2016-17

M.A.L.D.GOVT. DEGREE COLLEGE GADWAL.

Affiliated to Palamuru University, Mahabubnagar.

Department of Mathematics

STUDENT STUDY PROJECT

NAME OF TITLE: Uses of Geometry in Real life

NAME OF THE STUDENTS: Umera Taheeen III B.Sc (MPCs)
Karishma III B.Sc (MPCs)
Sumalatha III B.Sc (MPC)

Department of Mathematics

PRINCIPAL
M.A.L.D. Govt. Arts & Science College
GADWAL - 509 125

MALD Govt. Degree College, Gadwal
Jogulamba Gadwal (Dist), T.S.

SYNOPSIS OF STUDENT STUDY PROJECT.

Name of Title: Uses of Geometry in Real life.

Name of the Students: Umera Tahseen : III Bsc (MPCs)
Karishma : III Bsc (MPCs)
Sumalatha : III Bsc (MPC)

— x — x — x —

In this project work students briefly studied about Geometry, they observed how Geometry figures and formulas are used in Construction of college buildings, Dams, Bridges etc. They also discussed about how Contractors estimate cost of those constructions by using Geometrical formulas.

Students observed that in vedas, Shulba Sutras which explain about how to Contruct Fire Altar, Uses geometric figures.

Further in this project work students discussed about Geometric applications in Egypt Pyramid, Great wall of China Eiffel ~~to~~ tower, Tajmahal...etc, Constructions. They also discussed about history of Geometry, such as father of Geometry – Euclid, Rene descarts and other famous Scientists involved in development of Geometry

Vijay. V
Incharge in Mathematics
Govt. Degree College
GADWAL - 509125

గణిత శాస్త్రం ఎన్నో భాగాల అధ్యయనం. వాటిలో ఒక భాగం "జ్యామితి"

పరిచయం :-

వంతెనలు, ఆనకట్టలు, పాఠశాల భవనాలు, వసతి గృహాలు మరియు ఆసుపత్రులు వంటి పెద్ద పెద్ద కట్టడాలను మీరు గమనించి ఉంటారు. వీటన్నింటిని నిర్మించడం ప్రభుత్వానికి ఒక పెద్ద బాధ్యతతో కూడిన పని.

వీటిని నిర్మించేందుకు అయ్యే ఖర్చును ఎలా అంచనా వేస్తారో మీకు తెలుసా? దీనికి అగు సిమెంటు, కాంక్రీటు, కూలీ ఖర్చు వంటి వాటితో పాటు దాని ఆకారము మరియు పరిమాణాల పై ఆధారపడి ఉంటుంది.

ఒక కట్టడం యొక్క ఆకారం మరియు పరిమాణాలలో దాని పునాది, నిర్మాణ స్థల వైశాల్యం, గోడల పరిమాణం, పై కప్పు మరియు ఎత్తు మొదలగునవి కలిసి ఉంటాయి. వీటి నిర్మాణాలలో ఇమిడి ఉన్న జ్యామితీయ సూత్రాలను అర్థం చేసుకొనుటకు జ్యామితి యొక్క ప్రాథమిక భావనలను మరియు వాటి అనువర్తనాలను తెలుసుకోవాలి.

జ్యామితి సూత్రాలను మన దైనందిన కార్యక్రమాలైన చిత్రలేఖనం, హస్తకళలు, గదుల నేల మీద రాళ్ళను పరచడం, తోటలను పెంచడం, వంటి వాటిలో ఉపయోగిస్తామని మనకు తెలుసు.

ఈజిప్ట్‌లోని పిరమిడ్లు, చైనా కుడ్యము, భారతదేశంలో ఆలయాలు మరియు యజ్ఞవాటికలు, ఫ్రాన్స్‌లోని ఈఫిల్ వంటి ప్రసిద్ధ కట్టడాలు కొన్నింటిని జ్యామితి అనువర్తనాలకు ఉదాహరణలుగా చెప్పవచ్చు.

అయితే ఒక్క మాటలో చెప్పాలంటే జ్యామితి లేని జీవితాన్ని ఊహించుకోలేము. కాబట్టి జ్యామితి యొక్క మూలాలను అర్థం చేసుకోవడానికి జ్యామితి చరిత్రను, జ్యామితిని అభివృద్ధి పరిచిన తీరును, జ్యామితి వల్ల మనకు ఏ విధంగా ఉపయోగ పడుతుందో తెలుసుకుందాము.

చరిత్ర:-

నిర్మాణాల యొక్క ఆకారము మరియు పరిమాణాలను గణిత పరంగా అనేక విధాలుగా అధ్యయనం చేయవచ్చు. ఈ విధానాలు అన్నీ జ్యామితి పరిధిలోకి వస్తాయి.

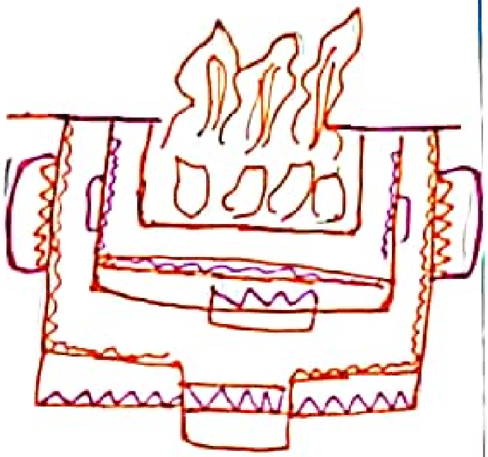
"జ్యామెట్రీ" అనే పదం ఆంగ్లపదం. ఇది గ్రీకు పదాలైన "జియో" అంటే భూమి మరియు "మెట్రియన్" అంటే కొలచుట అనే రెండు పదాల నుండి సంగ్రహించబడినది.

ప్రాచీన కాలంలో జ్యామితి అవశేషాలను సింధులోయ నాగరికత, బాబిలోనియా నాగరికత ప్రజా జీవనంలో గుర్తించవచ్చు. వారికి అధిక కోణ త్రిభుజాల గురించి తెలుసు "భక్షాలి" అను వ్రాత ప్రతి అనేక జ్యామితి సమస్యల ప్రస్తావనలో బాటు అక్రమాకార వస్తువుల "ఘనపరిమాణాలకు" సంబంధించిన లెక్కలను కలిగి వుంది.

క్రీ.పూ 2500 నాటికే వృత్తమును నిర్మించే సాధనాలున్నట్లుగా

సాక్ష్యాలు లభిస్తున్నాయి.

వైదిక సంస్కృతంలో గల శుల్బ సూత్రాలలో యజ్ఞవాటికలు మరియు హోమ గుండాలు నిర్మించుటలో ఇమిడి వున్న జ్యామితీయ సూత్రాలు పొందు-పరచబడినది. ఈ యజ్ఞవాటికల నిర్మా-ణం వెనుక గల అద్భుత మైన విషయం అవన్నీ వివిధ ఆకారాలలో ఉన్నప్పటికి అవి ఆక్రమించే వైశాల్యాలు సమానంగా ఉండుటయే.

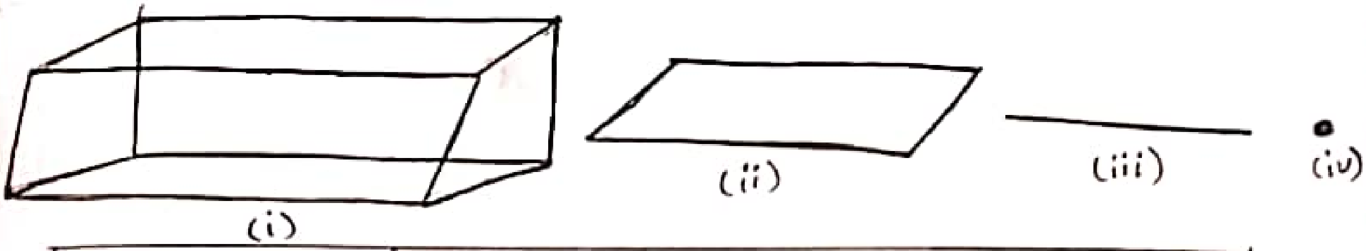


క్రీ.పూ 8వ శతాబ్దం నాటి బౌధాయనుడు సంకలనం చేసిన అత్యంత ప్రసిద్ధమైన బౌధాయన శుల్బ సూత్రాలులో దీర్ఘచతురస్ర భుజాలు, కర్ణాలకు పైథాగరస్ సిద్ధాంతమును ఆపాదించారు.

క్రీ.పూ 325-265 కాలంలో ఈజిప్టులోని అలెగ్జాండ్రియాకు చెందిన యూక్లిడ్ ఎలిమెంట్స్ పేరుతో 13 పుస్తకాలు సంకలనం చేసినాడు.

యూక్లిడ్ జ్యామితిని తాము జీవిస్తున్న ప్రపంచానికి ఒక నమూనాగా భావించారు. అయితే పరిసరాలలోని ఘనాలు, ఘనాల యొక్క అంత-రాళమును చూస్తే, ప్రతి ఘనము ఒక ఘనపరిమాణాన్ని. స్థలాని కలిగి యుండి ఒక చోటు నుండి మరొక చోటుకు కదల్చుటకు వీలగుతుంది.

ఈ తలాలకు మందం ఉండదు. తలాల యొక్క హద్దులుగా గాని,



ఘనాలు → 3-D	తలాలు/వక్రాలు → 2-D	రేఖలు → 1-D	బిందువులు (మితులు లేవు)

పైన చూపినట్లుగా పటం (i)లో ఇది ఒక దీర్ఘ ఘనం యొక్క పటము. దీనికి మూడు మితులు కలవు.

పటం-(ii)లో ఒక మితిని అంటే ఎత్తును కోల్పోతే. అప్పుడు తలాలు మాత్రమే మిగులుతాయి. అవి దీర్ఘ చతురస్రాలు

పటం-(iii)లో ఒక మితిని అంటే వెడల్పును కోల్పోతే అప్పుడు సరళ రేఖలు మిగిలిపోతాయి.

పటం-(iv)లో ఒక మితిని కోల్పోవలసి ఉంటే చివరగా బిందువులు మాత్రమే ఉంటాయి.

ఉదాహరణగా మనం పుస్తకం (or) ఒక బల్లను గమనిస్తే దాన్ని రేఖగా పోల్చవచ్చు.

వీటి సహాయంతో రేఖాఖండం, కోణము, త్రిభుజం వంటి వాటిని నిర్వచిస్తారు.

యూక్లిడ్ ఎలిమెంట్సులోని 1వ పుస్తకంలో 23 నిర్వచనాల జాబితాను రూపొందించాడు.

⇒ బిందువు అంటే ఎటువంటి భాగాలు లేనిది 

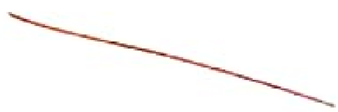
⇒ రేఖ అంటే వెడల్పు లేని పొడవు 

⇒ ఒక రేఖ యొక్క అంత్యాలు బిందువులు 

⇒ ఒక సరళ రేఖ అంటే బిందువుల సమంగా కలిగివున్న రేఖ 

⇒ ఒక తలం అంటే పొడవు మరియు వెడల్పులను కలిగివున్నది

⇒ తలం యొక్క అంచులు రేఖలు

⇒ ఒక సమతలం అంటే తుల్యంగా సరళరేఖలను దానిపై కలిగి వుండునది. 

ఈ విధంగా యూక్లిడ్ అను శాస్త్రవేత్త జ్యామితిని వివరించాడు. అందుకే "రేఖాగణిత పితామహుడు"గా యూక్లిడ్‌ని పిలుస్తారు. యూక్లిడ్ జ్యామితికి ఎంతో కృషి చేసారు.

ఇందులో "నిరూపక రేఖాగణితం" అను భాగం కలదు. దీనిని ఫ్రెంచ్ గణిత శాస్త్రజ్ఞుడు రెనెడెకార్టె (1596-1650) కనుగొన్నాడు. ఇతను

i) ఒక వాస్తవ వాస్తవ క్రమయుగ్మాన్ని ఒక తలంలోని ఒక బిందువుతో అనుసంధానం చేయవచ్చునని

(ii) ఒక త్రిమితీయ అంతరాళంలో ఏదైనా వక్రాన్ని x, y లలో ఒక సమీకరణం ద్వారా నిర్వచించవచ్చునని కనుగొన్నారు.

ఇప్పటి వరకు చరిత్ర గురించి తెలుసుకున్నాము. అదే విధంగా ఇప్పుడు జ్యామితి ఏ విధంగా ఉపయోగపడుతుంది? మన జీవితంలో ఎలా ఉపయోగపడుతుందని తెలుసుకుందాము.

జ్యామితి వల్ల మనకు కలిగే ఉపయోగాలు :-

ప్రయోజనాలు :-

* కంప్యూటర్ గ్రాఫిక్స్‌లలో :- జ్యామితి మీద ఆధారపడి ఉంటుంది. ఫోటోలను కంప్యూటర్‌లోకి తీసుకొని దాన్ని గ్రాఫిక్స్‌లను మనకు చూపించి వాటిలో కొన్ని రకాల స్లైడ్స్‌లలో రూపొందించబడుతుంది.

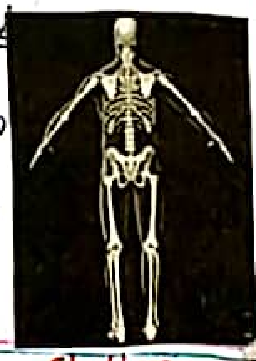
* కంప్యూటర్ - ఎయిడెడ్ డిజైన్స్ :-

దీనిలో కంప్యూటర్ ఒబ్జెక్ట్స్ మరియు ఇమేజ్‌కు కోసం కొన్ని సూచనలు, అదే విధంగా మిషన్స్ వాటిని తయారు చేయడానికి గల సూచనలతో తయారు చేయవచ్చు.

Ex:- Computer, బిల్డింగ్ కన్‌స్ట్రక్షన్స్.

ఇలా ఎన్నో రకాల వాటిలో జ్యామితిని ఉపయోగించేవారు.

* జ్యామితిని అన్ని రకాల గల వాటిలో ఉపయోగిస్తారు. ప్రపంచంలో ఏదైనా జ్యామితితోనే మొదలవుతుంది. ఎక్కువగా మనిషి కూడా జ్యామితి నిర్మాణాలనుతోనే మొదలవుతుంది పరిశీలనగా చూస్తే సరళరేఖ, త్రిభుజ-చతురస్రం, సర్కిల్ రకరకాల సూత్రాలలో జ్యామితి నిర్మాణాలను కలిగి వుంటాయి.

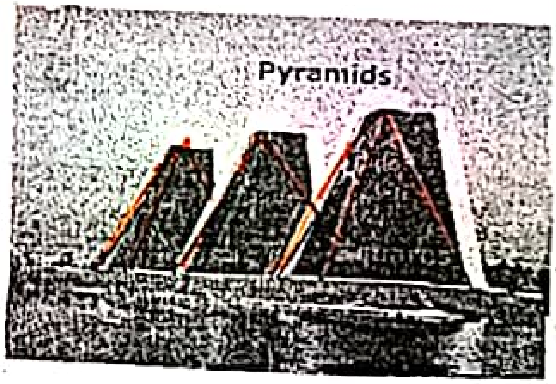


Skelton

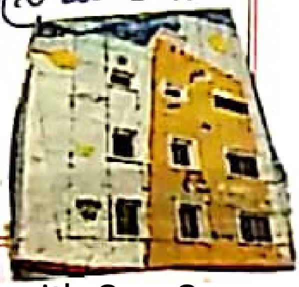
* ఇది స్క్రైక్రాపర్ అనునది దాని నిర్మాణం చూసినట్లయితే దాని కిటికీలు, తలుపులు సమాంతర చతుర్భుజములు కలవు. ఇందులో దీర్ఘచతురస్రం, చతురస్రములు కలవు బిల్డింగ్ దీనిలోనే నిర్మాణమును కలదు.

* ప్రక్కనున్న మన భూమిని చూసినట్లయితే అది గోళాకారంలో కలదు. ఇక్కడ జ్యామితీయ నిర్మాణం కలదు.

* పిరమిడ్స్ ప్రక్కన పటంలో చూడవచ్చు అయితే అలాగే బిల్డింగ్స్‌ను రూపొందించారు. అయితే ఇది ఇండియానా పొలిస్ లో కలవు. ఇందులో పిరమిడ్స్, చతురస్రాలు కలవు. ఇందులో 3D జ్యామితీయ ఆకారాలను కలిగినది. ఇందులో కిటికీలు టెంటెడ్ హీటెడ్ గ్లాస్ కలదు. బాల్కనీస్, గోడలు, విండోస్ అన్ని 3D జ్యామితీయ ఆకారాలను కలిగివుంది.



* ది Square అనే బిల్డింగ్ చూసినట్లయితే అది ఘనాలు, చతురస్రాలు, గోళం కలదు. బిల్డింగ్ పైన Sphere (గోళం) కలదు. బిల్డింగ్ పైన డోర్స్ దీర్ఘచతురస్రం, విండోస్ చతురస్రములుగా బిల్డింగ్‌ను రూపొందించారు.



* జ్యామితీయ సూత్రాలను బట్టి కార్లను కూడా రూపొందించవచ్చు.

అందులో ఒకటి చెవర్లెట SSR రోడ్‌స్టార్ పికప్. ఇందులో చక్రాలు, రిట్స్, సర్కిల్ లాగా, డోర్స్ దీర్ఘచతురస్రముగా లోపల డ్రైవ్ చేసే సీటు అనునది అర్ధ గోళం. Sides కూడా 1/4 వంతు

Car

గల గోళాములాగా, ఇంకా చూసినట్లయితే ఎన్నో రకాలు జ్యామితీయ సూత్రాలను కలిగి వుంది.

* రెడ్ ఇండియన్స్ నివసించు ఇళ్ళను చూసినట్లయితే ఇందులో కూడా జ్యామితీయ సూత్రాలను ఉపయోగించారు ఇది కట్టెలతో నిర్మితమై ఉంది. చతురస్రాలు, ట్రయాంగిల్స్‌ను చూడవచ్చు.

