	-84	1		
48	8	1	-24		264539520	-239660208	86112396	-16350012	1818369	-122472	4914	-108	1	
49	9	1	-27		-7142567040	6735365136	-2564694900	527562720	-65445975	5125113	-255150	7830	-135	1
50					-6888552483	6504823783	-2481647813	511741888	-63678995	5005477	-250319	7723	-134	1

ii. Using the values in the above table, we constructed terms of Newton's Forward Interpolation Formula with the formulas noted in each cell as shown below.

	D	E	F	G	H	I	J	K	L	M	N
56	Term	C	X	X^2	X^3	x^4	x^5	x^6	x^7	x^8	x^9
57	1	=C14									
58	2	=E41*U58	=F41*U58								
59	3	=E42*U59	=F42*$U59	=G42*$U59							
60	4	=E43*U60	=F43*U60	=G43*$U60	=H43*$U60						
61	5	=E44*U61	=F44*U61	=G44*$U61	=H44*$U61	=I44*$U61					
62	6	=E45*U62	=F45*U62	=G45*$U62	=H45*$U62	=I45*$U62	=J45*$U62				
63	7	=E46*U63	=F46*U63	=G46*$U63	=H46*$U63	=I46*$U63	=J46*$U63	=K46*$U63			
64	8	=E47*U64	=F47*U64	=G47*$U64	=H47*$U64	=I47*$U64	=J47*$U64	=K47*$U64	=L47*$U64		
65	9	=E48*U65	=F48*U65	=G48*$U65	=H48*$U65	=I48*$U65	=J48*$U65	=K48*$U65	=L48*$U65	=M48*$U65	
66	10	=E49*U66	=F49*U66	=G49*$U66	=H49*$U66	=I49*$U66	=J49*$U66	=K49*$U66	=L49*$U66	=M49*$U66	=N49*$U66
67											
68		=SUM(E57:E67)	=SUM(F57:F67)	=SUM(G57:G67)	=SUM(H57:H67)	=SUM(I57:I67)	=SUM(J57:J67)	=SUM(K57:K67)	=SUM(L57:L67)	=SUM(M57:M67	=SUM(N57:N67)

	P	Q	R	S	T	U
56						
57		NUMBERS	FACTORIALS	h	$\Delta^n y$	Coefficient
58		1	=FACT(Q58)	=I5	=D15	=T58/(R58*S58)
59		2	=FACT(Q59)	=POWER(S$58,Q59)	=E16	=T59/(R59*S59)
60		3	=FACT(Q60)	=POWER(S$58,Q60)	=F17	=T60/(R60*S60)
61		4	=FACT(Q61)	=POWER(S$58,Q61)	=G18	=T61/(R61*S61)
62		5	=FACT(Q62)	=POWER(S$58,Q62)	=H19	=T62/(R62*S62)
63		6	=FACT(Q63)	=POWER(S$58,Q63)	=I20	=T63/(R63*S63)
64		7	=FACT(Q64)	=POWER(S$58,Q64)	=J21	=T64/(R64*S64)
65		8	=FACT(Q65)	=POWER(S$58,Q65)	=K22	=T65/(R65*S65)
66		9	=FACT(Q66)	=POWER(S$58,Q66)	=L23	=T66/(R66*S66)
67						

iii. The output will be displayed as follows.

	C	D	E	F	G	H	I	J	K	L	M	N
56		Term	C	X	X^2	X^3	x^4	x^5	x^6	x^7	x^8	x^9
57		1	10.0000000000									
58		2	1.0000000000	-0.3333333333								
59		3	-11.0000000000	5.5000000000	-0.6111111111							
60		4	-33.0000000000	20.1666666667	-3.6666666667	0.2037037037						
61		5	-66.0000000000	45.8333333333	-10.6944444444	1.0185185185	-0.0339506173					
62		6	-100.0000000000	76.1111111111	-20.8333333333	2.6234567901	-0.1543209877	0.0034293553				
63		7	-101.0000000000	82.4833333333	-25.3123456790	3.8186728395	-0.3030692730	0.0121227709	-0.0001924249			
64		8	9.0000000000	-7.7785714286	2.6055555556	-0.4476851852	0.0432098765	-0.0023662551	0.0000685871	-0.0000008165		
65		9	377.0000000000	-341.5440476190	122.7203152557	-23.3006944444	2.5913901749	-0.1745370370	0.0070030293	-0.0001539127	0.0000014251	
66		10	1240.0000000000	-1169.3068783069	445.2491181658	-91.5886080084	11.3618827160	-0.8897557537	0.0442958390	-0.0013593432	0.0000234370	-0.0000001736
67												
68			1326.0000000000	-1288.8683862434	509.4570877425	-107.6726357861	13.5051418896	-1.0511069197	0.0511750305	-0.0015140725	0.0000248621	-0.0000001736

	P	Q	R	S	T	U	V
56							
57		NUMBERS	FACTORIALS	h	$\Delta^n y$	Coefficient	
58		1	1	3	-1	-0.333333333333	
59		2	2	9	-11	-0.611111111111	
60		3	6	27	33	0.203703703704	
61		4	24	81	-66	-0.033950617284	
62		5	120	243	100	0.003429355281	
63		6	720	729	-101	-0.000192424935	
64		7	5040	2187	-9	-0.000000816513	
65		8	40320	6561	377	0.000001425118	
66		9	362880	19683	-1240	-0.000000173607	
67							

iv. To get the polynomial for the given data we use the following formulas

	D	E	F	G	H	I	J	K	L	M	N
67											
68		=SUM(E57:E67)	=SUM(F57:F67)	=SUM(G57:G67)	=SUM(H57:H67)	=SUM(I57:I67)	=SUM(J57:J67)	=SUM(K57:K67)	=SUM(L57:L67)	=SUM(M57:M67)	=SUM(N57:N67)
69		=IF(E68>0,"+","-")	=IF(F68>0,"+","-")	=IF(G68>0,"+","-")	=IF(H68>0,"+","-")	=IF(I68>0,"+","-")	=IF(J68>0,"+","-")	=IF(K68>0,"+","-")	=IF(L68>0,"+","-")	=IF(M68>0,"+","-")	
70		=ABS(E68)	=ABS(F68)	=ABS(G68)	=ABS(H68)	=ABS(I68)	=ABS(J68)	=ABS(K68)	=ABS(L68)	=ABS(M68)	=ABS(N68)
71											