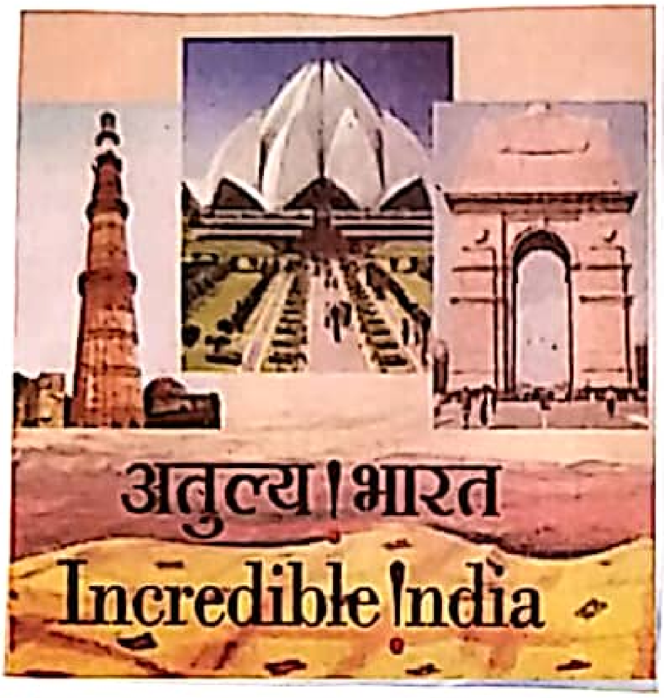
* మన దేశంలోని ఎన్నో ప్రముఖ కట్టడాలు వున్నాయి. అవి కూడా జ్యామితీయ (గం) సూత్రాలను తో నిర్మించారు. అందులో కొన్ని.

ముఖ్యంగా ఆగ్రాలో ఉన్న తాజ్‌మహల్

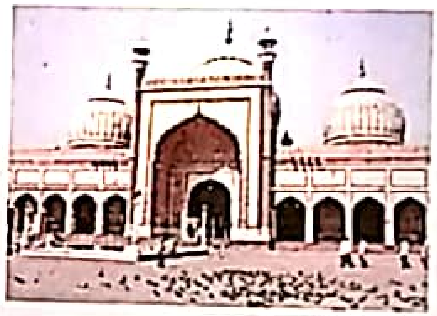
ఇది కూడా జ్యామితీయ సూత్రాల ఆధారంగా నిర్మించారు. ఇందులో అర్ధ స్థూపము మరియు అర్ధ గోళాలు కలవు. ఇందులో సరళ రేఖ ఇంకా ఉపయోగించి కట్టడం జరిగింది.

Tajmahal

* ఇంకా చార్మినార్, లోటస్ టెంపుల్ ఇంకా మన దేశంలో చాలా ప్రసిద్ధమైన కట్టడాలు, పురాతన గుళ్ళు చాలా వున్నాయి అందులో (కాను) అన్నింటిలోను జ్యామితీయ సూత్రాలు ఆధారంగా కట్టడం జరిగింది.



Redfort



Aeroplan

Charminar

Effil tawar

Aeroplan, House వంటివాటిలో కూడా నిర్మించడంలోనూ జ్యామితీయ సూత్రాలు ఇమిడి వున్నాయి.

* జ్యామితి మెడికల్ ఇమేజింగ్‌లో కూడా వాడుతారు. అనగా కొన్ని పరికరాలను తయారు చేయడానికి జ్యామితీయ సూత్రాలను ఉపయోగించారు. సిటి స్కాన్ యంత్రాన్ని స్టెతస్కోప్ రకరకాల వాటిలోను ఉపయోగించడం జరుగుతుంది. ఇప్పటికి కూడా తయారు చేస్తారు.

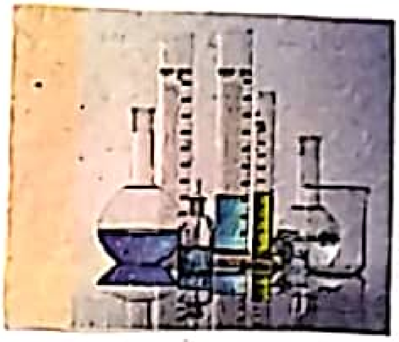
* రోబోటిక్స్ లలో జ్యామితీయ సూత్రాలను కూడా వాడుతారు. దాని చేయి ఎలా ఉండాలో ప్లాన్ చేసుకోవడం జ్యామితీయ నిర్మా-ణాలను ఆధారంగా కూడా (రోబోటిక్స్) లో రోబోట్లలో ఉపయో-గిస్తారు.

* ప్రోటీన్ నిర్మాణంలో కూడా జ్యామితీయ నిర్మాణాల ఆధారంగా కూడా ఉపయోగపడుతాయి. ప్రోటీన్స్ యొక్క ఆకారం మరియు భాగాలు ఎలా కదులుతాయో తెలుస్తుంది. కంప్యూటర్స్‌కు, జ్యామితికి దగ్గరగా ఉంటాయి.

* మెకానికల్ ఇంజనీరింగ్ లో మరియు ప్రకృతిక ఇంజనీరింగ్ లోను ఈ జ్యామితీయ నిర్మాణాల ఆధారంగా యంత్రాలను తయారు చేస్తారు. దాని యొక్క డిజైన్లలో, ఆర్కిటెక్చర్లలో కూడా ఉపయోగిస్తారు.

* ఇంకా ఫిజిక్స్, కెమిస్ట్రీ, బయాలజీలోను. కొన్ని శాస్త్రాల లో కూడా జ్యామితీయ నిర్మాణాలను ఉపయోగిస్తాము. సైన్స్‌కు

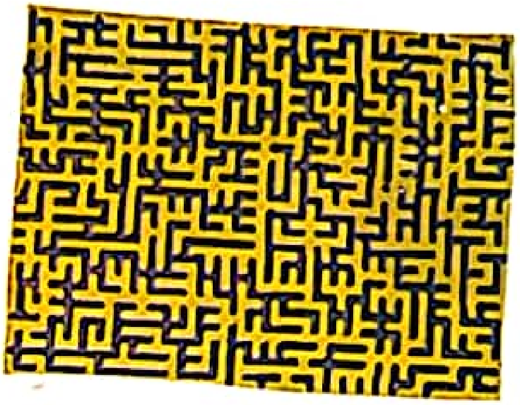
సంబంధించిన వాటిలోనూ, జ్యామితి మరియు క్రొత్తగా గల
స్పీసీస్‌లోను కలదు. పరివర్తనలుగా గల జ్యామితి మరియు సంబంధి-
-చిన వాటికి సిమెట్రి (symmetry) లలో వుంటాయి.



* కొన్ని చిన్న చిన్న వస్తువులలో కూడా కలదు. అందులో పక్కన
చూపినట్లయితే బోల్టులు. అవి ఆ ఆకారంలో ఎందుకు
-న్నాయో వివరించబడింది.

* గణితానికి, భౌతిక శాస్త్రానికి చాలా దగ్గరగా వుంటుంది.
గణితము, భౌతిక శాస్త్రాలు చాలా చాలా దగ్గరగా వుంటాయి. అందు
-లో కొన్ని కలవు. వీటిలో జ్యామితీయ నిర్మాణాలు కూడా
కలవు.

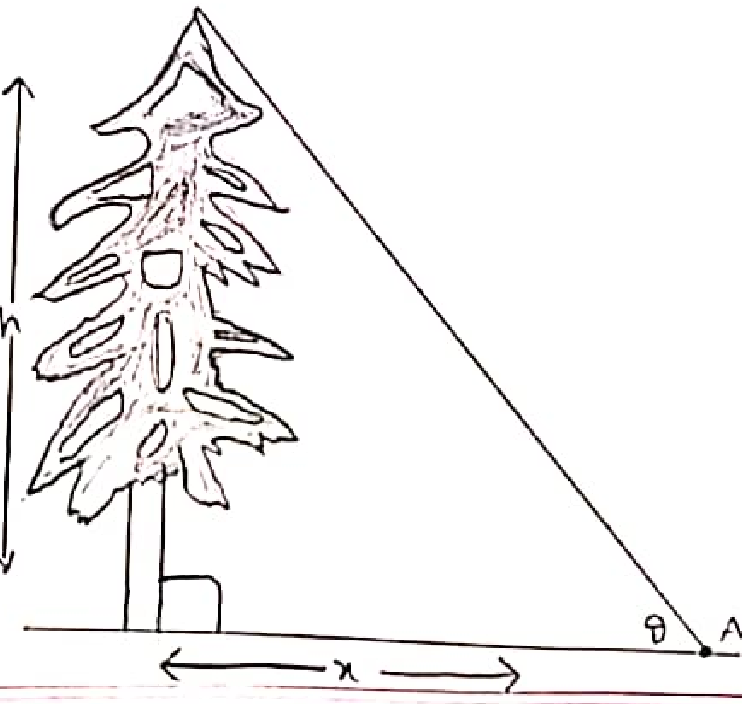
* ఇది రిఫ్లెక్షన్ సిమ్మెట్రిలగా కలదు ఇందులో అధిక కోణాలు కలదు మూడు వైపులా ఒకటేటి కలదు. అందులో ప్రతి ప్రాంతం కూడా ఒకటే ఉంటుంది.



* ఇది రొటేషన్ సిమ్మెట్రి ఈ పటంలో చూపినట్లుగా వుంటుంది. ఒకటే ఇందులో ఎటు వైపు చూసిన [కలదు] ఒకటే కలదు. జ్యామితీయ నిర్మాణాలను కలదు.

ఇలా ఈ విధంగా చాలా రకాల్లో జ్యామితీయ నిర్మాణా-లనుసరించి రూపొందించబడ్డాయి. మన ఇంటా వాడే వస్తువులు కొన్ని జ్యామితీయ సూత్రాల ఆధారంగా నిర్మింపబడ్డాయి.

ప్రక్కన చూపినట్లయితే చెట్టు మొక్క ఎత్తును భూమి నుండి చెట్టుకు పైపున చూసినపుడు ఎంత డిగ్రీలలో వుండే దాని మనం తెలుసుకోవచ్చు. దీనిని పైథాగరస్ సిద్ధాంతం నుండి కనుకొనవచ్చు.



అభిప్రాయం :-

జ్యామితి ఎలా ఉపయోగపడుతుందో ఈ ప్రాజెక్టు ద్వారా నేను తెలుసుకున్నాను. అయితే ఇందులో జ్యామితి నిర్మాణాల ఆధారంగా రూపొందించబడినవి అని అర్థమయ్యింది. దీనిలో భౌతిక శాస్త్రం, రసాయన శాస్త్రం ఇలా అన్ని రకాల శాస్త్రాలలో కూడా జ్యామితిని ఉపయోగిస్తారని నేను మొదటి సారిగా తెలుసుకున్నాను. అయితే కొన్ని భౌతిక శాస్త్రం ఆధారంగా కూడా ఉంటాయి. నిజ జీవితంలో మనం వాడే ప్రతి వస్తువు అంటే పెన్ను, పెన్సిల్ ఇలా అన్ని రకాలలోను సరళరేఖ, వృత్తం, అన్ని ఆకారంలో కలదు. అని తెలుసుకున్నాను. ఈ జ్యామితి అనేది చాలా ఉపయోగకరమైనది. ఈ జ్యామితి వల్ల నేను చాలా నేర్చుకున్నాను. కాబట్టి దీని ఉపయోగకరాన్ని నేను నా ఫ్రెండ్స్‌కు కూడా తెలియజేస్తాను.

ఈ సమాచారాన్ని నేను నెట్ నుండి, న్యూస్ పేపర్ నుండి సేకరించాను. అలాగే బుక్స్ నుండి కూడా సేకరించాను.

Maths
3 - Gadwal-

MALD GOVT. ART'S & SCIENCE COLLEGE

DEPARTMENT OF MATHEMATICS

Students Study Project – 2017-18

APPLICATIONS OF DIFFERENTIAL CALCULUS

Students

1. V. Ramakrishna	III B.Sc MPCS
2. T.Latha	III B. Sc MPCS
3.K. Sai Kiran	II B.Sc MPCS
4. B. Anil Kumar	I B.sc MPCS
5. Pavan Kalyan Goud	I B.Sc MPCs
6. Sai Teja	I B. Sc MPCS

Project Guide

V. Manoj Kumar
Asst. Prof. of Mathematics

PRINCIPAL
M.A.L.D. Govt. Degree College
GADWAL.

APPLICATIONS OF DIFFERENTIAL CALCULUS

Statement of the problem: It is universally acknowledged and accepted openly that there is no branch of knowledge on this planet without the right inclusion of mathematics. It is also believed by all that we need to learn mathematics simultaneously with our mother tongue right from our childhood.

In the process of learning mathematics we get ourselves encountered with certain concepts, formulas and solutions to the problems in specific manner. But the bottom line of topic is that – we don't find specific linkup at UG level of education in our system to the problems in our day-to-day life.

Here we are trying to relate or attribute some of the concepts of differential calculus with real life situations (or) applications.

AIMS AND OBJECTIVES: Our main aim of this study project is to find clear solutions to various concepts of differential calculus to that of real life problems by adopting mathematical concepts thoroughly and practically.

After going through this study project it is believed that student could be capable of understanding more clearly and lucidly these concepts. It is also believed that students could be capable of applying this practical knowledge of differential calculus with that of real life situations and students could lot of interest in learning mathematic with real life exposure.

REVIEW OF LITERATURE: Differential calculus applications are used in various fields such as architecture, designing, graphing, constructions of roads and rail tracks, algorithms, computer programming, optimization etc.

RESEARCH METHODOLOGY: Heuristic method, Project method, Analytical method and Problem solving method.

FINDINGS: Mathematics is a science of measurement, quantity and magnitude, it is exact (complete, correct (or) true in every way), precise (definitely (or) fixed (or) strictly stated), systematic and a logical subject.

Calculus is a branch of mathematics that has two branches in itself – differential and integral calculus.

Differential calculus involves finding the derivatives of functions in order to discover the behavior and rate of change of a function. The graph of said function can then be computed, analyzed and predicted.

Integral calculus involves the usage of anti differentiating and is used to find the areas or volumes under/around a certain function

Now we will discuss some topics in **Differential calculus** and their applications in daily life

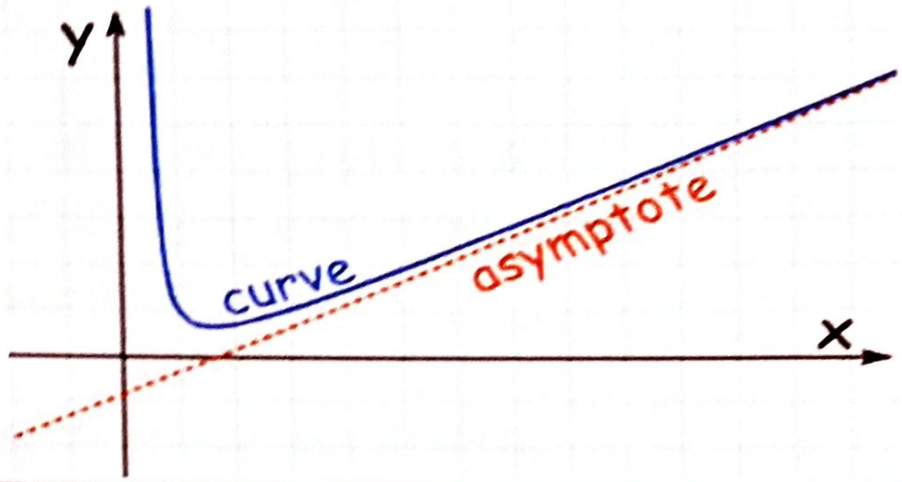
(1)Asymptotes

(2)Curvature

(3)Optimization (Max. and Min. values of a function)

Asymptotes: Asymptote of a curve is a line to which the curve converges. In other words, the curve and its asymptote get infinitely close, but they never meet.

Graph:



Mathematically we define it as

" A straight line is said to be an asymptote of an infinite branch of a curve, if as the point P recedes to infinity along the branch, the perpendicular distance of P from the straight line tends to zero (or)

A non vertical line with equation $y = mx + b$ is called an asymptote of the graph of $y = f(x)$ if the difference $f(x) - (mx + b)$ tends to 0 as x takes on arbitrarily large positive values or arbitrarily large negative values.

There are three types of asymptotes

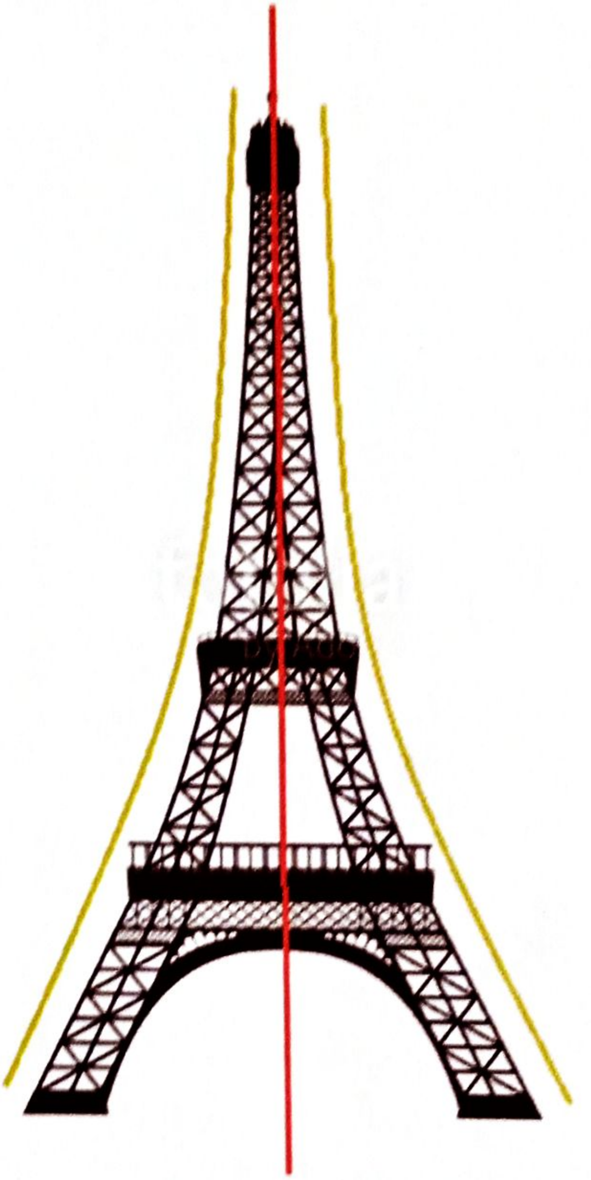
1) Horizontal asymptote.
2) Slant asymptote.
3) Vertical asymptote.

We apply the concept of asymptotes in architecture, designing for example.....

EIFFEL TOWER

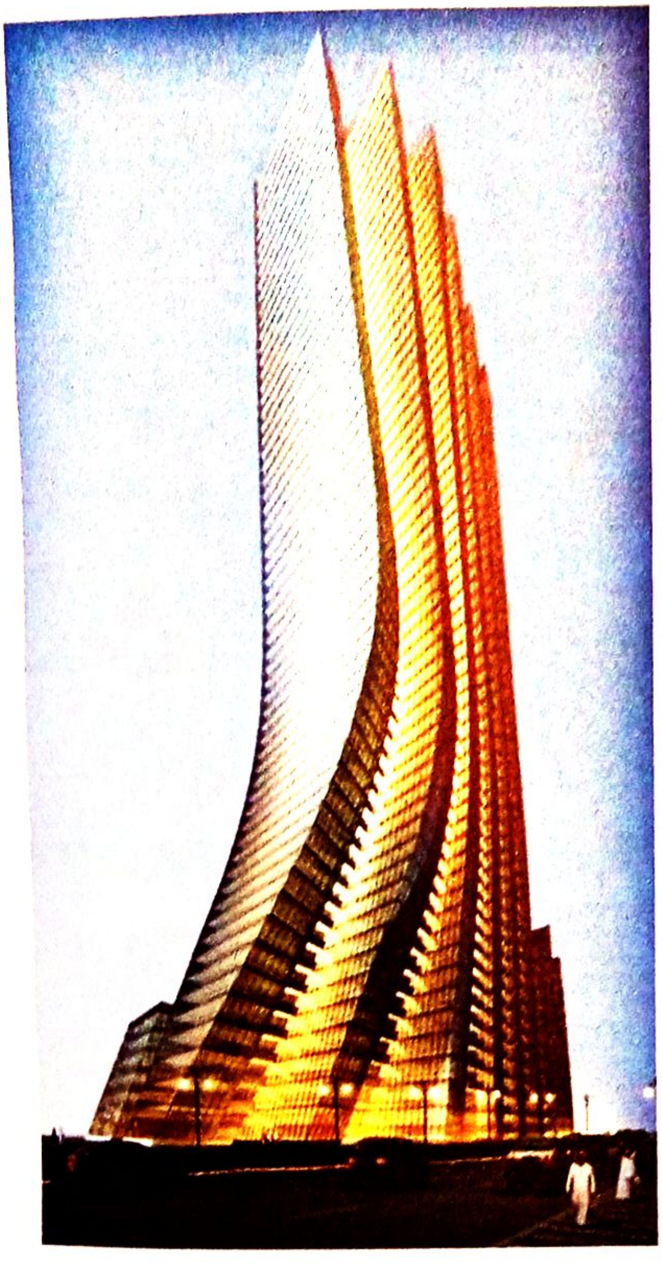
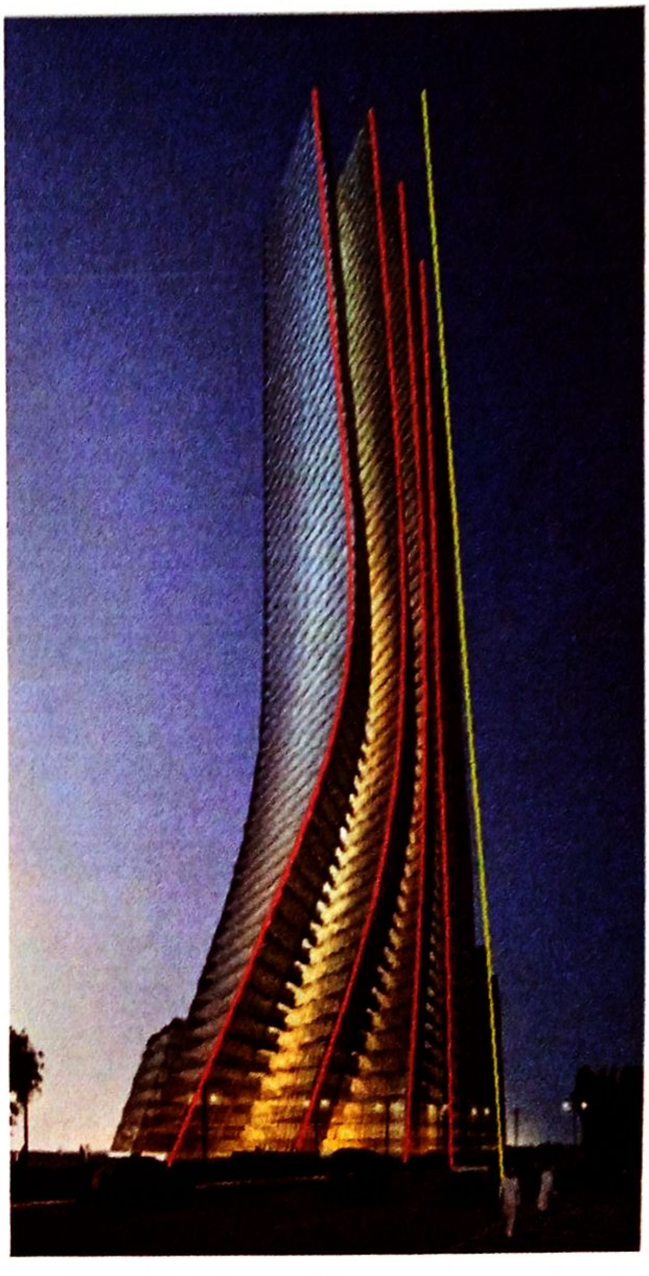
We can observe asymptotes concept as

(Vertical asymptote)

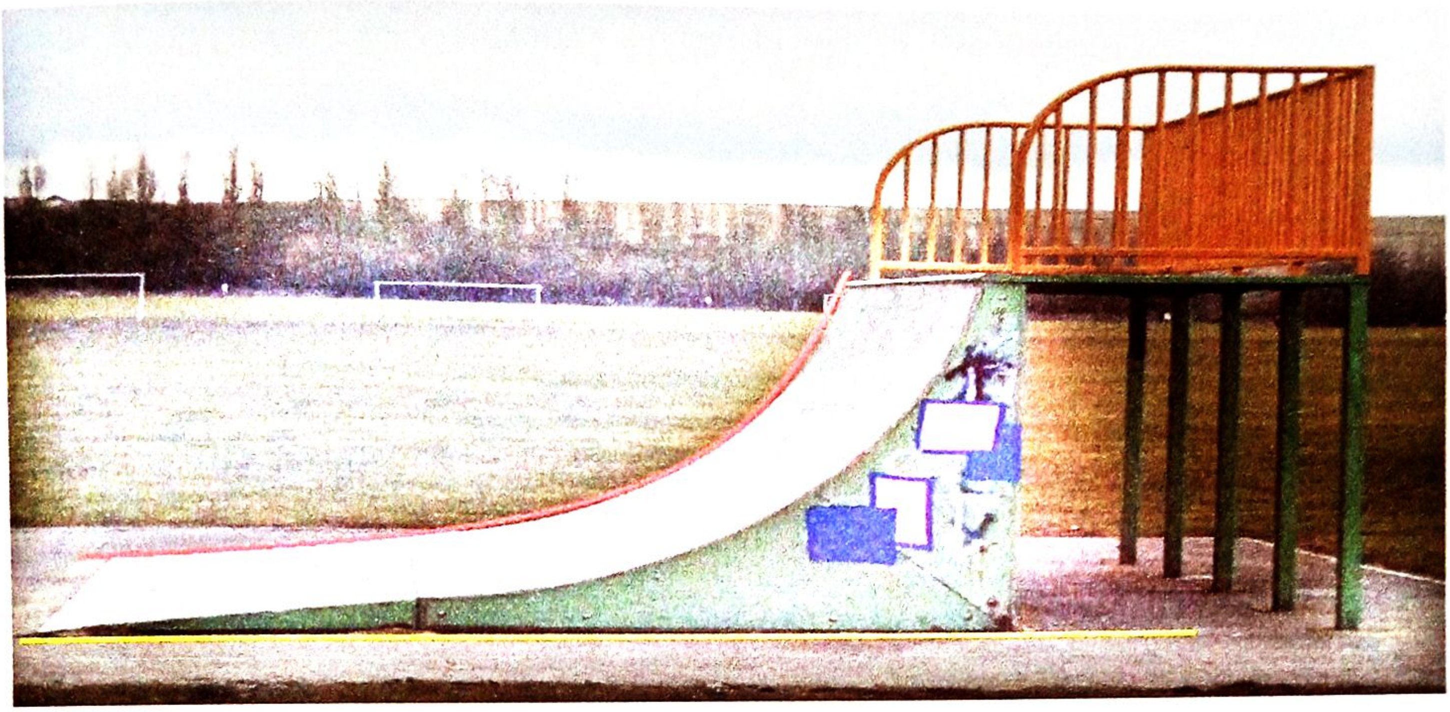


Some more examples

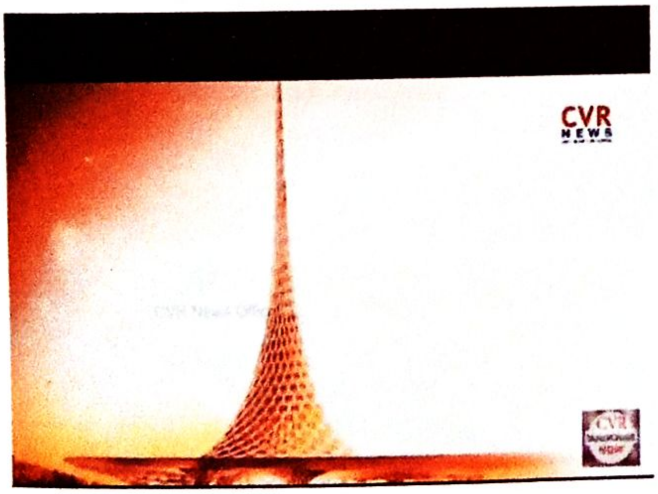
Slant Asymptote

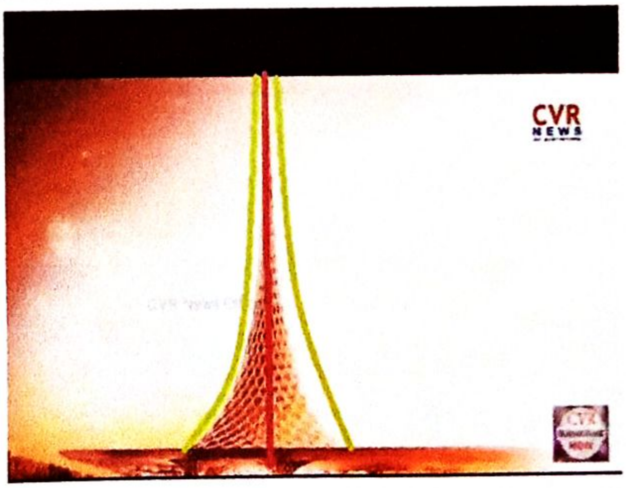
 

Horizontal Asymptote



This is the proposed AMARAVATHI assembly design





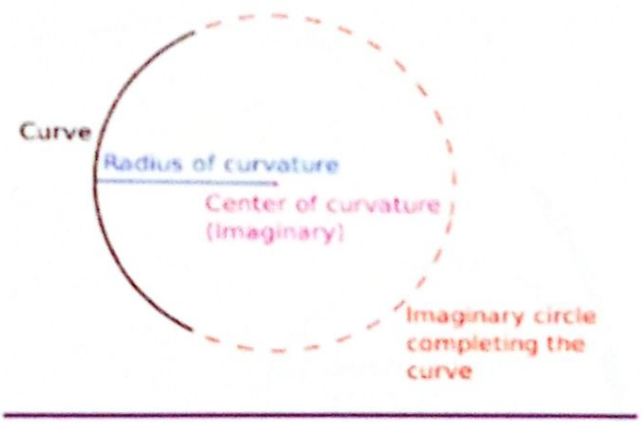
Asymptote concepts are used to find approximations to complex equations and to analyze algorithms running time

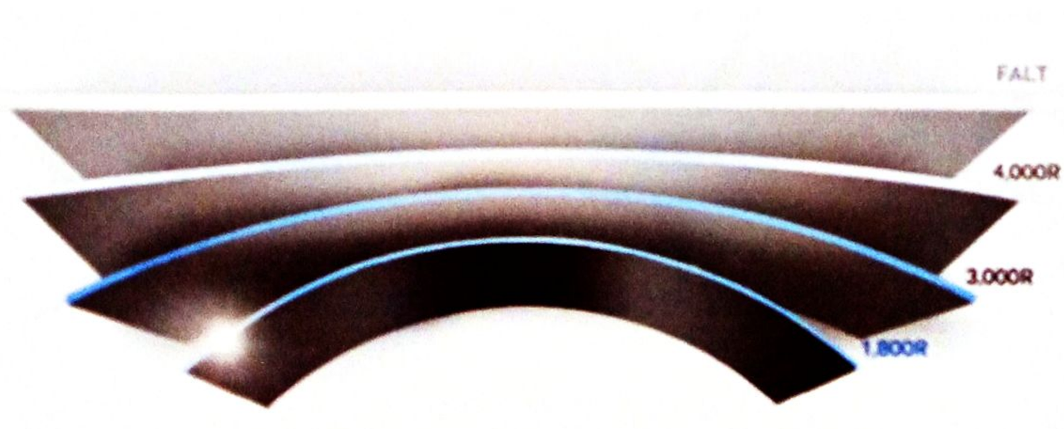
Curvature: Bending of a curve is known as curvature.

- ➢ A straight-line has no curvature
- ➢ A circle has constant curvature

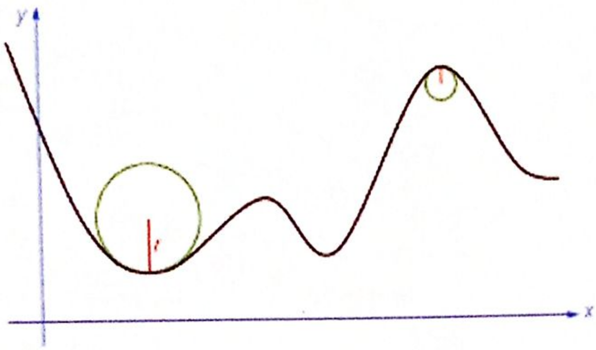
Radius of curvature: If we imagine a circle, taking curve as a sector, then radius of that circle is known as Radius of curvature to that curve at that point.

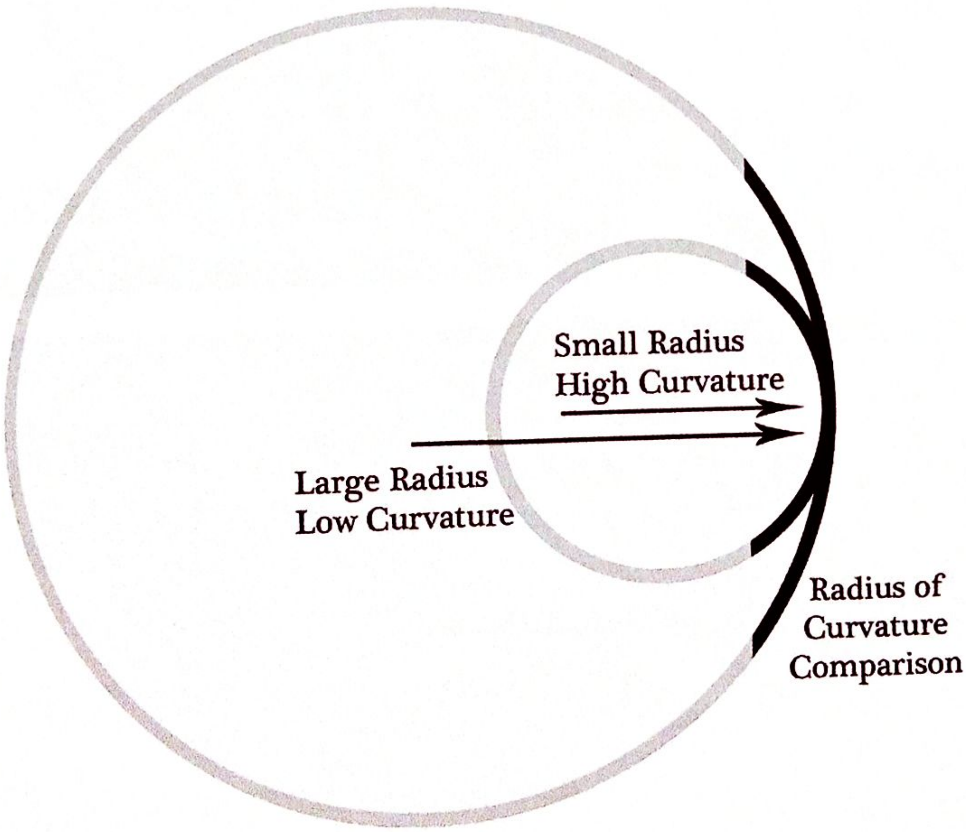
Graph:



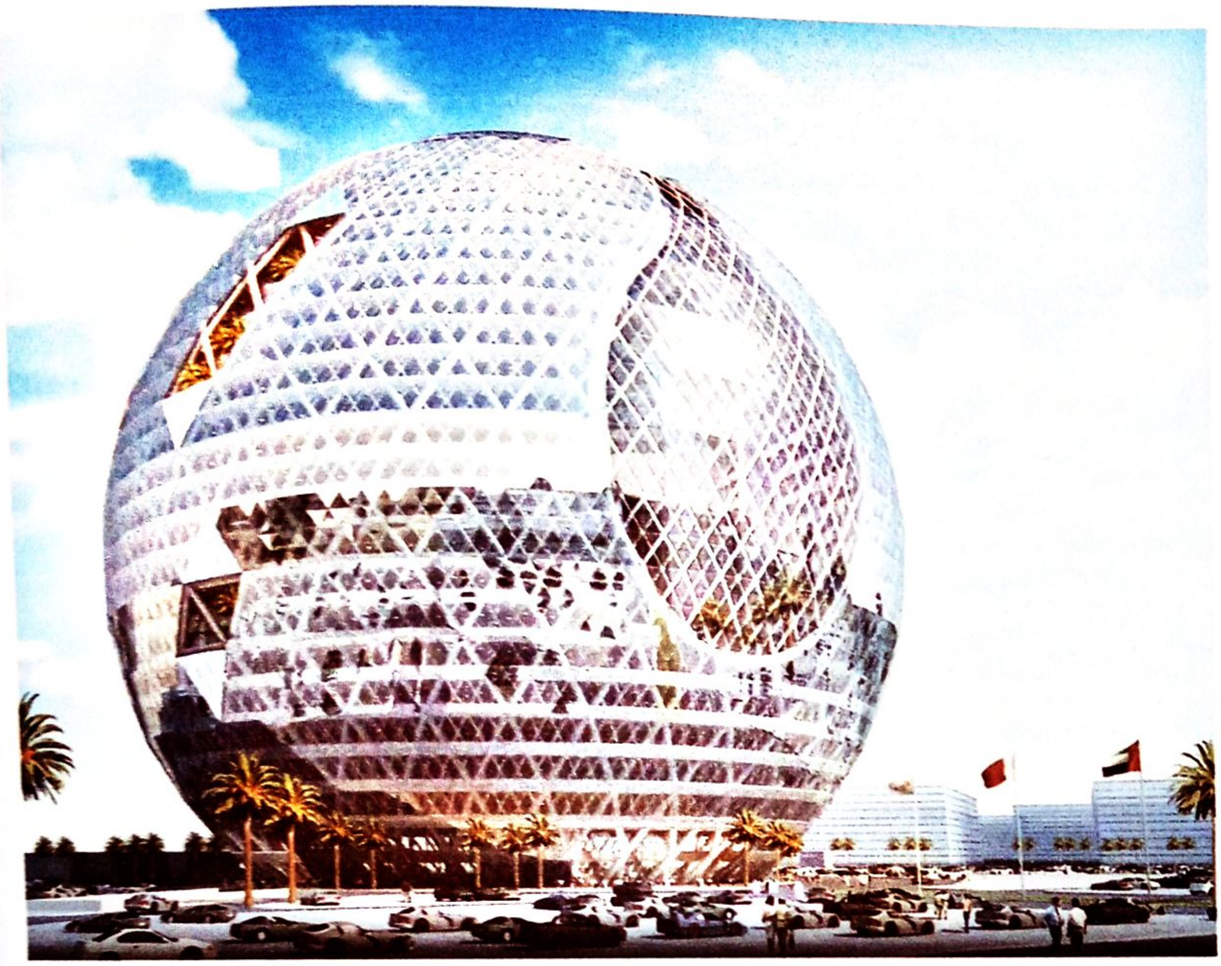


It may be remarked here that more the sharpness of bending, less is the radius of curvature and hence more is the curvature (and vice versa).