In E72 we used the following formula to get the polynomial

=CONCATENATE("(",N68,")",N56,M69,"(",M70,")",M56,L69,"(",L70,")",L56,K69,"(",K70,")",K56,J69,"(" ,J70,")",J56,I69,"(",I70,")",I56,H69,"(",H70,")",H56,G69,"(",G70,")",G56,F69,"(",F70,")",F56,E69,E70)

Final output displayed as

	D	E	F	G	H	I	J	K	L	M	N
67											
68		1326.0000000000	-1288.8683862434	509.4570877425	-107.6726357861	13.5051418896	-1.0511069197	0.0511750305	-0.0015140725	0.0000248621	-0.0000001736
69		+	-	+	-	+	-	+	-	+	
70		1326.0000000000	1288.8683862434	509.4570877425	107.6726357861	13.5051418896	1.0511069197	0.0511750305	0.0015140725	0.0000248621	0.0000001736

And required polynomial is

	C	D	E	F	G	H
71						
72			(-1.7360705094621E-07)x^9+(0.000024862069758046)x^8-(0.00151407245314424)x^7+(0.0511750304831581)x^6-(1.05110691967688)x^5+(13.5051418895748)x^4-(107.672635786139)X^3+(509.457087742504)X^2-(1288.86838624339)X+1326			

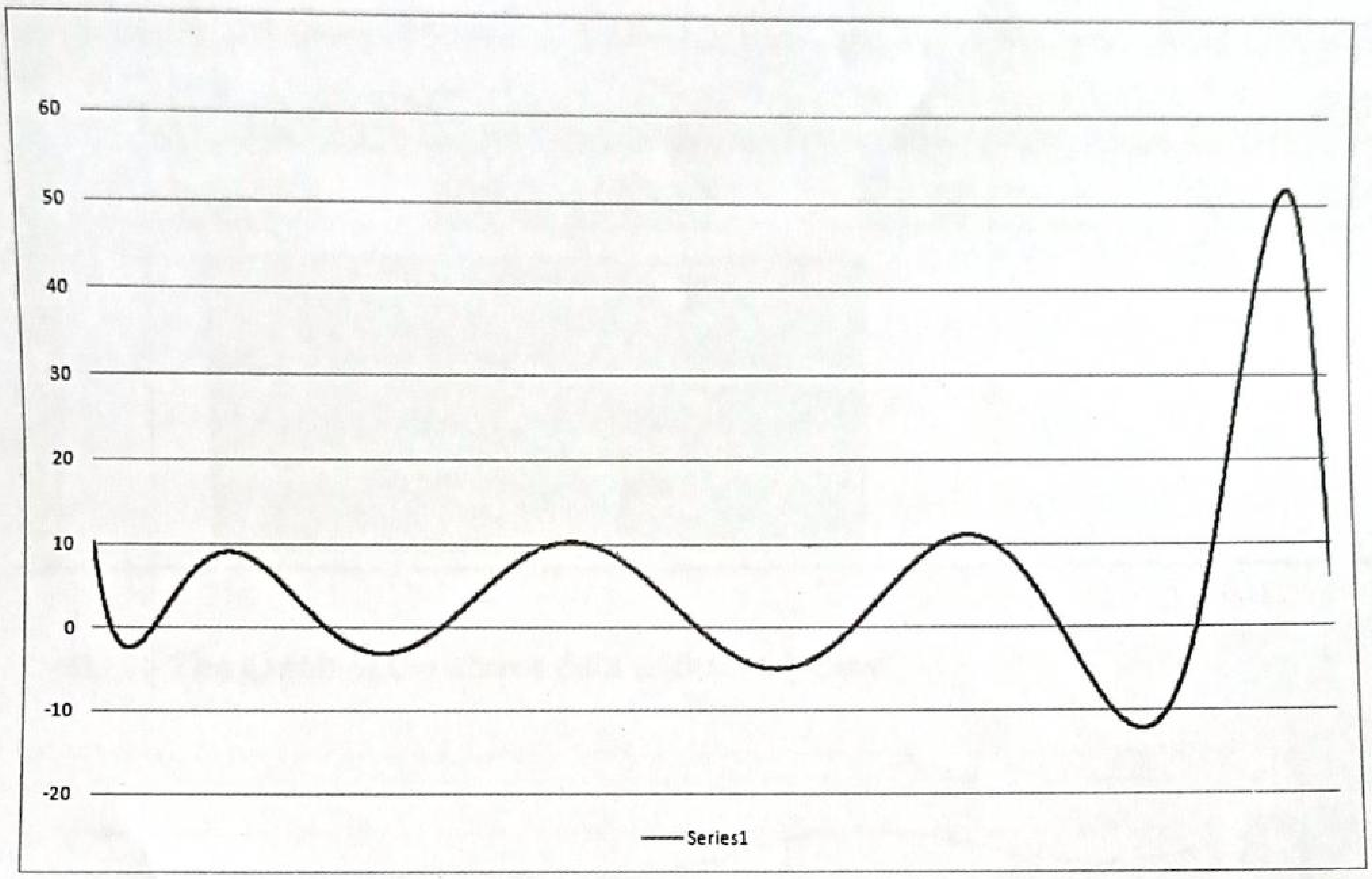
5. <u>Findings & Limitations</u>

i. Using the program prepared in this project, one can get the result of intermediate values instantly, after entering the data and the value at which result is required.