Applications: Architecture – tracks – design.



Sanchi stupa, at Madhya pradesh

Railway traces

In construction of roller coaster we use curvature concepts to control the speed of cars

Radius of curvature concept applied to measure the thermal stress in the semiconductor structures.

Optimization

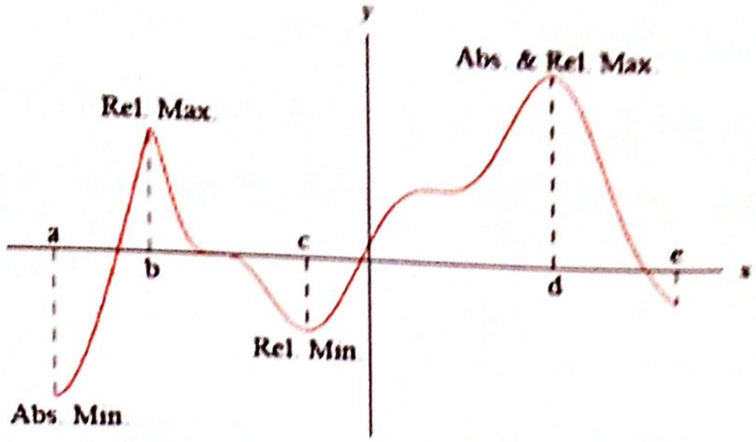
Maximum and minimum of a function: The terms maximum and minimum refers to extremes values of a function.

Maximum means upper bound or largest possible quantity. The absolute maximum of a function is the largest number contained in the range of the function, i.e. $f(a) \geq f(x), x \in$ Domain

In terms of its graph, the absolute maximum of a function is the value of the function that corresponds to the highest point on the graph.

Conversely, minimum means lower bound or least possible quantity. The absolute minimum of a function is the smallest number in its range and corresponds to the value of the function at the lowest point of its graph, i.e. $f(a) \leq f(x), x \in$ Domain.

In terms of its graph, the absolute minimum of a function is the value of the function that corresponds to the lowest point on the graph.



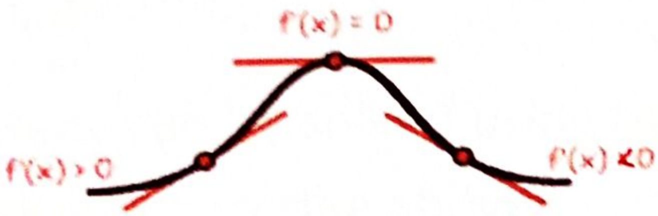
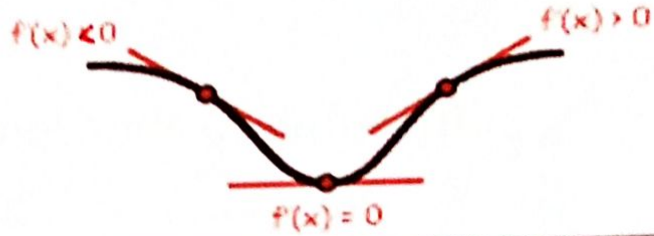
Finding the maxima and minima, **both absolute and relative,** of various functions represents an important class of problems solvable by use of differential calculus.

The theory behind finding maximum and minimum value of a function is based on the fact that the derivative of a function is equal to the slope of the tangent.

When the values of a function increases as the value of independent variable increases, the lines that are tangent to the graph of the function have positive slope and the function is said to be increasing, i.e. $f'(x)>0$

Conversely when the value of the function decreases with increasing value of the independent variable, the tangent line have negative slope and the function is said to be decreasing, i.e. $f'(x)<0$.

Precisely at the point where the function changes from increasing to decreasing or decreasing to increasing, the tangent line is horizontal (has slope zero) and the derivative is zero, i.e. $f'(x) = 0$.

Maxima and minima & the derivative	
Maximum	**Minimum**
$f'(x)$ positive on the left $f'(x)$ negative on the right	$f'(x)$ negative on the left $f'(x)$ positive on the right
	

In order to find maximum and minimum points, first find the values of the independent variable for which the derivative of the function is zero, i.e. $f'(x) = 0$, these points are called <u>**critical points**</u>, and then substitute them in the original function to obtain the corresponding maximum or minimum value of the function.

If $f''(c)<0$ then 'c' is a point of maximum

If $f''(c)>0$ then 'c' is a point of minimum, where 'c' is a critical point

There are numerous practical applications in which it is desired to find the maximum and minimum value of a particular quantity.

Such applications exits in economics, business and engineering.

Many can be solved using the methods of differential calculus describe above.

For example in many manufacturing business it is usually possible to express profit as a function of the number of units sold. Finding a maximum for these functions represents a straight forward way of maximizing profits. In other case, the shape of a container may be determined minimizing the amount of a material requires to manufacture it. The design of piping systems is often based on minimizing pressure drop which in turn minimizes required pump sizes and reduces cost. The shapes of steel beams are based on maximizing strength.

<u>Optimization</u>: The action of making the best or most effective use of a situation or resource (or) optimization is the process of finding the greatest or least value of a function for some constraint, which must be true regardless of the solutions.

Optimization is nothing but finding maximum or minimum of a function in given domain. Here we will find solutions to some problems by using the concept of max. and min. of functions

We follow three steps to find solutions of given problems

1) Express the given data in the form of equations, if there are more than one variable then write other variables in terms of first variable by using suitable equations
2) Find the derivative of resulting equation and equal it to zero to find critical (or) stationary values of function.
3) Find the second derivative and substitute stationary value in that equation to check either it is max. or min.

Business Applications:

The goal of business management is to

- Maximize Profit
- Maximize Revenue
- Minimize Cost

Since maximums and minimums occur when the derivative is zero, our goal is to find values for which marginal revenue, marginal profit, marginal cost = 0.

EXAMPLE 1:

If the price for a product is p, the revenue (in thousands of rupees) is approximated by

$$R = -0.05p^{2} + 0.98p + 18$$

What price maximizes revenue?

Solution: $R'(p) = -0.1p + 0.98 = 0$

$$\Rightarrow 0.1p = 0.98$$

$$\Rightarrow p = 9.8$$

Question: How do we know that $p = 9.8$ gives a maximum (and not a minimum)?

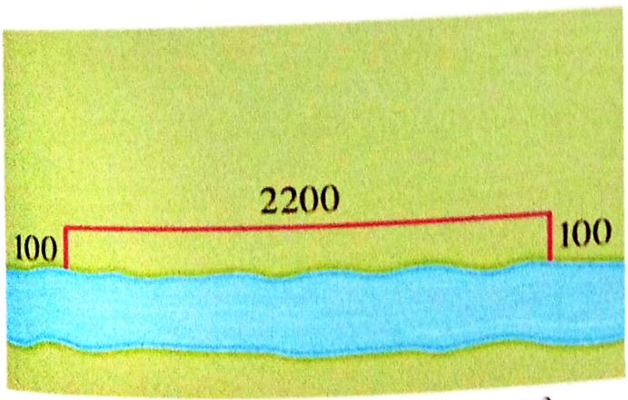
To tell whether this critical point is a max or a min,

Use the second derivative test $R''(p) = -0.1$

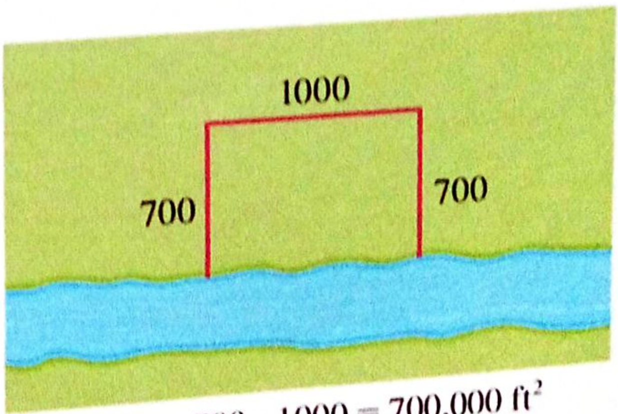
And a negative second derivative implies $p = 9.8$ is a Maximum

EXAMPLE 2: A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

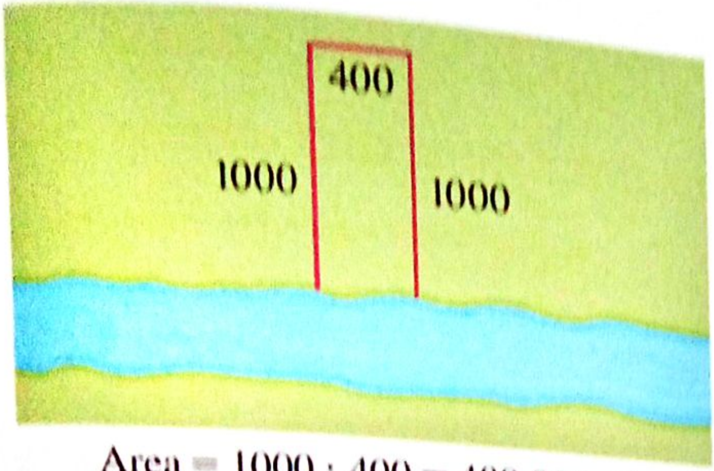
Solution: Note that the area of the field depends on its dimensions:



Area = 100 · 2200 = 220,000 ft^2

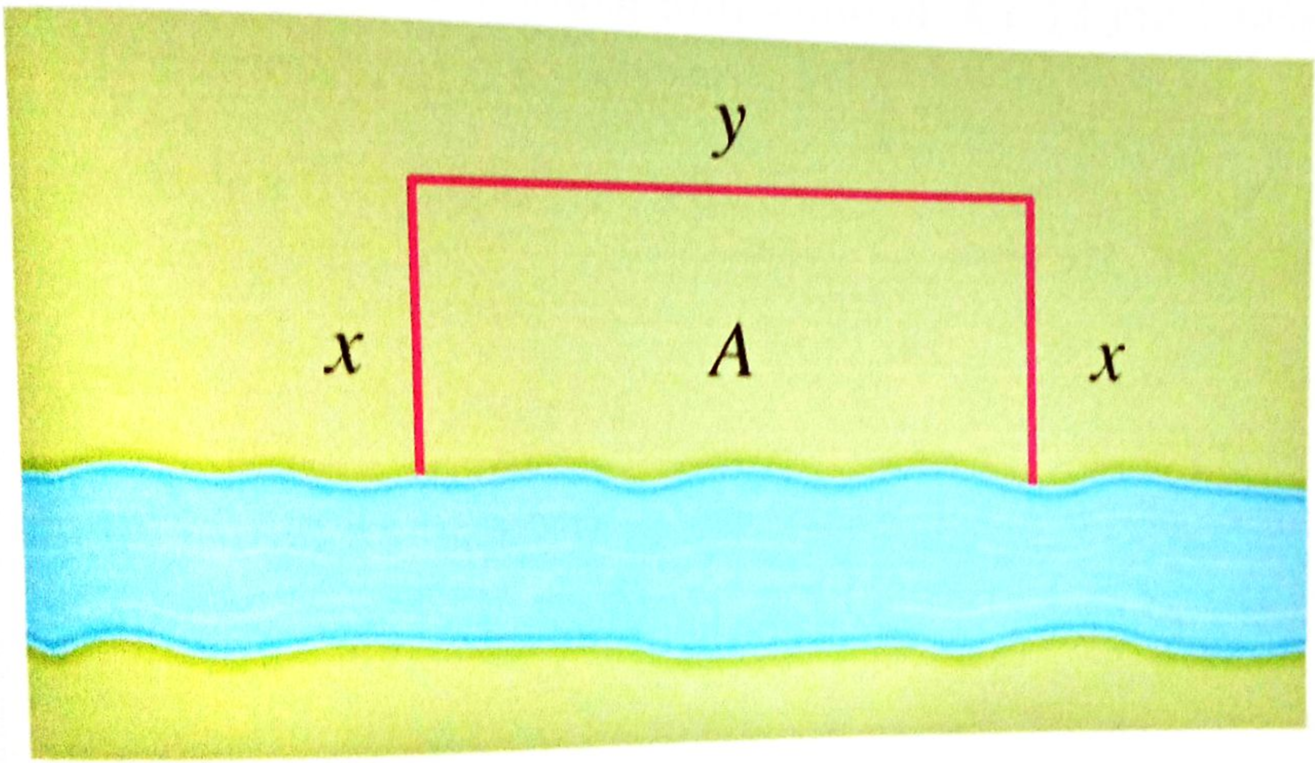


Area = 700 · 1000 = 700,000 ft^2



Area = $1000 \cdot 400 = 400{,}000$ ft^2

To solve the problem, we first draw a picture that illustrates the general case:



The next step is to create a corresponding mathematical model:

Maximize: $A = xy$

Constraint: $2x + y = 2400$

We now solve the second equation for y and substitute the result into the first equation to express A as a function of one variable:

$2x + y = 2400$

$\Rightarrow y = 2400 - 2x$

$\Rightarrow A = xy = x(2400 - 2x) = 2400x - 2x^2$

To find the absolute maximum value of $A = 2400x - 2x^2$, We first show that $0 \leq x \leq 1200$.

Indeed, $y \geq 0 \Rightarrow 2400 - 2x \geq 0$

$\Rightarrow 2400 \geq 2x$

$\Rightarrow 1200 \geq x$

On the other hand, $x \geq 0$.

Combining these two inequalities gives $0 \leq x \leq 1200$.

The derivative of A(x) is

$$
\begin{aligned}
A'(x) &= (2400x - 2x^2)' \\
&= 2400x' - 2(x^2)' \\
&= 2400 \cdot 1 - 2 \cdot 2x \\
&= 2400 - 4x
\end{aligned}
$$

so to find the critical numbers we solve the equation

$$2400 - 4x = 0$$

$$\Rightarrow 2400 = 4x$$

$$\Rightarrow x = 2400 \div 4 = 600$$

To find the maximum value of A(x) we evaluate it at the end points and critical number,

$A(0) = 0,$

$A(600) = 2400 \cdot 600 - 2 \cdot 600^2 = 720,000,$

$A(1200) = 0$

Also, $A''(x) = (2400 - 4x)'$

$$= -4 < 0$$

Therefore A(x) has maximum value at $x = 600$

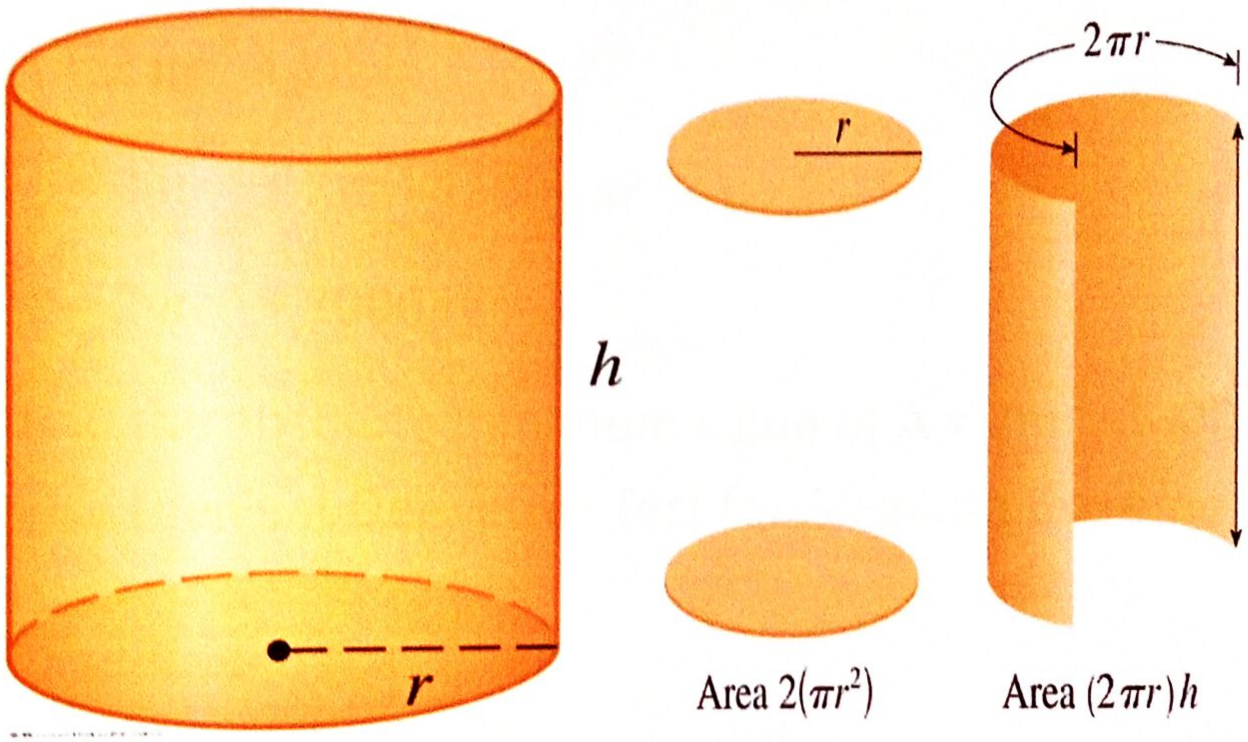
The maximum value is $A(600) = 720,000 \text{ ft}^2$ and

the dimensions are

$x = 600$ ft, $y = 2400 - 2 \cdot 600 = 1200$ ft.

EXAMPLE 3: A manufacturer needs to make a cylindrical can that will hold 1.5 liters of liquid. Determine the dimensions of the can that will minimize the amount of material used in its construction.

Solution: We first draw a picture:



The next step is to create a corresponding mathematical model

Minimize: $A = 2\pi r^2 + 2\pi rh$

Constraint: $V = \pi r^2 h = 1500$

It is easy to see that $A'(r) < 0$ for all $0 < r < (750/\pi)^{1/3}$ and $A'(r) > 0$ for all $r > (750/\pi)^{1/3}$.

Therefore the minimum value of the area must occur at

$r = (750/\pi)^{1/3} \approx 6.2035$ cm.

and this value is $A((750/\pi)^{1/3}) \approx 725.3964\ \text{cm}^2$.

Finally, the height of the can is

$h = 1500/\pi r^2$

$= 1500/\pi(750/\pi)^{2/3} = 2(750/\pi)^{1/3} = 2r \approx 12.4070$ cm

CONCLUSIONS:

Differential Calculus is not a study of concepts which is purely restricted to class room. We can see lot of applications in our day-to-day life.

"Mathematics is a science whose subject matter is special forms and quantitative relationships of the real world"

"Mathematics is a language in which GOD has written the world" ---- Galileo.

2018-19

M.A.L.D.GOVT. DEGREE COLLEGE GADWAL.

Affiliated to Palamuru University, Mahabubnagar.

Department of Mathematics

STUDENT STUDY PROJECT

NAME OF TITLE: Applications of Differential Equations in real life

NAME OF THE STUDENTS:

K. Laxmi III B.Sc (MPCs)
Poojari Deepika I B.Sc(MPCs)
Uma I B.Sc(MPCs)
Sarveshwari I B.Sc (MPCs)
M. Sharada I B.Sc (MPCs)

PRINCIPAL
M.A.L.D. Govt. Arts & Science College
GADWAL - 509 125

In charge
Department of Mathematics

M.A.L.D. Govt College Gadwal

Department of Mathematics

Project Work

Topic: **Applications of Differential equations in Real life**

Team:

1) K. Laxmi
2) Poojari Deepika
3) Uma
4) Sarveshwari
5) M. Sharada

Differential Equations

Objective: The main aim of this course is to introduce the students (we) to the techniques of solving differential equations and to train to apply their skills in solving some of the problems of engineering and science.

Outcome: After learning the cource the students (we) will be equipped with the various tools to solve few types differential equations that arise in several branches of science.

Deffination of Differential Equation:

An equation involving independent and dependent variables and the derivatives or differentials of one or more dependent variables with respect to one or more independent variables is called "Differential equation".

There are two types of Differential equations

1) ordinary differential equations.

2) partial differential equation.

1) Ordinary differential equation :

A differential equation which involves derivatives with respect to a single independent variable is known as ordinary differential equation.

Example :

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \cos x$$

$$\frac{d^3y}{dx^3} + y = 0$$

2) Partial differential equation :

A differential equation which contains two or more independent variables and partial derivatives with respect to them is called a partial differential equation.

Example :

$$\frac{\partial^2 y}{\partial x^2} = \left(\frac{\partial^2 y}{\partial x^2}\right)^2$$

$$\frac{\partial^3 y}{\partial x^3} + k\left(\frac{\partial y}{\partial x}\right)^2 = 0$$

* Order of differential equation :-

The order of the highest derivative involving in the differential equation is called the "order of differential equation".

* Degree of differential equation :

The degree of differential equation is the power of the highest order derivative present in the equation after the differential equation has been made free from the radical and fractions as far as the derivatives are concerned. It is called Degree of differential equation.

* Defferentiation Formulas

⇒ Here i am giving some differentiation formulas

1. $\frac{d}{dx}(x) = 1$

2. $\frac{d}{dx}(ax) = a$

3. $\frac{d}{dx}(x^n) = nx^{n-1}$

4. $\frac{d}{dx}(\cos x) = -\sin x$

5. $\frac{d}{dx}(\sin x) = \cos x$

6. $\frac{d}{dx}(\tan x) = \sec^2 x$

7. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

8. $\frac{d}{dx}(\sec x) = \sec x \tan x$

9. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x . \cot x$

10. $\frac{d}{dx}(\log x) = \frac{1}{x}$

11. $\frac{d}{dx}(e^x) = e^x$

12. $\frac{d}{dx}(a^x) = (\log a)a^x$

13. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

14. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

15. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

First order differential equations:

An equation $x, y, \frac{dy}{dx}$. This tipe of equation is known as First order differential equation.

* Solving methods of first order differential equations

They are.

① variable saparable method.

② Homogeneous method.

③ Exact Differential equation

④ Linear Differential equation

⑤ Bernolli method.

* Variable seperable method

$$f(x)dy = f(y)dx$$

Example problem:

Solve $(e^y+1)\cos x\, dx + e^y \sin x\, dy = 0$

$$(e^y+1)\cos x\, dx = -e^y \sin x\, dy$$

$$\frac{\cos x\, dx}{\sin x} = \frac{-e^y}{1+e^y} dy$$

Let Integrate on both sides

$$\int \frac{\cos x}{\sin x} dx = -\int \frac{e^y}{1+e^y} dy$$

$$\log \sin x = -\log(1+e^y) + \log c$$

$$\log \sin x + \log(1+e^y) = \log c$$

$$\cancel{\log}(\sin x.(1+e^y)) = \cancel{\log} c$$

$$\sin x(1+e^y) = c$$

$$\therefore \sin x(1+e^y) = c.$$

DEPARTMENT OF

Page No.

Date

Application of Differential equations in Real Life:

⇒ An Important Application is the "population" models.

⇒ Whether its the growth of Human population all the number of predator prey Relationship of the growth of micro organisms.

⇒ We have population which is constantly changing with respect time and this can be very well dipticted use thing Differential equation.

⇒ And one such model is the

Logistic Population Growth Model :

If P is the Population and t is the time then the rate of change of p with respect t is directly proposnal to the product of P and the difference between the carrying capacity and P.

$$\boxed{\frac{dP}{dt} = kp(N-p)}$$

where: p - population at time t

N - Carrying capacity

t - time

k - population growth rate

What is the carrying capacity: It is nothing but the maximum sustainable population.

On solving the differential equation. we get

$$\boxed{P(t) = \frac{N}{\left(\frac{N-P_0}{P_0}\right)e^{-kNt}+1}}$$

⇒ Using that we can findout the population for any given value of time.

Population: A population census is the process of collecting, compiling, analyzing and disseminating demographic, social, cultural and economic data relating to all persons in the country like India, with at a particular time in ten years interval. Conducting

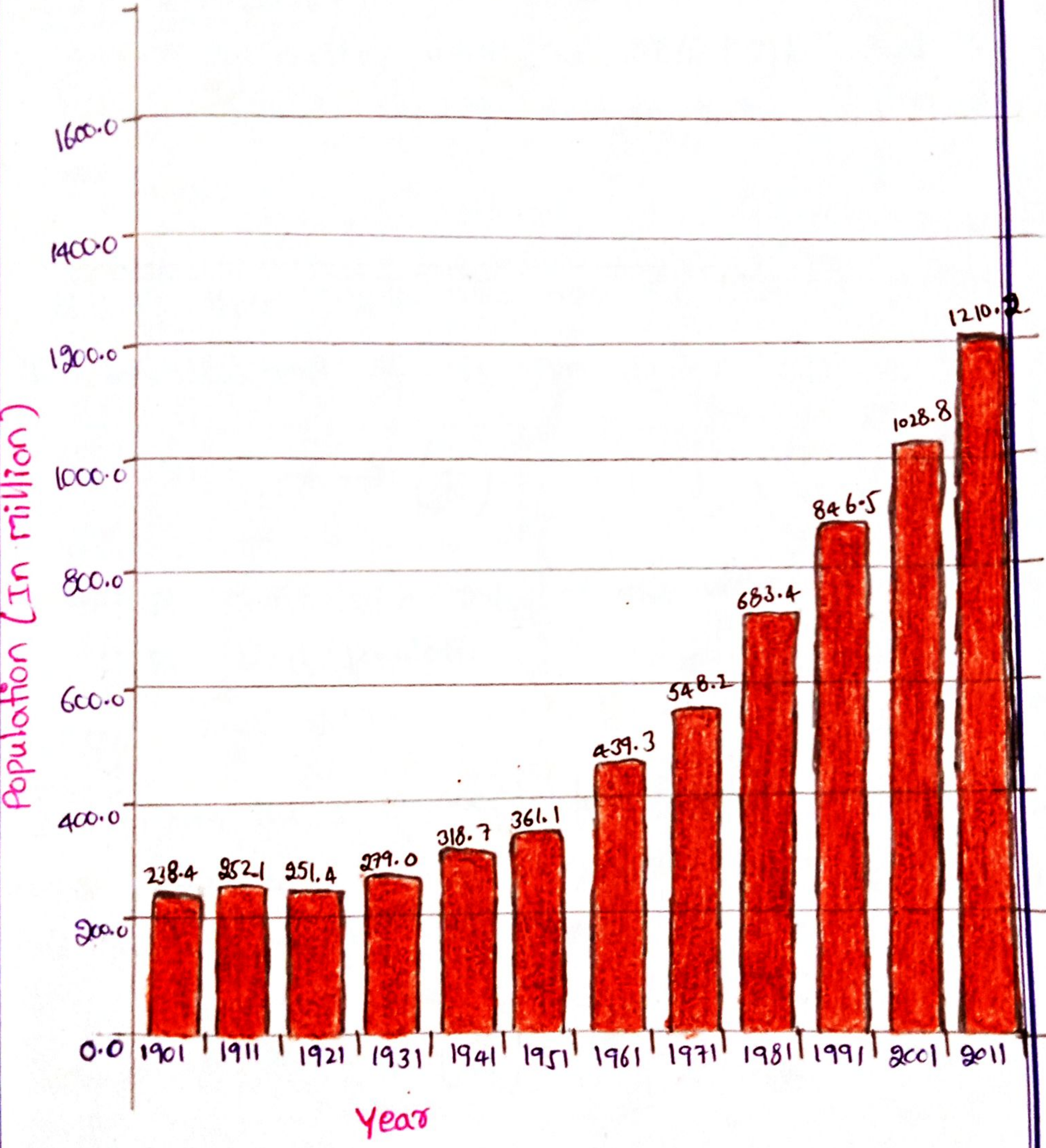
like India, with great population census in a country is information collected diversity of physical features, is information collected avaliable to through census on houses, amenities available and cultural characte the households, socia economic and cultural the ristics of the population makes India census the richest and the only source for planners, reasarch scholars, administrators and other data users. The planning and execution of India census is challenging and fascinating.

The current population of India is 1,396,525,308 based on projections of the latest United Nations data. The UN estimates the July 1, 2021 population at 1,393,409,038.

DEPARTMENT OF

Page No.

Date



DEPARTMENT OF

Page No.

Date

SEIR Model :-

⇒ Seir is the anether model. The SEIR MODEL that is being currently used to Analyse that covid Pandamic.

S-Susceptible :

That is those people who are the risk of getting the infection and $\frac{ds}{dt}$ is given by $-\beta s\left(\frac{1}{N}\right)$.

$$\boxed{\frac{ds}{dt} = -\beta s\left(\frac{1}{N}\right)}$$

where : β – Transmission rate of the virus

N – Total population.

E-Exposed

Now, From these susceptable people you have some people who will be expossed to the virus and that is denoted by Exposed.

$$\boxed{\frac{dE}{dt} = -\beta s\left(\frac{1}{N}\right) - \sigma E}$$

and σ is the infection Rate.

I - INFECTED

Now, Some of these exxposed people will actaally get Infected with the virus, and that is

$$\boxed{\frac{dI}{dt} = \sigma E - \gamma I}$$

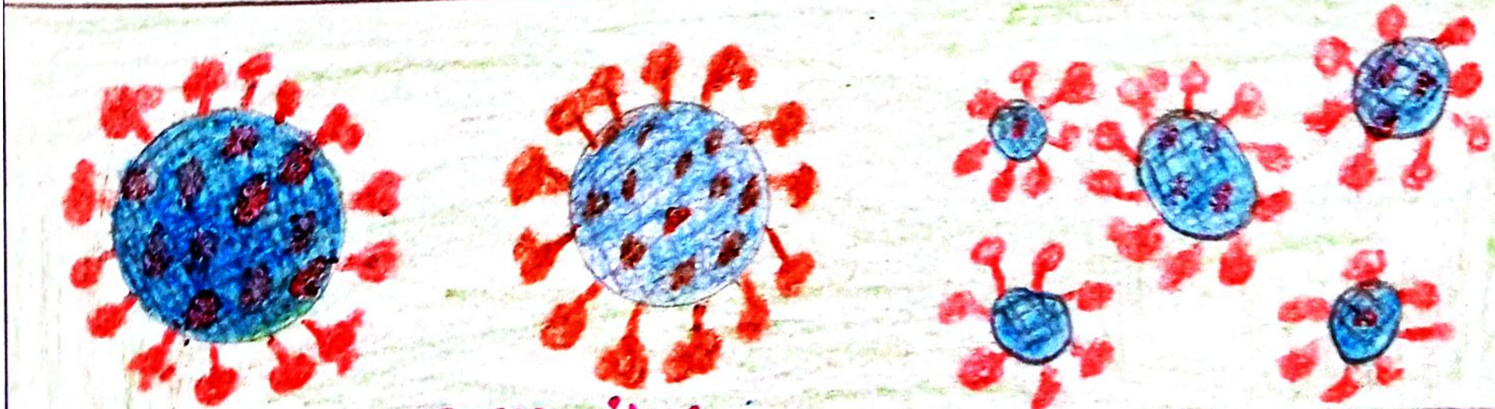
here γ = Recovery Rate.

R - RECOVARED (Removed)

Last but not the least you have R that is Removed that is those infected people who aither die and Completely recovared and this is the differential equation.

$$\boxed{\frac{dR}{dt} = \gamma I}$$

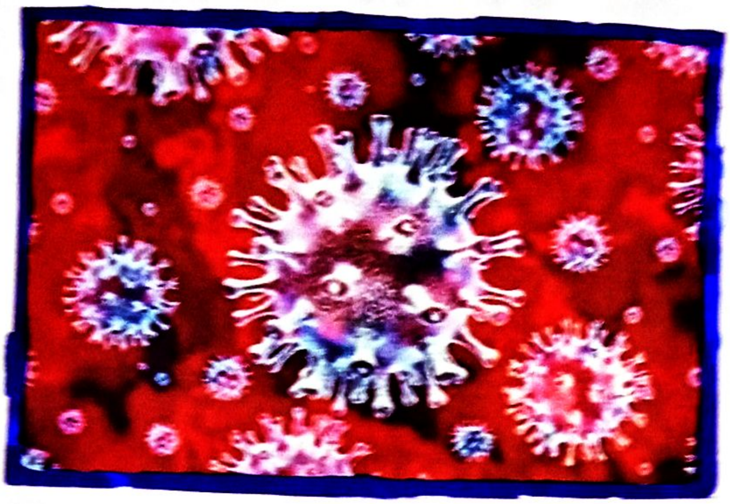
So, Friends this only underlines how in valuable this data is for all those People who are currently fighting with the Pandamic so differential equations and dowtedly is very valuable for population models.



DEPARTMENT OF

Page No.

Date



Covid -19 : The global pandamic caused by serrere acute respiratory syndrome (corona virus disese) is the major concern of manking now. The tiny virus has wiped out million of lives and changed the total economic scendio of world. The present review is formulated with a view to study and analyse the epidemics of first and second wave of covid -19 in relation to its eeffects on human health.

The world threatening covid -19 disease is swee-
ping throughout the world. Second wave of covid-19
daily case numbers have exploded since last octo-
ber 2020 in India. Rising of daily covid -19 cases
has worstly impact on people. The scenario is getting
aweful day by day due to people's carelessness.
limitted availability of vaccine etc. Many resea-
rcher's analysis that the major country U.S. and
Europe, including the proposed method.