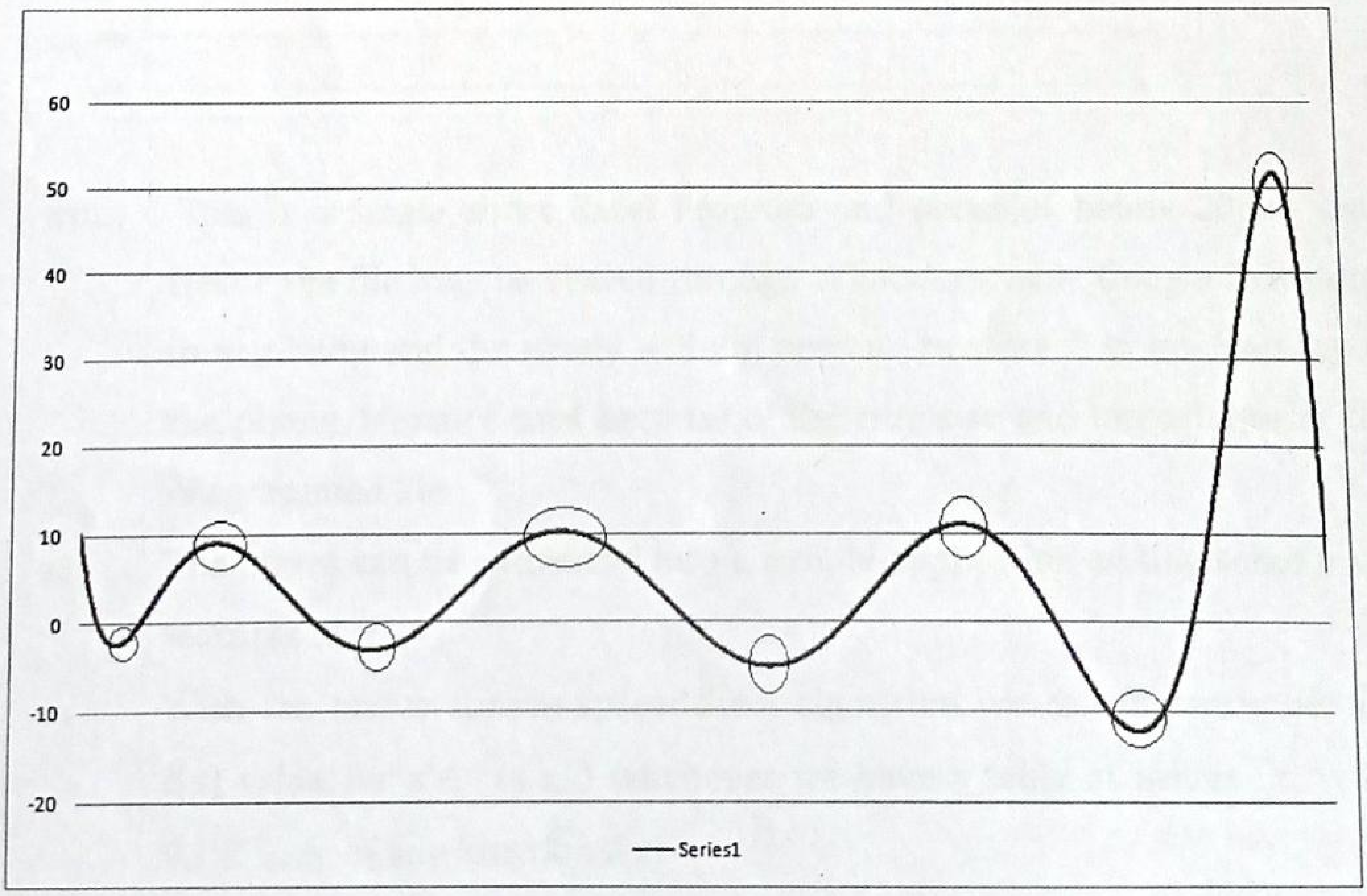
ii. This project will also give the polynomial to the given data, which is an algebraic form of the data.

iii. The above algebraic Polynomial can be used to draw graphs and by finding its derivative we can find local minima & maxima of the data, which has significant applications like Target and stop loss in stock market and other financial markets.

iv. For the data used above we got the following graph using the polynomial of the data.



v. We can see the local maxima, local minima points in the above graph clearly.



vi. If we change the data in Data panel, we can get the Polynomial and graphs instantly.