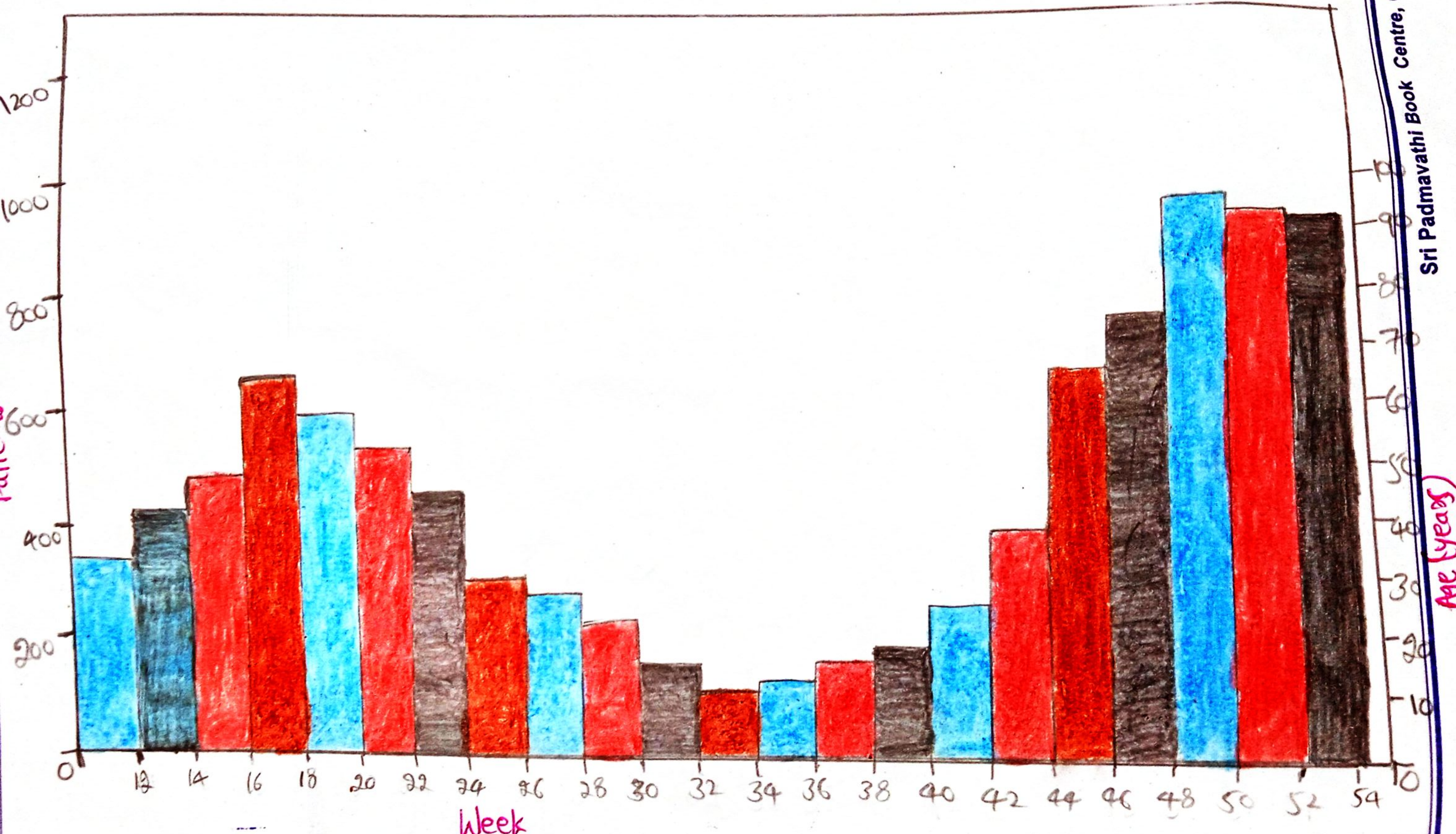


Fig: The hospitalized patients in the first wave and second wave of covid-19 Discharged.

Date

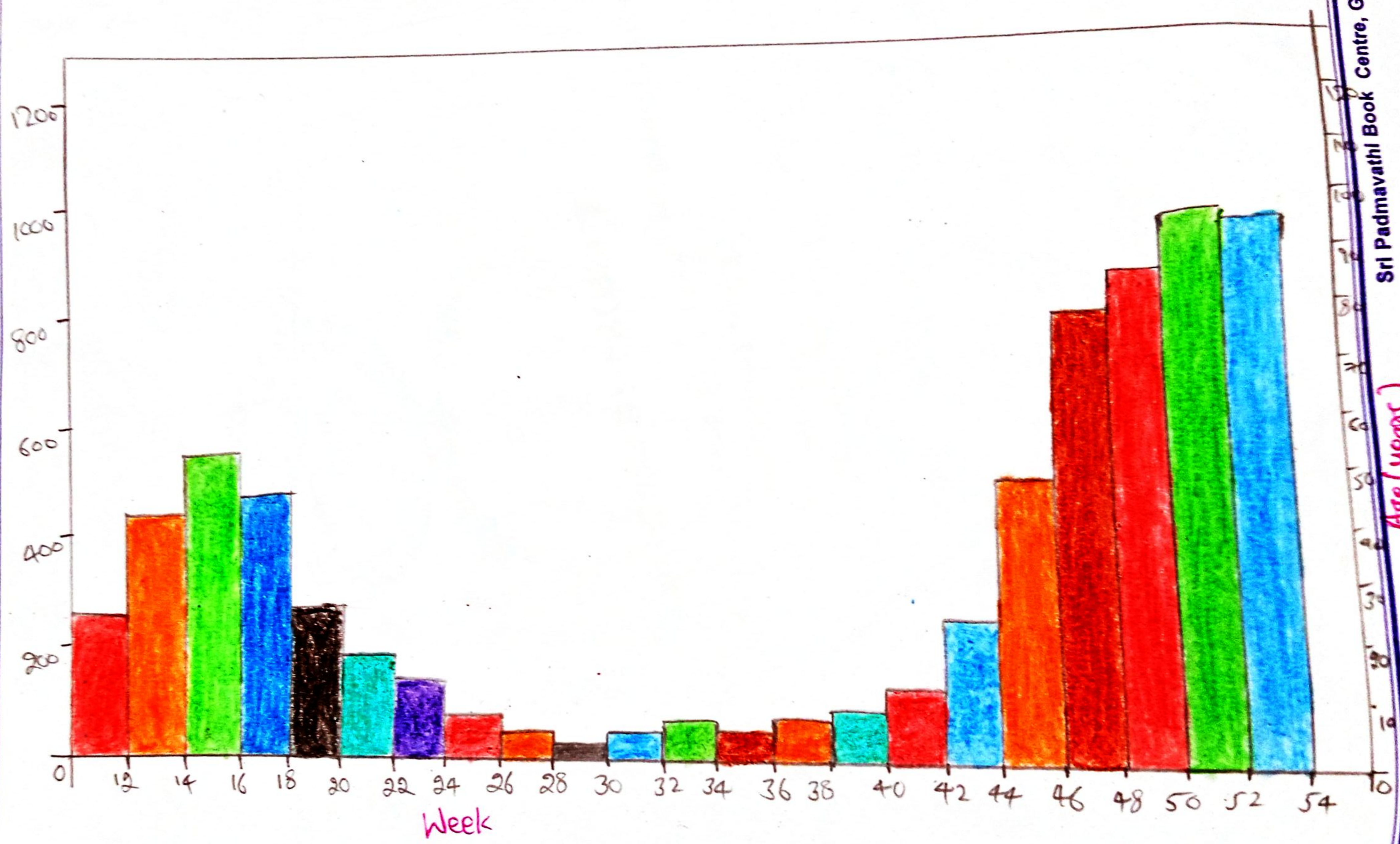


Fig: The haspitalized Patients in the first wave and second wave of covid-19 Died

2018-19

M.A.L.D.GOVT. DEGREE COLLEGE
GADWAL.

Affiliated toPalamuru University, Mahabubnagar.

Department of Mathematics

STUDENT STUDY PROJECT

NAME OF TITLE: Group Homomorphism

NAME OF THE STUDENTS: G. Ramyashree
Sumayya
Nasreen
Swathi
B. Jayalaxmi

In charge
Department of Mathematics

PRINCIPAL
M.A.L.D. Govt. Arts & Science College
GADWAL - 509 125

MALD GOVT ARTS & SCIENCE COLLEGE

DEPARTMENT OF MATHEMATICS

Student Study Project -

Group Homomorphism

Students :

1. G. RamyaSree
2. Sumayya
3. Nasreen
4. Swathi
5. B. Jayalaxmi

A+

Project Guide.

V. Manoj Kumar.

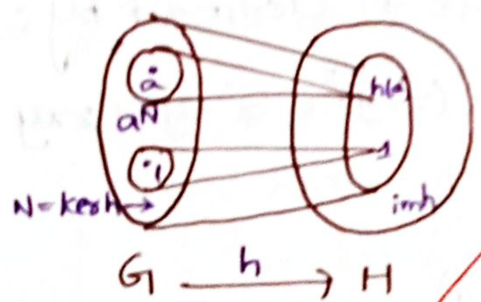
Asst. prof. of Mathematics.

:- Group Homomorphisms :-

In mathematics, given two groups, $(G,*)$ & $(H,\cdot)$, a group homomorphisms from $(G,*)$ to $(H,\cdot)$ is a function $h: G \to H$ such that for all u and v in G it holds that

$$h(u * v) = h(u) \cdot h(v)$$

Where the group operation on the left hand side of the equation is that of G and on the right hand side that of H.



A mapping from one algebraic system to a similar algebraic system with certain conditions is called a Homomorphism

* Homomorphism :-

Let G and $\bar{G}$ be two groups. A mapping $\phi: G \to \bar{G}$ is a homomorphism if

$$\phi(ab) = \phi(a)\phi(b) \text{ for all } a,b \in G$$

If $*$ and $\circ$ are the binary operation on G and $\bar{G}$ respectively then, $\phi: G \to \bar{G}$ is a homomorphism implies $\phi(a * b) = \phi(a) \circ \phi(b)$

$\forall a, b \in G$

* Examples :-

i) Consider the groups $(R^+, \cdot)$ and $(R, +)$. If $\phi : R^+ \to R$ is defined by $\phi(a) = \log_2 a$, for $a \in R^+$, then

$$\phi(a \cdot b) = \log_2 (a \cdot b) = \log_2 a + \log_2 b$$
$$= \phi(a) + \phi(b), \ \forall \ a, b \in R^+.$$

$\therefore$ Thus ϕ is a homomorphism.

ii) consider $\phi : (R, +) \to (R, +)$ defined by : $\phi(x) = x^2$, $\forall x \in R$, then $\phi(x+y) = (x+y)^2 = x^2 + y^2 + 2xy$

but $\phi(x) + \phi(y) = x^2 + y^2$

Thus $\phi(x+y) \neq \phi(x) + \phi(y)$

$\therefore$ ϕ is not a homomorphism on R

* <u>Homomorphism image</u> :- Let G and $\bar{G}$ be two groups and $\phi : G \to \bar{G}$ be a homomorphism. Then the homomorphism image of ϕ is defined as

$$\phi(G) = \{ b \in \bar{G} : b = \phi(a) \}$$

Here $\phi(G) \subseteq \bar{G}$.

* <u>Kernel of ϕ</u>: Let G and $\bar{G}$ be two groups and $\phi : G \to \bar{G}$ be a homomorphism. Then the kernel of ϕ is defined to be the set: $\ker \phi = \{ a \in G : \phi(a) = \bar{e} \}$, where $\bar{e}$ is the identity in $\bar{G}$. Here, $\ker \phi \subseteq G$

* Properties of Homomorphism:

Let ϕ be a homomorphism from a group G to a group G', & Let a be the elements of G then

1) $\phi(e) = e'$ where e, e' are identities in G & G' respectively

2) $\phi(a^n) = (\phi(a))^n \ \forall\ n \in \mathbb{Z}$ & hence $\phi(a^{-1}) = (\phi(a))$

3) If $|a|$ is finite then $|\phi(a)|$ divides $|a|$

4) $\ker\phi$ is a subgroup of G

5) The homomorphic image $\phi(G)$ is a subgroup of G'. $(\phi(G) \subseteq G')$

1) **Proof :-** Let $\phi : G \to G'$ be a homomorphism, e, e' identities in G & G', consider $\phi(e.e) = \phi(e).\phi(e)$ (ϕ is homomorphism)

$$\phi(e) = \phi(e).\phi(e) \quad \text{(Identity law)}$$
$$e'\phi(e) = \phi(e).\phi(e) \quad \text{(Identity in } G')$$
by R.C.L
$$e' = \phi(e)$$

2) **Proof :-** for $n = 2$

$$\phi(a.a) = \phi(a).\phi(a)$$
$$\phi(a^2) = (\phi(a))^2$$

$\therefore$ for $n=2$ The statement is true

Assume statement is true for $n = k$

i.e $\phi(a^k) = (\phi(a))^k$.

Consider $\phi(a^{k+1}) = \phi(a^k.a) = \phi(a^k).\phi(a)$
$$= (\phi(a))^k.\phi(a)$$
$$= (\phi(a))^{k+1}.$$

for $n = k+1$, statement is true by Mathematical induction.

3) Proof :-

Let $|a|$ is finite, Let $|a|=n$ $\qquad$ $|a|=n$

$\exists$ least +ve integer $n \ \exists \ a^n = e$ $\qquad$ $a^m = e$

$\phi(a^n) = \phi(e)$ $\qquad$ n/m

$(\phi(a))^n = e'$

$|\phi(a)|/n$

4) Proof:- $\phi : G \to G'$

$\ker\phi = \{a \in G / \phi(a) = e'\}$

$\ker\phi \leq G$

by Prop ① $\quad \phi(e) = e'$

$e \in \ker\phi$

$\therefore \ker\phi \neq \phi$

Let $a, b \in \ker\phi = \phi(a) = e', \phi(b) = e'$

consider $\phi(ab^{-1}) = \phi(a) \cdot \phi(b^{-1})$

$= \phi(a) \cdot (\phi(b))^{-1}$

$= e' \cdot (e')^{-1}$

$= e' \cdot e'$

$\phi(ab^{-1}) = e'$

$a, b \in \ker\phi \quad \phi(ab^{-1}) = e'$

$ab^{-1} \in \ker\phi$

$\ker\phi \leq G$

5) Proof:- $\phi(G) = \{\phi(a) / a \in G\}$

$e' = \phi(e) \in \phi(G)$ $\qquad$ $\phi(a \cdot b) = \phi(a) \cdot \phi(b)$

$e' \in \phi(G)$

$\phi(G) \neq \phi$

Let $x, y \in \phi(G) \Rightarrow \exists \ a, b \in G$

$x = \phi(a), \ y = \phi(b)$

$xy^{-1} = \phi(a) \cdot (\phi(b))^{-1}$

$= \phi(ab^{-1}) \in \phi(G)$

$x, y \in \phi(G) \Rightarrow xy^{-1} \in \phi(G) \qquad \therefore \phi(G) \leq G'$

$\phi: G \to G/de'$

* **Theorem :-** Let ϕ be a homomorphism from $G \to G'$ then $\ker\phi$ is a normal subgroup of G.

Proof:- $\phi: G \to G'$ is a homomorphism

by defn $\ker\phi = \{a \in G / \phi(a) = e'\}$

$\phi(e) = e' \Rightarrow e \in \ker\phi$

$\ker\phi \neq \phi$

Let $a, b \in \ker\phi = \phi(a) = e'$, $\phi(b) = e'$

consider $\phi(ab^{-1}) = \phi(a) \cdot \phi(b^{-1})$

$= \phi(a) \cdot (\phi(b))^{-1}$

$= \phi(a) \cdot (e')^{-1}$

$= e' \cdot e'$

$\therefore \phi(ab^{-1}) = e'$

$\therefore \ker\phi \subseteq G$

Let $h \in \ker\phi \Rightarrow \phi(h) = e'$

consider $\phi(xhx^{-1}) = \phi(x) \cdot \phi(hx^{-1})$

$= \phi(x) \cdot \phi(h) \cdot \phi(x^{-1})$

$= \phi(x) \cdot e' \cdot (\phi(x))^{-1}$

$= \phi(x) \cdot (\phi(x))^{-1}$

$\phi(xhx^{-1}) = e'$

$xhx^{-1} \in \ker\phi$; $\forall x \in h$, $h \in \ker\phi$

$\ker\phi \Delta G$

Natural homomorphism :-

Let 'G' be a group & K be the normal subgroup of G then the mapping $\phi: G \to \frac{G}{K}$ is always a homomorphism defined by $\phi(a) = Ka \ \forall a \in G$

$\phi: G \to \frac{G}{K}$ is $\phi(a) = Ka$, $\forall a \in G$

$\forall a, b \in G$ $\phi(ab) = Kab$

$= (Ka)(Kb)$

$= \phi(a)\,\phi(b)$

* Isomorphism :-

Let G and G' be two groups, a homomorphism $\phi: G \to G'$ is said to be an isomorphism. if ϕ is both one-one & onto

$\Rightarrow$ If there is an isomorphism from G onto G' then we say that G and G' are isomorphic & write it as $G \approx G'$

* **Theorem :-** Let G and G' be two groups. A homomorphism ϕ from G to G' with kernel k is an isomorphism if & only if $\ker\phi = \{e\}$

Proof :- $\phi: G \to G'$ be homomorphism

ϕ is onto

ϕ is isomorphism $\Longleftrightarrow \ker\phi = \{e\}$

$\Rightarrow$ Let $\phi: G \to G'$ be onto homomorphism

Assume ϕ is isomorphism

$\Rightarrow \phi$ is one-one

$\ker\phi = \{e\}$

$\ker\phi = \{a \in G \,/\, \phi(e) = e'\}$

$a \in \ker\phi$

$\Rightarrow \phi(a) = e'$

$\Rightarrow$ ϕ is homomorphism by (1)

$\phi(e) = e'$

$\phi(a) = \phi(e)$ (ϕ is one-one)

$a = e$

$\ker\phi = \{e\}$

$\Leftarrow$ Let $\ker\phi = \{e\}$

$\phi: G \to G'$ be onto homomorphism

Claim :- ϕ is one-one

Let $\phi(a) = \phi(b)$

$\phi(a).(\phi(b^{-1})) = e'$

$\phi(ab^{-1}) = e'$

$ab^{-1} \in \ker\phi$

$ab^{-1} = 1$

$(ab^{-1})b = eb$

$a(b^{-1}b) = b$

$ae = b$

$a = b$

ϕ is one-one

$\therefore$ ϕ is isomorphism

Fundamental theorem of Homomorphism :-

Let G & G' be two groups and $\phi : G \to G'$ be a homomorphism then $\frac{G}{\ker\phi} \approx \phi(G)$

Let G & G' be the two groups & $\phi : G \to G'$ be homomorphism

$k = \ker\phi = \{ x \in G / \phi(x) = e' \}$

$\ker\phi \trianglelefteq G$

we can construct $\frac{G}{\ker\phi} = \{ak / a \in G\}$

ϕ is homomorphisms $\Rightarrow \phi(G) \leq G'$

$f : \frac{G}{\ker\phi} \to \phi(G)$

defined by $f(ak) = \phi(a)$ for $ak \in \frac{G}{k}$

Claim :- f is well defined

$ak = bk$

$a^{-1}(ak) = a^{-1}(bk)$

$ek = a^{-1}bk$

$k = a^{-1}bk$ by (1) $a^{-1}b \in k \Rightarrow \phi(a^{-1}b) = e'$

$\phi(a^{-1})\phi(b) = e$

$\phi(a)(\phi(a))^{-1}.\phi(b) = \phi(a)$

$e'\phi(b) = \phi(a) \Rightarrow \phi(b) = \phi(a)$

$f(ak) = f(bk)$

$\therefore f$ is well defined

Claim: f is one-one

$ak, bk \in \frac{G}{K}$

$f(ak) = f(bk)$

$\phi(a) = \phi(b)$

$(\phi(a))^{-1} \cdot \phi(a) = (\phi(a))^{-1} \cdot \phi(b)$

$e' = \phi(a^{-1}) \cdot \phi(b) = \phi(a^{-1}b)$

$a^{-1}b \in \ker\phi = K$

$a^{-1}b \in K$

$a^{-1}bK = K \Rightarrow a(a^{-1}b)K = aK$

$ebK = aK$

$aK = bK$

$\therefore f$ is one-one

Claim :- $y \in \phi(G)$

$y = \phi(a)$ for some $a \in G$

$\exists\, a, K \ni f(aK) = \phi(a) = y$

$\therefore f$ is onto

Claim :-

f is homomorphism

$\forall\, ak, bk \in \frac{G}{K}$

$f(ak, bk) = f(abk)$

$= \phi(ab)$

$= \phi(a) \cdot \phi(b)$

$f(ak, bk) = f(ak) \cdot f(bk)$

$\therefore f : \frac{G}{\ker\phi} \to \phi(G)$ is homomorphism, one-one, & onto.