Data

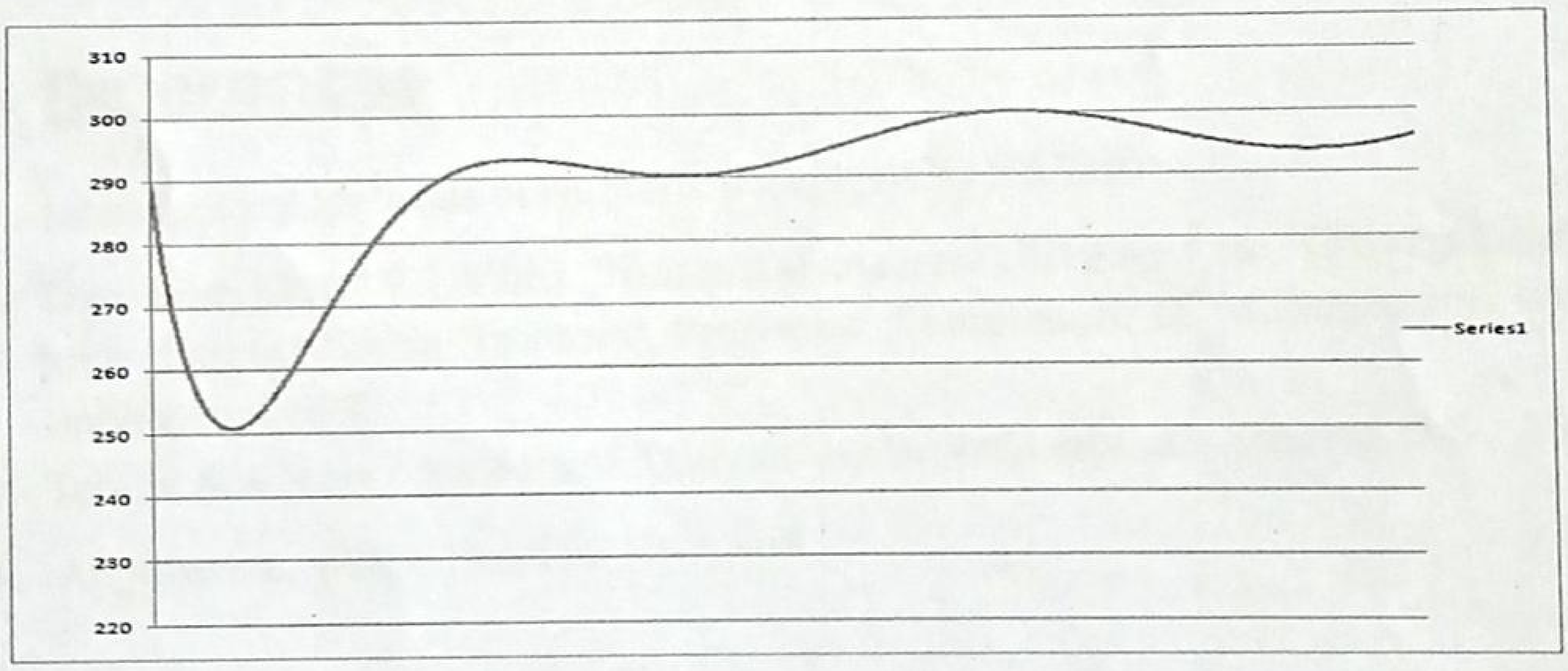
x	1	6	11	16	21	26	31	36	41	46
y	289.6	282.7	291	295.4	299.3	293.7	303.35	297	293.5	291.5

x= 7 | f(7)= 289.33063 | h= 5 | p= 1.2

Polynomial of the given data

(-1.76747795414463E-09)x^9+(3.68891428571431E-07)x^8-(3.24418937566139E-05)x^7+(0.0015645772088889)x^6-(0.0451047279614817)x^5+(0.794270542702227)x^4-(8.3618656672548)X^3+(48.9447753655978)X^2-(132.914524772003)X+381.180916756481

vii. The graph of the above data is shown below.



viii. This is a single sheet Excel Program and occupies below 200kb size. Hence the file may be shared through WhatsApp, mail, Google Drive etc., to any body and the needy will not hesitate to store it in his memory of the phone/Memory card because of the tiny size and importance of the programmed file.

ix. The above can be converted into a mobile application adding some more features to it.

x. With the above simple spreadsheet algorithm we can instantly get any f(x) value for $x \in (x_0, x_9)$ whenever we have a table of values $(x_i, y_i)i = 0,1,2, \ldots .9$ of any function f(x)

xi. Using the above algorithm we can accommodate exactly 10 (x_i, y_i)s only. We need to extend the above algorithm if more or less (x_i, y_i)s available.

6. Conclusion

One can instantly find out any intermediate value of any data using the above spreadsheet based programme. The program can be executable in MS- Excel or any other similar software's like Libreoffice calc.

The above program is executable even in smart phones where spreadsheets are available. It occupies below 200kb space and hence it can be used in any android/smart phone without consuming much space. Thus the Excel file programmed through this project is very handy and gives instant results at the fingertips of the needy.

7. References

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3. Telugu Academy " Numerical Analysis" by Dr.Vedanabhatla Srinivas,M.Sc., Ph.D
4. Applications of MS-Excel by Microsoft.

Student Study Project
(2017-18)

Title of the project: Ramanujan's Contribution to Mathematics World

Group: B.SC (MPC)

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1

Ramanujan's contribution to Mathematics World:-

Srinivasa Ramanujan was on of India's gratest Mathematical geniuses. He made substantial contributions to the analytical theory of numbers and worked on elliptic functions, continued fractions, and infinite series

Ramanujan was born in his grand mother's house in Erode a small village about 400km southwest of Madras (now chennai) when Ramanujan was a year old his mother took him to the town of Kumbkonam, about 160 km nearer madras. His father worked in kumbakonam as a clerk in a cloth merchant's shop In December 1889 he contracted small pox.

When he was nearly five years old, Ramanujan enterd the primary school in kumbakonam although he would attend several different primary schools before entering the town High school in kumbakonam in January 1898. At the town High school Ramanujan was to do well in all his school subjects and showed himself an able all round scholar. In 1900 he began to work on his own on Mathematics summing geometric and arithemetic series

Ramanujan was shown how to solve cubic equations in 1902 and we went on to find his own method to solve the quatric The following year not knowing that the quintic could not be solved by radicals he tried (and of course faild) to solve the quintic

It was in the town High school that Ramanujan came across a mathematics book by G.S carr called synopsis of elementary results in pure mathematics. This book, with its very concise style allowed Ramanujan to teach himself mathematics, but the style of the book was to have a rather unfortunate effect on the way Ramanujan was later to write down mathematics since it provided the only model that the had of written mathematical arguments The book contained theorems formulae and short proofs It also contained an index to papers on pure mathematics which had been published in the European Journals of learned societies during the first half of the 19th century. The book published in 1886 was of course well out of date by the time Ramanujan used it.

By 1904 Ramanujan had begun to undertake deep research He investigated the series $\Sigma(\frac{1}{n})$ and calculated Euler's constant to 15 decimal places. He began to study the Bernoulli numbers. although this was entirely his own independent discovery.

Continuing his mathematical work Ramanujan studied continued fractions and divergent series in 1908. At this stage He became seriously ill again and underwent an operation in April 1909 after which he took him some considerable time to recover. He married on 14 July 1909 when his mother arranged for him to marry a ten year old girls Ammal.

Ramanujan continued to develope his Mathematical ideas and began to pose problems and solve problems in the Journal of the Indian Mathematical society. He developed relations between elliptic Modular equations in 1910. After publication of a brilliant reasearch paper on Bernoulli numbers in 1911 in the Journal of the Indian mathematical society he gained recognition for his work. Despite his lack of a university education he was becoming well known in the madras area as a Mathematical genius.

In 1911 Ramanujan approached the founder of the Indian Mathematical society for advice on a Job. After this he was appointed to his first Job, a temporary post in the Accountant General's office in Madras It was then suggested that he approach Ramachandra Rao who was a collecter at Nellore. Ramachandra Rao was a founder member of the Indian Mathematical Society who had helped start the

Mathematics library.

Ramanujan compiled around 3,900 results consisting of equations and identities one of his most treasured findings was his infinite series for pi, this series forms the basis of many algorithms we used today He gave several fascinating formulas to calculate the digits of pi in many unconventional ways.

⇒ He discovered along list of new ideas to solve many challenging mathematical problems. which gave theory is purely based on intuition and natural talent and remains unrivalled to this day.

*⇒ He elaborately described the mock theta function which is a concept in the realm of modular form in Mathematics considered an enigma till sometime back, it is now recongnized as Holomorphic parts of mass forms

*⇒ one of Ramanujan's note books was discovered by george Andrew in 1976 in libray at trinity college Later the contents of this note book were published as a book.

*⇒ 1729 is known as the Ramanujan number it is the sum of the cubes of two numbers 10 and 9.

For instance, 1729 results from adding 1000 (the cube of 10) and 729 (the cube of 9) This is the smallest number that can be expressed in two different ways as it is the sum of these two cubes. Interestingly, 1729 is a natural number following 1728 and preceding 1730.

*⇒ Ramanujan's contributions stretch across mathematics field. including complex analysis, number theory. infinite series, and continued fractions.

Ramanujan's other notable contributions include hyper geometric series the Riemann series the elliptic integrals the theory of divergent series and the functional equations of the zeta function

Student study Project
(2017-18)

Title of the project: "Pi"

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About π (Pie)

$$\pi = \frac{\text{circumference of the circle}}{\text{diameter of the circle}}$$

the number π is a Mathematical constant. It is defined as the ratio of a Circles Circumferasee to its diameter. and is also has various equivalent definitions. It appears in many formulas in all areas of Mathematics and physics. The earliest known use of the Greek letter π to represent the ratio of a circle's circumference to its diameter was by welsh mathematician william Jones in 1706 It is approximately equal to 3.14159 It has been represented by the Greek π since the mid-18th century and is spelled out as "pi" It is also refferred to as "Archimedes constant"

Being an irrational number, π cannot be expressed as a common fraction although fractions such as 22/7 are commonly used to approximate it equivalently its decimal representation never ends and never settls in a permanently repeating pattern Its decimal digits appear to be randomly distributed and are conjectured to satisfy a specific kind of statistical randomness.

It is known that π is a trancedental number. it is not the root of any polynomial with rational coefficiants. the trancedence of π implies that it is impossible to solve that ancient challenge of squaring the circle with a compass and straightedge.

Ancient civilizations including the Egyptian's and Babylonians required fairly accurate approximations of π for practical computations Around 250 BC the Greek mathematician Archimedes created an algorithm to approximate π with arbitrary accuracy In the 5th century AD, chinese mathematics approximated π to seven digits while Indian mathematics madea fix digit approximation. both using geometrical techniques the first exact formula for π, based on infinite series. was discovered a millennium later, when in the 14th century the madhava - Leibniz series was discovered in indian Mathematics. the invention of calculus soon led to the calculation of hundreds of digits of π, enough for all practical scientific computations. Never the less in the 20th and 21st centuries mathematics and computer scientists have pursued new approches that, when combined with increasing computational power, extended the decimal representation of π to many trillions of digits The primary motivation for these computations is as a test case to develop efficient algorithms to calculate numeric series, as well as the quest to break records. the extensive calculations involved have also been used to test super computers an high-precision multiplication algorithms . Because its most elementary definition relates to the circle π is found in many formulae in trigonometry and gemetrig especially those concerning circles ellipses, and spheres in more modern mathematical analysis the number is instead defined using the spectral properties of the real number system, as an eigenvalue or a period without any reference to geometry It appears the refer in area of mathematics and sciences having little to do with geometry of of circles such as number theory and statistics as well as in almost all areas of physics the ubiquity of π makes it one of the most widley known mathematical constant - both inside and out side the scientific community several books devoted to π have been published and record-setting calculations of the digits of π often result in news headlines Adepts have succeeded in memorizing the value of π to over

70,000 digits

* Student student project *

(2016-17)

Project name: RAMANUJAN BIOGRAPHY

Group :- MPC [I/m]

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Srinivasa Ramanujan

Born : 22 December 1887
Erode, Madras Presidency, British Raj
(Now Tamil Nadu, India)

Died : 26 April 1920 (aged 32)
kumbakonam, Madras Presidency, British Raj

Residence : kumbakonam, Madras Presidency Madras,
Madras presidency London, United kingdom

Nationality : Indian

Fields : Mathematics

Institutions : Trinity college, cambridge Govt Arts college (no degree)

Alma mater : Pachaiyappa's college (no degree)
Trinity college, cambridge (BSc, 1916)

Thesis : Highly Composite Numbers (1916)

Acadamic advisors : G.H. Hardy
J.E. Littlewood
Landau-Ramanujan constant
Mock theta functions
Ramanujan conjecture
Ramanujan prime

Known for : Ramanujan-soldner. constant
Ramanujan-theta function
Ramanujan's sum
Rogers- Ramanujan identities
Ramanujan's master theorem

Influences : G. S. Carr
Influenced : G.H. Hardy
Notable awads : Fellow of the Royal society

Early life

Ramanujan's home on Sarangapani Sannidhi street, Kumbakonam.