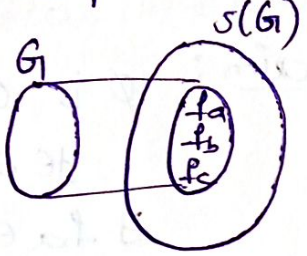
$\therefore f$ is isomorphism.

Cayley's theorem :-

Every group is isomorphic to some permutati-on group.

Proof:- Let G be a group.

A bijective function from G to G is a permutation on G



$f_a : G \to G$

defined by $f_a(x) = ax \ \forall x$

claim :- f_a well defined

Assume $x = y$, $a \in G$

$ax = ay$

$f_a(x) = f_a(y)$

$\therefore f_a$ is well defined

f_a is one-one

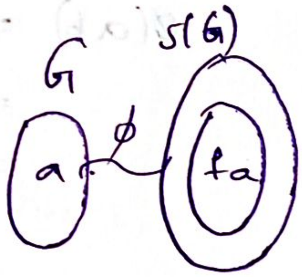
$f_a(x) = f_a(y)$

$ax = ay \Rightarrow a^{-1}(ax) = a^{-1}(ay)$

$\Rightarrow ex = ey$

$x = y$

$\therefore f_a$ is one-one



f_a is onto

$y \in G$

$f_a(x) = y$

$ax = y$

$x = a^{-1}y$

$\therefore f_a$ is onto

$f_a : G \to G$ is bijective

$\therefore f_a$ is permutation

laim:- ϕ is well defined

Assume $a = b$

$ax = bx \ \forall x$

$f_a(x) = f_b(x) \ \forall x$

$f_a = f_b$

$\phi(a) = \phi(b)$

$\therefore \phi$ is well defined

ϕ is one - one

$\phi(a) = \phi(b)$

$f_a = f_b$

$x . f_a = x f_b \Rightarrow f_a x = f_b x$

$ax = bx$

$a = b$

$\therefore$ ϕ is one-one

Claim: ϕ is onto

$y \in S(G)$

$\exists f_a \in S(G)$

$y = f_a$

for $a \in G$

$\exists a \in G, \phi(a) = f_a$

$\phi(a) = y$

$\therefore$ ϕ is onto

Claim: ϕ is homomorphism

$\phi(ab) = f_{ab}$

$= f_{ab}(x) \ \forall x$

$= abx$

$= a(bx)$

$= a(f_b(x))$

$= f_a(f_b(x))$

$= f_a f_b$

$= \phi(a) . \phi(b)$

$G \approx$ subgroup of permutation group.

Automorphisms :-

~~Let G be Group. An~~

Let 'G' be a group. an isomorphism of G onto itself is called Automorphism.

Ex: The mapping $\phi: C \to C$ defined by $\phi(a+ib) = a-ib$

claim: ϕ is homomorphism.

$z_1, z_2 \in C$

$\phi(z_1+z_2) = \overline{z_1+z_2}$

$= \bar{z}_1 + \bar{z}_2 = \phi(z_1) + \phi(z_2)$

ϕ is one-one

$\ker\phi = \{z \in C / \phi(z) = 0\}$

$= \{z \in C / \bar{z} = 0\}$

$= \{0\}$

$\therefore \phi$ is one-one

ϕ is onto for each $z \in C$

$\therefore \phi$ is onto

$\therefore \phi: C \to \bar{C}$ is an automorphism.

Theorem :-

Let G be a group the set of all automorphisms on G forms a group under the compositions of mappings.

Proof: G be a group

Automorphism of $G = \{f : / f: G \to G \text{ an Iso}\}$

$I: G \to G$ defined by

$I(x) = x \quad \forall x \in G$

$\forall\, x, y \in G \quad I(xy) = xy$

$= I(x) I(y)$

$\therefore$ $I \in Aut(G)$ & $aut(G) \neq 0$

1) $f \circ g \in Aut(G)$

$f: G \to G$, $g: G \to G$

$f \circ g: G \to G$

$\therefore f \circ g \in Aut(G)$

2) Functions are associative, automorphisms are also functions

$\therefore$ automorphisms are associative

3) Identity function is an identity in $aut(G)$

4) Let $f \in Aut(G) \Rightarrow f$ is bijective $\Rightarrow f^{-1}$ exists and also it is homomorphism

$\therefore f^{-1} \in Aut(G)$

$\therefore (Aut\,G, \circ)$ is a group.

* **Inner Automorphisms:**

Let G be a group and $a \in G$ the function $\phi_a: G \to G$ defined by $\phi_a(x) = axa^{-1}$ $\forall x \in G$ is called an Inner automorphism of G induced by a

* **Group of Inner automorphisms:**

Let G be a group the set of all Inner automorphisms on G forms a group under the composition of Inner automorphisms It is called the group of Inner automorphism denoted by $Inn(G)$

If $G = \{a, b, c \ldots\}$

then $Inn(G) = \{\phi(a), \phi(b), \phi(c) \ldots\}$

2018-19

M.A.L.D.GOVT. DEGREE COLLEGE GADWAL.

Affiliated to Palamuru University, Mahabubnagar.

Department of Mathematics

STUDENT STUDY PROJECT

NAME OF TITLE: Eigen values and Eigen vectors

NAME OF THE STUDENTS:
Neha III B.Sc (MPC)
Jyothi III B.Sc (MPC)
Hemalatha III B.Sc (MPC)
Bheemeshwari III. B.sc (MPC)
Kaveri III B.sc (MPC)

In charge
Department of Mathematics

PRINCIPAL
M.A.L.D. Govt. Arts & Science College
GADWAL - 509 125

MALD GOVT ARTS & SCIENCE COLLEGE

DEPARTMENT OF MATHEMATICS.

Student Study Project

Eigen Values & Eigen Vectors.

Students :

1. Neha
2. Jyothi
3. Hemalatha
4. Bheemeshwari
5. Kaveri

Project Guide :-

V. Manoj Kumar

Asst. prof. of Mathematics.

Eigen values and Eigen vectors :-

Introduction to Eigen values :-

linear equation $Ax=b$ comes from steady state
roblems. Eigen values have their greatest importan
e in dynamic problems. the solution of $du/dt=Au$
s changing with time growing or decaying or
scillating. we can't find it by elimination.
This chapeter enters a new part of linear
gebra, based on $Ax=\lambda x$. All matrices in this
apeter are square.

A good model comes from the powers A, A^2, A^3
... of a matrix. suppose you need the hundre
th power A^{100}. the starting matrix A becomes
unrecognizable after a few steps. and A^{100} is very
close to [6.6; 4.4]:

$$\underset{A}{\begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}} \quad \underset{A^2}{\begin{bmatrix} .70 & .45 \\ .30 & .55 \end{bmatrix}} \quad \underset{A^3}{\begin{bmatrix} .650 & .525 \\ .350 & .475 \end{bmatrix}} \cdots$$

A^{100} was found by using the Eigen values of A.
not by multiplying 100 matrices. those Eigen value
(here they are 1 and 1/2) are new way to see into
the heart of a matrix. to explain eigen values
we, first explain eigen vector. Almost all vector
change direction, when they are multiplied A.
certain exceptional vectors x are in the same
direction as Ax. those are the Eigen vectors

Multiple an Eigen vectors by A. and the vector Ax is a number λ times the original x.
the basics equation in $Ax = \lambda x$. the number λ is an eigen value of A.
the eigen value tells wheather the special vector x is stretched or shrunk or reversed or left unchanged - when it is multiplied by A.
We May find $\lambda = 2$ (or) $1/2$ or -1 or 1. the Eigen value λ could be zero ; then $Ax = 0x$ mean that this Eigen vector x is in the Null space.

If A is the Identity matrix. every vector has $Ax = x$. All vectors are Eigen vectors of I. All Eigen values lambda are $\lambda = 1$. This is unusual to say the least. Most 2 by 2 matrices have two Eigen vector directions and two Eigen values. We will show that $\det(A - \lambda I) = 0$

Example :1 :-

the matrix A has two Eigen values $\lambda = 1$ and $\lambda = 1/2$. Look at $\det(A - \lambda I)$ $A = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}$

$$\det \begin{bmatrix} .8-\lambda & .3 \\ .2 & .7-\lambda \end{bmatrix} = \lambda^2 - \frac{3}{2}\lambda + \frac{1}{2} = (\lambda - 1)(\lambda - \frac{1}{2})$$

I factored the quadratic into $\lambda - 1$ time $\lambda - \frac{1}{2}$ to see the two Eigen values $\lambda = 1$ and $\lambda = \frac{1}{2}$
for the number. the matrix $A - \lambda I$ becomes singular (zero determinant) the Eigen vectors x_1 and x_2 are in the null spaces of $A - I$.

and $A - \frac{1}{2}I$

$(A - I)x_1 = 0$ in $Ax_1 = x_1$ and the first eigen vectors is $(.6, .4)$

$(A - \frac{1}{2}I)x_2 = 0$ in $Ax_2 = \frac{1}{2}x_2$ and the second eigen vector $(1, -1)$ $x_1 = \begin{bmatrix} .6 \\ .4 \end{bmatrix}$ and $Ax_1 = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}\begin{bmatrix} .6 \\ .4 \end{bmatrix}$

$= x_1$ ($Ax = x$ means that $\lambda_1 = 1$)

$x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $Ax_2 = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} .5 \\ -.5 \end{bmatrix}$ (.. this $\frac{1}{2}x_2$

b so $\lambda_2 = \frac{1}{2}$)

If x_1 is multiplied again by A, we still get x_1. every power of will given $A^n x_1 = x_1$. multiplying x_2 and A have $\frac{1}{2}x_2$. and if we multiply given we get $(\frac{1}{2})^2$ this x_2

When A is squared, this eigen vector stay the same. the eigen values are squared.

this pattern keep going, because the eigen vectors stay in their own directions (Fig. 6.1) and never get mixed. the eigen vectors of A^{100} are the same x_1 and x_2. the eigen values of A^{100} are $1^{100} = 1$ and $(\frac{1}{2})^{100}$ = very small number

$\lambda = 1$ $\quad Ax_1 = x_1 = \begin{bmatrix} .6 \\ .4 \end{bmatrix}$ $\quad \lambda^2 = 1$ $\quad A^2 x_1 = (1)^2 x_1$

$\lambda^2 = .25$ $\quad A^2 x_2 = (.5)^2 x_2 = \begin{bmatrix} .25 \\ -.25 \end{bmatrix}$

$Ax_2 = \lambda_2 x_2 = \begin{bmatrix} .5 \\ -.5 \end{bmatrix}$

$x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$Ax = \lambda x$

$A^2 x = \lambda^2 x$

Fig. 6.1 the eigen vectors keep their direction. A^2 has eigen values 1^2 and $(.5)^2$ other vector do change direction but all other vectors are combination of the two eigen vectors. the first column of A is the combination $x_1 + (.2)x_2$

separate into eigen vectors $\begin{bmatrix} .8 \\ .2 \end{bmatrix} = x_1 + (.2)x_2 = \begin{bmatrix} .6 \\ .4 \end{bmatrix} + \begin{bmatrix} .2 \\ -.2 \end{bmatrix}$

Multiplying by a given (.7 .3). the first column of A^2, do it separately for x_1 and $(.2)x_2$ of course $Ax_1 = x_1$. and Multiple x_2 by its eigen value $\frac{1}{2}$ Multiply each x_i by λ_i $A\begin{bmatrix} .8 \\ .2 \end{bmatrix} = \begin{bmatrix} .7 \\ .3 \end{bmatrix}$ is $x_1 + \frac{1}{2}(.2)x_2 = \begin{bmatrix} .6 \\ .4 \end{bmatrix} + \begin{bmatrix} .1 \\ -.1 \end{bmatrix}$

each eigen vector is multiplied by its eigen vectors cors eigen value when we Multiply by A. 6000 was not exact we left out $(.2)(\frac{1}{2})^{99}$ which wouldn't show up for 30 decimal places

the eigen vectors x_1 is a steady state that does'nt change (because $\lambda_1 = 1$) the eigen vector x_2 is a decaying mode that virtually disappears. (because $\lambda_2 = .5$) the higher the power of A, the closer its column approach the steady state. we mention that this particular A is a Markov matrix. its entries are positive and every column adds to 1. those facts guarantee that the largest eigen value is $\lambda = 1$ (as we found) it eigen vector $x_1 = (.6, .4)$ is the steady state. which all column of A^k will approach. section 8.3 show how markov matrices appear in applications like to the for projection we can spot the steady state $(\lambda = 1)$ and the null space $(\lambda = 0)$

example (2) :- the projection $P = \begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix}$ has Eigen values $\lambda = 1$ and $\lambda = 0$

its Eigen vector are $x_1 = (1,1)$ and $x_2 = (1,-1)$ for those vectors $px_1 = x_1$ (steady states) and $px_2 = 0$ (Null space) ·This example illustrates Markov matrices and singular matrices and (most improtant) symmetric matrices. All have special λ's and x's

1· Each column of $p = \begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix}$ also to 1 so $\lambda = 1$ is an Eigen value·

2· p is singular so $\lambda = 0$, is an Eigen value

3· p is symmetric ·so its Eigen vectors $(1,1)$ and $(1,-1)$ are perpendicular

The only eigen values of a projection matrix are 0 and 1· the Eigen vectors for $\lambda = 0$ (which means $px = 0x$) fill up the Null space. the Eigen vectors for $\lambda = 1$ (which mean $px = x$) fill up the column space. the Null space is projections keeps the column space and destory the Null spaces

project each part $v = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ projects onto $pv = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

Special properties of a matrix lead to Special eigen values and Eigen vectors,

this is a major theme of this chapter (it is captured in a table at the very end)
projection have $\lambda = 0$ and 1 permutations have all $|\lambda| = 1$ the next matrix R (a reflection and the same time a permutation) is also special

example :- (3) the reflection matrix $R = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ has Eigen values 1 and -1

the Eigen vector (1,1) is unchanged by R. the second eigen vector is (1,-1) its signs are reversed by R. A matrix with no negative entries can still have a negative eigen value the eigen vectors for R are the same as for P. because reflection = 2 (projection) - I".

$R = 2P - I$ $\quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = 2 \begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Here is the point If $Px = \lambda x$ then $2Px = 2\lambda x$. the Eigen value are doubled when the matrix is doubled, Now subtract $Ix = x$, the result is $(2P - I)x = (2\lambda - I)x$ when a matrix is shifted I, each λ is satisfied. 1 no change in Eigen vectors.

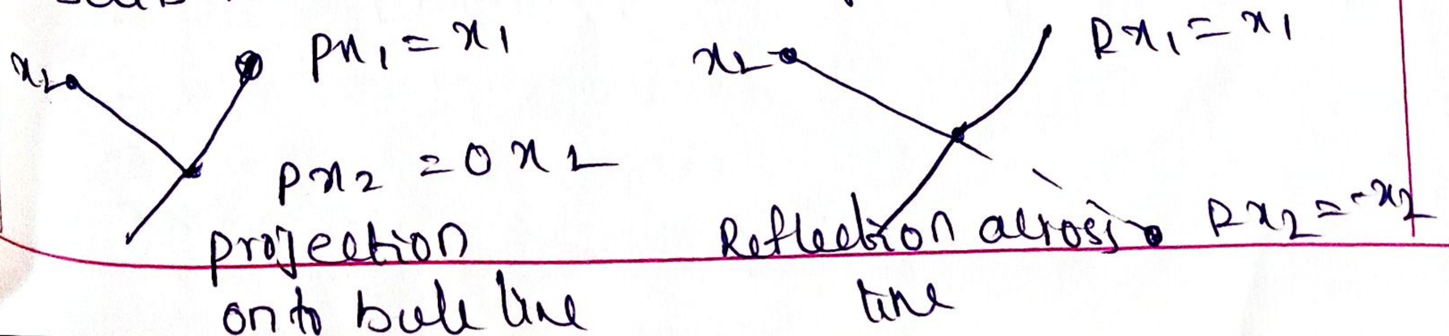


fig. 6.2 projection P have Eigen values 1 and 0. Reflection R have $\lambda = 1$ and -1. A typical x changes direction, but not the eigen vectors x_1 and x_2

key idea. the Eigen value of R and P are related exactly as the matrices are related. the eigen values of $R = 2P - I$ are $2(1) - 1 = 1$ and $2(0) - 1 = -1$

the eigen values R^2 and λ^2, In this case $R^2 = I$

Check $(1)^2 = 1$ and $(-1)^2 = 1$

The equation for Eigen values :-

for prejection and reflection we found λ's and x s by geometry $Px = x$, $Px = 0$ $Rx = -x$. NOW we use determinants and linear algebra This is the key calculation in the chapeter almost every applications starts by solving $Ax = \lambda x$

first move λx to the eigen to the left side write the equation $Ax = \lambda x$ as $(A - \lambda I)x = 0$ the matrix $A - \lambda I$ times the Eigen vector x is the zero vector, the eigen vectors make up the Null space of $A - \lambda I$, when we know an eigen value λ. we find an eigen vectors by solving $(A - \lambda I)x = 0$.