Ramanujan was born on 22 December 1887 into a Tamil Brahmin Iyengar family in Erode, Madras Presidency (now Tamil Nadu), at the residence of his maternal grandparents. His father, K. Srinivasa Iyengar, worked as a clerk in a sarishop and hailed from Thanjavur district. His mother, Komalatammal was a housewife and also sang at a local temple. They lived in a small traditional home on Sarangapani Sannidi street in the town of Kumbakonam. The family home is now a museum. When Ramanujan was a year and a half old, his mother gave birth to a son, Sadagopan, who died less than three months later. In December 1889, Ramanujan contracted smallpox, but unlike the thousands in the Thanjavur distict who died of the disease that year, he recovered. He moved with his mother to her parent's house in Kanchipuram, near Madras (now Chennai). His mother gave birth to two more children, in 1891 and 1894, but both died in infancy.

On 1 October 1892, Ramanujan was enrolled at the local school. After his maternal grandfather lost his job a court official in kanchipuram, Ramanujan and his mother moved back to Kumbakonam and he was enrolled in the Kangayan Primary School.

Since Ramanujan's father was at work most of the day, his mother took care of the boy as a child. He had a close relationship with her. From her, he learned about tradition and puranas. He learned to sing religious Songs, to attend pujas at the temple, and to maintain Particular eating habits - all of which are part of Brahmin culture. At the Kangayan Primary school, Ramanujan Performed well. He passed his primary examinations he entered Town Higher Secondary school, where he encountered formal mathematics for the first time.

It was in 1910, after a meeting between the 23-year-old Ramanujan and the founder of the Indian Mathematical society, V. Ramaswamy Aiyer, also known as Professor Ramaswami, that Ramanujan started to get recognition with in the mathematics circles of Madras, Subsequently leading to his inclusion as a researcher at the University of Madras.

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Life in England

Ramanujan (centre) with other scientists at Trinity College

Whewell's Court, Trinity College, Cambridge

He was elected "for his investigation in Elliptic functions and the theorey of Numbers." On 13 October 1918, he was the first Indian to be elected a Fellow of Trinity college, cambridge.

Mathematical achievements

In mathematics, there is a distinction between having an insight and having a proof. Ramanujan proposed an abundance of formulae that could be investigated later in death. G.H Hardy said that Ramanujan's discovaries are unsually rich and that there is often more to tham than intially meets the eye. As a byproduct of his work, new directions of research were opened up. Examples of the most interesting of these formulae include the intriguing infinite series for π.

In 1918 Hardy and Ramanujan studied the partition function P(n) extensively. They gave a non convergent asymptotic series that permits exact computation of the number of partitions of an integer. Hans Rademacher, in 1937, was able to refine their formula to find an exact convergent series solution to this problem. Ramanujan and Hardy's work in this area gave rise to a powerful new method for finding asymptotic formulae called the circle method.

In the last year of his life, Ramanujan discovered mock theta functions. For many years these functions were a mystery, but they are now known to be the holomorphic parts of harmonic weak Maass forms.

Ramanujan's home state of Tamil Nadu celebrates 22 December (Ramanujan's birthday) as "state IT Day". A stamp picturing Ramanujan was released by the Govt of India in 1962, the 75th anniversary of Ramanujan's birth - commemorating his achievements in the field of number theory, and a new design was issued on 26 December 2011, by the India Post.

In 2011, on the 125th anniversary of his birth, the Indian Government declared that 22 December will be celebrated every year as National Mathematics Day. Then Indian Prime Minister Manmohan singh also declared that the year 2012 would be celebrated as the National Mathematics day.

Illness and death

Throughout his life, Ramanujan was plagued by health problems. His health worsened in England. He was diagnosed with tuberculosis and a severe vitamin deficiency, and was confined to a sanatorium. In 1919 he returned to Kumkonam, Madras Presidency, and soon thereafter, in 1920, died at the age of 32. After his death, his brother Tirunarayanan chronicled

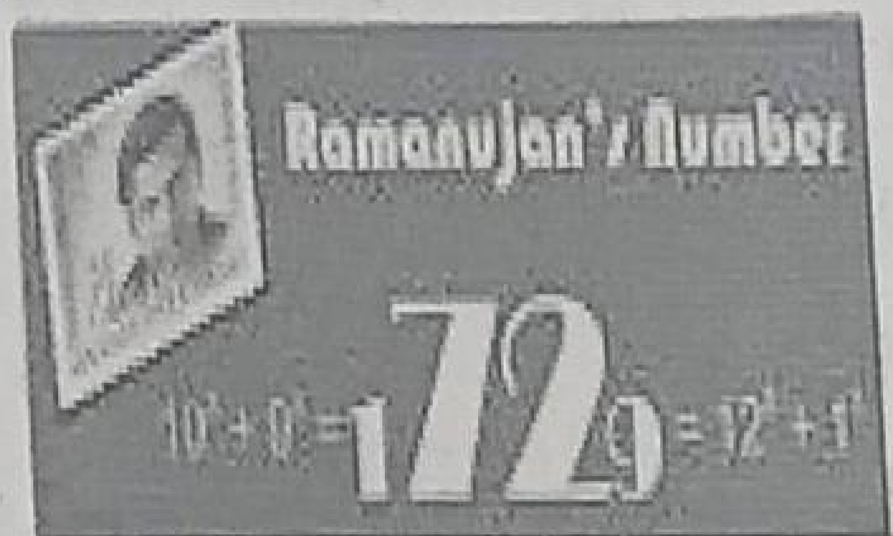
Hardy – Ramanujan number 1729

Main Article : 1729 (number)

The number 1729 is known as the Hardy – Ramanujan number after a famous visit by Hardy to see Ramanujan at a hospital. In Hardy's words.

I remember once going to see him when he was ill at Putney. I had ridden in taxi cap number 1729 and remalked that the number seemed to me rather a dull one, and that I hoped it was not an unfavarable omen. "No" he replied, it is a very interesting number, it is the smallest number expressible as the sum of two cubes in two different ways.

Immediately before this anecdote, Hardy quoted Littlewood as saying, 'Every Positive integer was one of [Ramanujan's] personal friends.



The two different ways are

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

Generalizations of this idea have created the notion of "taxicab numbers".

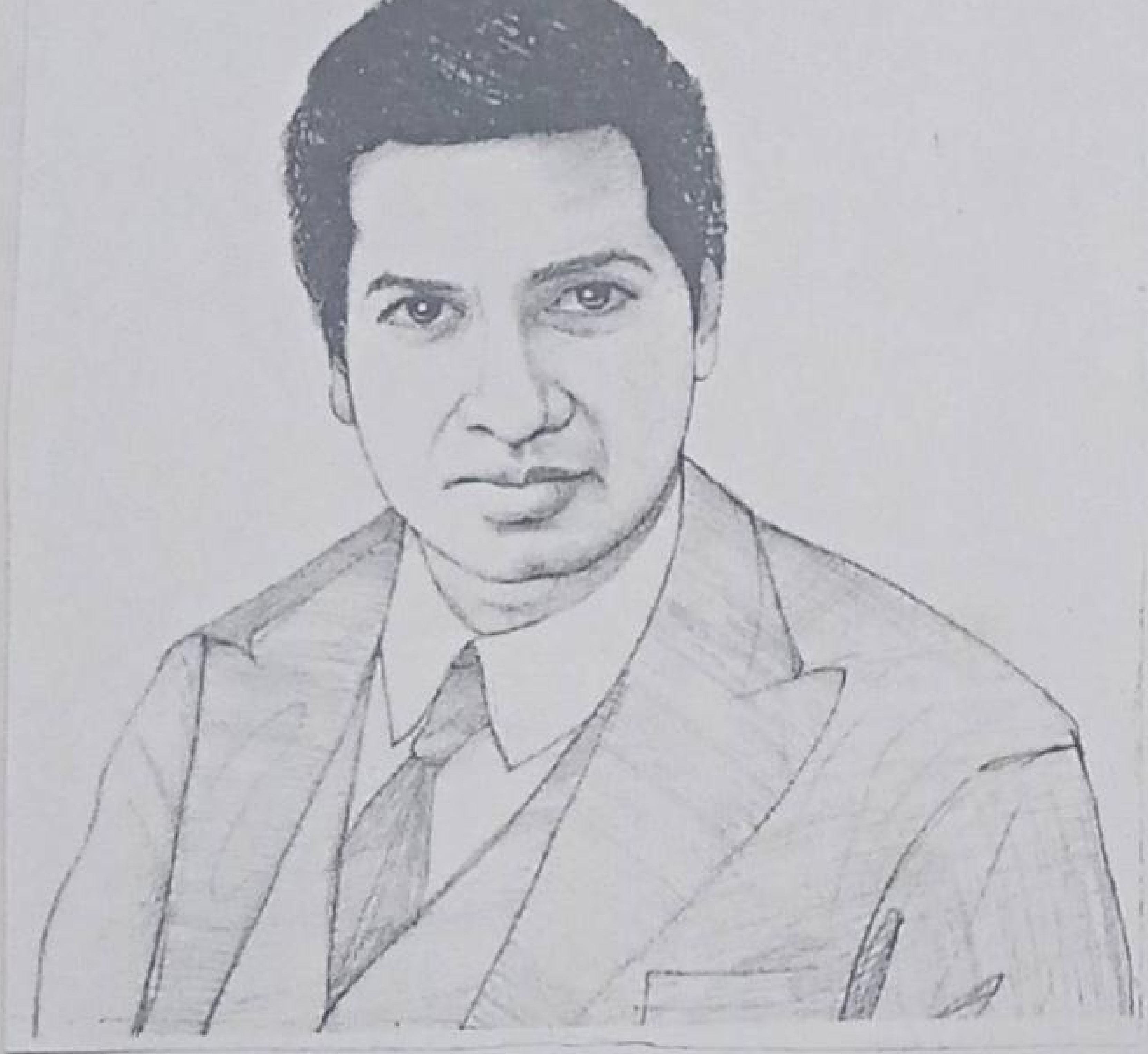
Posthumous recognition

Further information :

List of things named after Srinivasa Ramanujan. Bust of Ramanujan in the garden of Birla Industrial & Technological Museum.

Scanned by TapScanner

Ramanujan's remaining handwritten notes consisting of formulae on singular moduli, hypergeometric series and continued fractions and compiled them. Ramanujan's widow, S. Janaki Ammal, moved to Bombay ; in 1950 she returned to chennai (formerly Madras), where she lived in Triplicane untill her death in 1994 at the age 95



A 1994 analysis of Ramanujan's medical records symptoms by Dr. D.A. Young concluded that it was much more likely he had hepatic amoebiasis, an illness then widespread in Madras, rather than tuberculosis. He had two episodes of dysentery before he left India. When not properly treated, dysentery can lie dormant for years and lead to hepatic amoebiasis. Amoebiasis was a treatable and often curable disease at the time.

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Group - MPCS (BSC)

2nd Year

About Srinivasa Ramanujan

Srinivasa Ramanujan

Childhood & Early Life

Srinivasa Ramanujan was born on 22 December 1887 in Erode, Madras presidency, to K. Srinivasa Iyengar and his wife Komalatammal. His family was a humble one and his father worked as a clerk in a sari shop. His mother gave birth to several children after ramanujan, but none of them survived infancy.

Ramanujan contracted smallpox in 1889 but recovered from the potentially fatal disease. While a young child, he spent considerable time in his maternal grandparents home.

He started his schooling in 1892. Initially he did not like school though he soon started

Excelling in his studies, especially mathematics. After passing out of kangayan primary school, he Enrolled at town Higher Secondary School in 1897. He soon discovered a book on advanced trigonometry written by S.L. Loney which he mastered by the time he was 13. He proved to be brilliant student and won several merit certi-ficates and academic awards.

In 1903, he got his hands on a book called 'A synopsis of Elementary results in pure and Applied Mathematics' by G.S. Carr which was a collection of 5000 theorems. He was thoroughly fascinated by the book and spent months studying it in detail. This book is credited to have awakened the mathematical genius in him.

By the time he was 17, he had independently developed and investigated the Bernouli numbers and had calculated the Euler - Mascheroni constant up to 15 decimal places. He was now no longer interested in any other subject, and totally immersed himself in the study of mathematics only.

He graduated from Town Higher Secondary School in 1904 and was awarded the K. Ranga-natha Rao prize for mathematics by the school's headmaster, Krishnaswami Iyer.

He went to the Government Arts college, Kumba-konam, on scholarship. However, he was so preocc-upied with mathematics that he could not focus on any other subject, and failed in most of them. Due to this, his scholarship was revoked.

He later enrolled at Pachaiyappa's college in Madras where again he excelled in mathematics, but performed poorly in other subjects. He failed to clear his fellow of Arts exam in December 1906 and again a year later. Then he left college

without a degree and continued to pursue indep-
-endent research in mathematics.

Later years

After dropping out of college, he struggled to make a living and lived in poverty for a while. He also suffered from poor health and had to undergo a surgery in 1910. After recuperating, he continued his search for a job.

He tutored some college students while desper-
-ately searching for a clerical position in Madras. Finally he had a meeting with deputy collector V. Ramaswamy Aiyer, who had recently founded the Indian Mathematical Society. Impressed by the young man's works. Aiyer sent him with letters of introduction to R. Ramachandra Rao, the district collector of Nellore and the secretary of the Indian Mathematical society.

Rao, though intially skeptical of the young man's abilities soon changed his mind after Ramanujan discussed elliptic integrals, hypergeometric series, and his theory of divergent series with him. Rao agreed to help him get a job and also promised to finan-cially fund his research.

Ramanujan got a clerical post with the Madras port Trust, and continued his research with the financial help from Rao. His first paper, a 17- page work on Bernoulli numbers, was published with the help of Ramaswamy Aiyer, in the 'journal of the Indian Mathematical society' in 1911.

The publication of his paper helped him gain attention for his works, and soon he was popular among the mathematical fraternity in India. Wishing to further Explore research in mathematics, Ramanujan began a correspondence with the acclaimed English mathematician, Godfrey H. Hardy in 1913.

Hardy was very impressed with Ramanujan's work and helped him get a special scholarship from the university of Madras and a grant from Trinity College, Cambridge. Thus Ramanujan travelled to England in 1914 and worked alongside Hardy who mentored and collaborated with the young Indian.

In spite of having almost no formal training in mathematics, Ramanujan's knowledge of mathematics was astonishing. Even though he had no knowledge of the modern developments in the subject, he effortlessly worked out the Riemann series, the elliptic integrals, hypergeometric series, and the functional equations of the zeta function.

However, his lack of formal training also meant that he had no knowledge of doubly periodic functions, the classical theory of quadratic forms, or cauchy's theorem. Also, several of his theorem on the theory of prime numbers were wrong.

In England, he finally got the opportunity to interact with other gifted mathematicians like his mentor, Hardy and made several further advances, especially in the partition of numbers. His papers were published in European journals, and he was awarded a Bachelor of science degree by research in march 1916 for his work on highly composite numbers. His brilliant career was however cut short by his untimely death.

Major works

Considered to be a mathematical genius, Srinivasa Ramanujan, was regarded at par with the likes of Leonhard Euler and carl Jacobi. Along with Hardy, he studied the partition function p(n) extensively and gave a non-convergent asymptotic series that permits exact computation of the number of partitions of an integer. Their work led to the development of a new method for finding asymptotic formulae, called the circle method.

Awards & Achievements

He was elected a fellow of the Royal society in 1918, as one of the youngest fellows in the history of the Royal society. He was elected "for his investigation in elliptic functions and the theory of Numbers".

The same year, he was also elected a fellow of trinity college - the first Indian to be so honored.

Personal Life & Legacy

He was married to a ten-year-old girl named janakiammal in July 1909 when he was in his early 20s. The marriage was arranged by his mother. The couple did not have any have children.

Ramanujan suffered from various health problems, throughout his life. His health declined considerably while he was living in England as the climatic conditions did not suit him. Also, he was a vegetarian who found it extremely difficult to obtain nutritions vegetarian food in England.

He was diagnosed with tuberculosis and a severe vitamin deficiency during the late 1910s and returned home to madras in 1919. He never fully recovered and breathed his last on 26 April 1920, aged just 32.

His birthday 22 December, is celebrated as 'state IT Day" in his home state of Tamil Nadu. On the 125th anniversary of his birth, India declared his birthday as 'National Mathematics Day'.