Eigen first If $(A - \lambda I)x = 0$ has non zero sol. $A - \lambda I$ is not invertible the determination of $A - \lambda I$ must be zero. this is how to recognize an Eigen value λ.

2019 - 2020

M.A.L.D.GOVT. DEGREE COLLEGE GADWAL.

Affiliated to Palamuru University, Mahabubnagar.

Department of Mathematics

STUDENT STUDY PROJECT

NAME OF TITLE: GOLDEN RATIO

NAME OF THE STUDENTS:

1. C. Sudheer Bsc[mpcs] - I [19033024468009]
2. S. Pavan Bsc[mpcs] - I [19033024468038]
3. G. Naveen Kumar Bsc[mpcs] - I [19033024468014]

In charge
Department of Mathematics

"MALD GOVT DEGREE COLLEGE

GADWAL - 509125"

: Student study project :

A NOTE ON GOLDEN RATIO

Research Guide

* Google
* Sri. Manoj kumar sir
* Library

By

1. C. Sudheer B.Sc(MPCS)-I
2. S. Pavan BSc (MPCS) I
3. G. Naveen kumar BSC(MPCS) III

Golden ratio in Simple terms:-

It is "the ratio of a line seg-
ment cut into two pieces of different lengths such"
that the ratio of the whole segment to that of the
longer segment is equal to the ratio of the longer
segment to the shorter segment ...

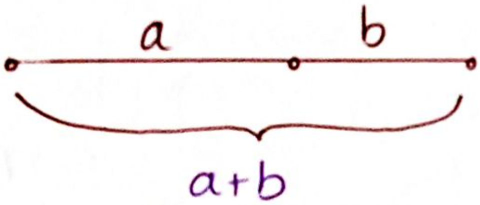
Uses of study Golden ratio:-

The composition is important
for any image, whether it's to convey important inf-
ormation or to create an aesthetically pleasing photo
-graph. The Golden ratio can help crate a composi-
-tion that will draw the eyes to the importance
of elements of the photo."

-: Golden Ratio :-

Golden Ratio :-

In mathematics, two quantities are in the golden ratio if thier ratio is the same as the ratio of their sum to the larger of the two quantities. Expressed algebraically, for quantities a and b with $a > b > 0$.



a+b is to a as a is to b

Line segments in the golden ratio

Representations :-

Decimal : 1.618 033 988 749 894....

Formula : $\phi = \frac{1+\sqrt{5}}{2}$

Continued fraction : $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}$

Binary : 1.1001 1110 0011 0111 011....

Hexadecimal : 1.9E37 79B9 7F4A 7C15..

The Golden ratio through history :-

2400 years at least is a long period of the fascination with the golden ratio not only by mathematics but also by biologists, philosophers, artists, architects, musicians and even mystics.

The golden ratio was first studied by the ancient Greek because of it's frequent appearance in geometry.

The development of the idea of the golden ratio is usually attributed to Pythagoras (580-497 BC) and his students. The symbol of pythagoras school was the regular pentagram. plato (428-347 BC) saw the world in terms of perfect geometric proportions and symmetry. His ideas were based on platonic solids: a cube for earth, a tetrahedron for fire, an octahedron for air and icosahedrons for water, some of them are related to the golden ratio.

Euclid (c.325-c. 265 BC) in his Elements gives the first known definition of "Extreme and mean ratio", i.e., the golden ratio, as we call it today. In several propositions in Elements and their proofs the golden ratio is used.

Uses of golden ratio in daily life:-

This ideal ratio is used by many because of it's apparent love of human eye. The golden ratio has been said to be the most appealing ratio, and is therefore used frequently. Every thing from commercial advertising companies, to painters, to even doctors incorporate this "magical" ratio into their work.

Used in nature:-

The golden ratio is sometimes called the "divine proportion" because of it's frequently in natural world. The number of petals on a flower, for instance, will often be a Fibonacci number. The seeds of sunflowers and pine cones twist in opposing spirals of Fibonacci number.

Golden ratio in human body:-

These include the shape of the perfect face and also the ratio of height of the navel to the height of the navel to the height of the body.... If you consider enough of them then you are bound to get numbers close to the value of the golden ratio (around

1.6(3).

Importance of golden ratio:-

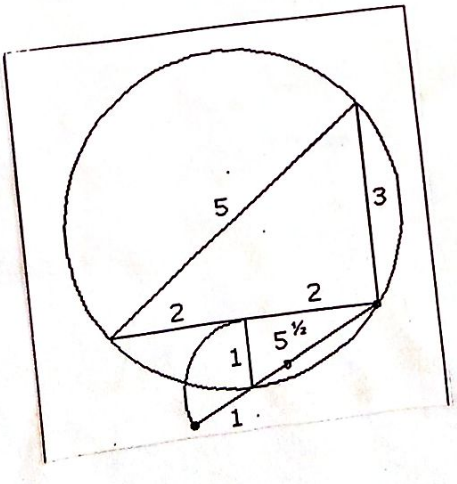
The "Composition is important for any image, whether it's to convey important information or to create an aesthetically pleasing photography. The Golden Ratio can help create a composition that will draw the eyes to the important elements of the photo. //

— A cosmic constant known as the "golden ratio" is said to be found in the shape of hurricanes, elephant tusks and even in galaxies. Now researches say this ratio is also seen in the topology of space-times affecting the entire universe as a whole.

Constructions of the Golden ratio:-

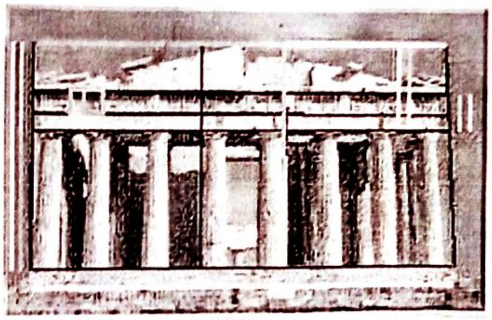
The Golden Ratio by Gabries Borsia:-

Gabriel Borsia has discovered ~~be~~ another interesting way to construct the Golden ratio. He associated the right triangle $1:2:\sqrt{5}$ with the right triangle 3-4-5 as on figure.



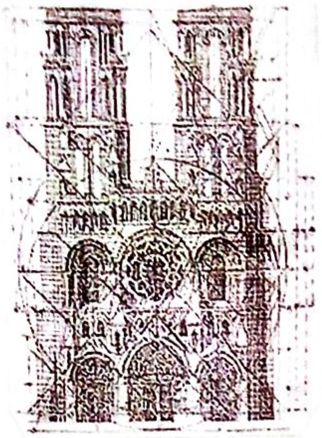
History of gold ratio:-

Throughout history, the ratio for length to width of rectangles of 1.61803 39887498948482 0 has been consider the most pleasing to the eye. This ratio was named by the golden ratio by the Greeks. In the world of mathematics, the numeric value is called "phi", named for the Greek sculptor phidias. The space between the coulmns form golden rectangles. There are golden rectangles throught this structure which is found in Athenens, Greece.

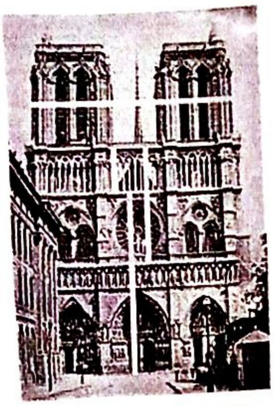


* Gothic:-

One of the most popular monuments from this period is Notre Dame Cathedral in paris (built 1163-1346). F. Macody Lung in his book Ad Quadratum (1919) claims that this church, cathedral of chartres (early 12th century), the Notre-Dame of Laon (12th-13th century) were designed according to the golden ratio.



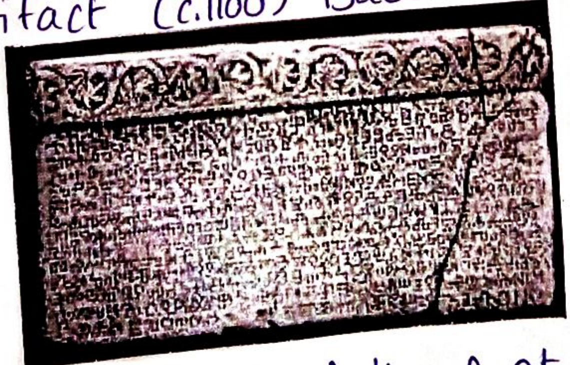
Notre-Dame of Laon



Notre-Dame of paris.

Few examples in Croatia:-

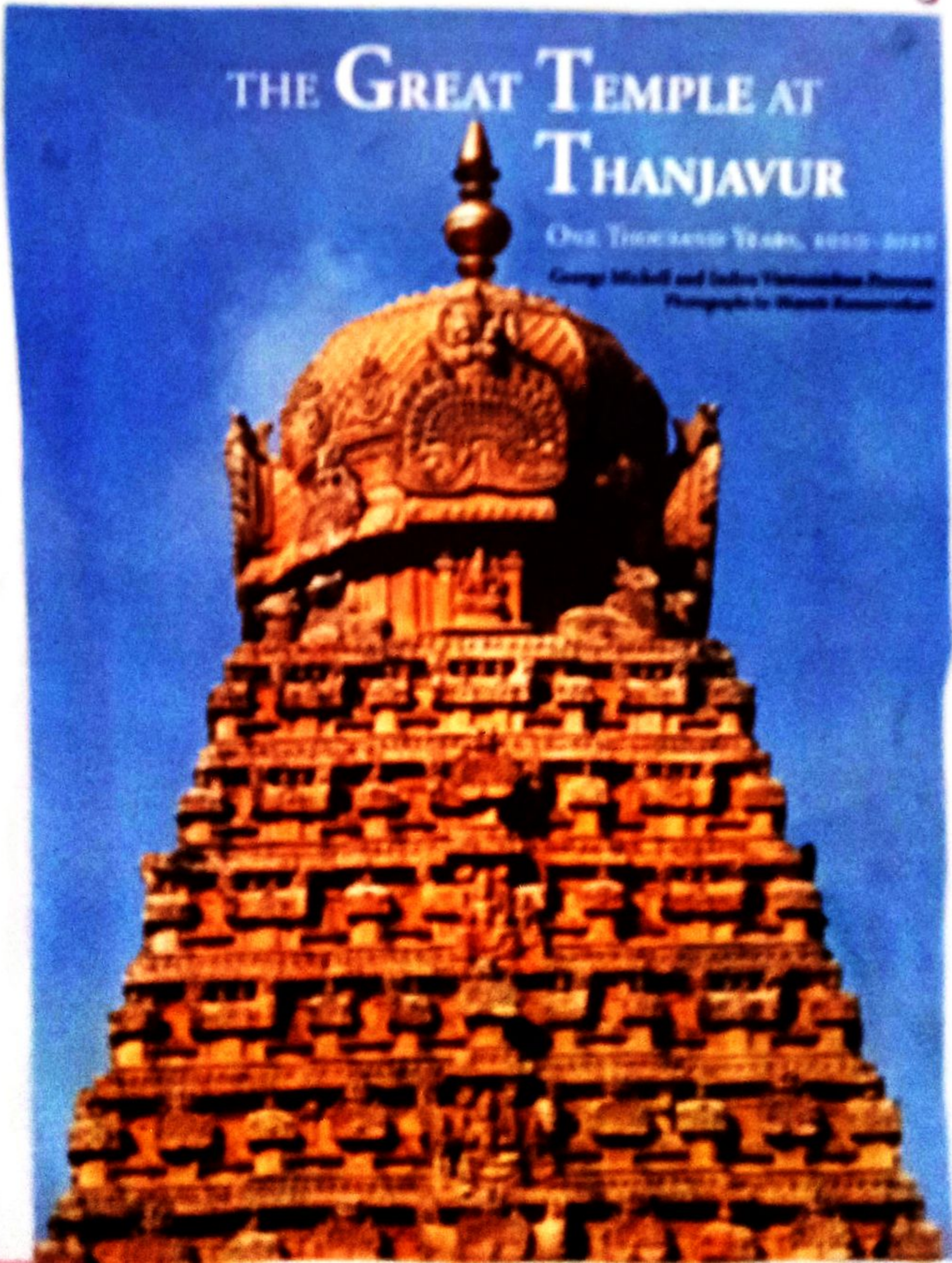
So far we have found just two examples of the exploitation of the golden ratio. The first is the oldest Croatian artifact (c.1100) Bascanka ploca.



Furthermore, the windows at the front of a Secondery school (19th) century are golden rectangles.

-: Brihadisvara Temple, Thanjavur :-

Brihadisvara Temple (originally known as 'peruvudaiyar kovil') locally known as Thanjai Periya kovil, and also called as, Raja Rajeshwaram, is a Hindu Temple dedicated to Shiva located in South bank of Cauvery river in Thanjavur, tamil Nadu, India. It is one of the largest South Indian temples an exemplary example of a fully realized Tamil Architecture. It is called as Dakshina Meru (meru of South). Built by Tamil king Raja Raja chola I between 1003 and 1010 AD, the temple is a part of the "UNESCO World Heritage site known



" Great Living chola temples", along with the Chola dynasty era angaikonda cholapuram temple and Airavatesvara temple that are about 70 kilometers (43 mi) and 40 kilometers (25 mi) to it's northeast respectively.

Ther original monuments of this 11th-century temple were built around a moat. It included gopura, the main temple, it's massive tower inscriptions, frescoes and sculptures predominantly related to ~~Shaiv~~ Shaivism.

but also of Vaishnavism and Shaktism traditions of Hinduism. The temple was damaged in it's history and some artwork is now missing. Additional mandapam and monuments were added in the centuries that followed. The temple now stands amidst fortified walls that were added after the 16th century. ~~In this~~ temple construction of architectures used golden Ratio.

2019-20

M.A.L.D.GOVT. DEGREE COLLEGE GADWAL.

Affiliated to Palamuru University, Mahabubnagar.

Department of Mathematics

STUDENT STUDY PROJECT

NAME OF TITLE: Group of cosets and Normal subgroups

NAME OF THE STUDENTS: Pallavi II B.sc (M.P.cs)
Rajitha II B.sc (M.P.cs)
Sandhya II B.sc (M.P.cs)
Nishadh II B.sc (M.P.cs)
shailaja II B.sc (M.P.cs)

In charge
Department of Mathematics

PRINCIPAL
M.A.L.D. Govt. Arts & Science College
GADWAL - 509 125

MALD GOVT ARTS & SCIENCE COLLEGE

DEPATMENT OF MATHEMATICS

Student Study project

Group of cosets and Normal subgroups

Students :-
1. pallavi
2. Rajitha
3. Sandhya
4. Nishadh
5. Shailaja

project Guide :-

V. Manoj Kumar

Asst. prof. of Mathematics

cosets and Noormal subgooups

Introoduction :- Given a subgooup H of a gooup (G . *). it is Possible to constouct subsets of G using $a \in G$. These subsets aoe called "cosets of H" - in fact left and oight cosets. cosets and noomality (some cosets may coincide. foo example . if a is in the centeo of G . then $aH = Ha$) on the otheo hand. the subgooup N is noomal if and only if $gN = Ng$ foo all g in G...

Since eveoy oight coset is a left coset. theoe is no need to distinguish "left cosets" from "oight cosets"

G

0	4	H
1	5	1+H
2	6	2+H
3	7	3+H

G is the group $(\mathbb{Z}/8\mathbb{Z}, +)$, the integers mod 8 under addition. The subgroup H contains only 0 and 4, and is isomorphic to $(\mathbb{Z}/2\mathbb{Z}, +)$. There are four left cosets of H : H itself, $1+H$, $2+H$, and $3+H$ (this is the additive group).

Defination :-

Let H be a subgroup of a group $(G, *)$ and $a \in G$. Then the set $aH = \{a * h : h \in H\}$ is called a left coset of H in G, generated by a, and the set : $Ha = \{h * a : h \in H\}$ is called a right coset of H in G generated by a in G.

Properties of cosets :-

Theorem :- Let H be any subgroup of a group G, and let $a, b \in G$ then :

1) $a \in aH$
2) $aH = H$ if and only if $a \in H$.
3) $aH = bH$ if and only if $a \in bH$.
4) $aH = bH$ (or) $aH \cap bH = \phi$
5) $aH = bH$ if and only $a^{-1}b \in H$
6) $|aH| = |bH|$

7) aH is a subgroup of G if and only if $a \in H$.

8) $\bigcup_{a \in G} aH = G$ (or) $\bigcup_{i=1}^{r} a_i H = G$, for some i.

Proof :- Let G be a group, H be a subgroup of G.

Let $a \in G$ and e be the identity in G.

1. If e is the identity in G then e is also identity in H.

Thus $a = ae \in aH$ ($\because e \in H$)

$\Rightarrow a \in aH$.

2. Let $aH = H$. To prove that $a \in H$.

from ① $a \in aH \Rightarrow a \in H$

conversely let $a \in H$. To prove that $aH = H$.

consider $ah \in aH$, for $h \in H$

$a \in H, h \in H \Rightarrow ah \in H$

$\Rightarrow aH \subseteq H$.

Also for $a \in H$, from ① $a \in aH$

Hence $H \subseteq aH$

Thus $aH = H$.

3. Let $aH = bH$

since $a \in aH$, we have $a \in bH$

$\Rightarrow b^{-1}a \in b^{-1}bH$

$\Rightarrow b^{-1}a \in eH \Rightarrow b^{-1}a \in H$

$\Rightarrow b^{-1}aH = H$

$\Rightarrow bb^{-1}aH = bH$

$\Rightarrow eaH = bH$

$\Rightarrow aH = bH.$

4. For any two cosets aH and bH, if $aH \cap bH = \phi$, then there is nothing to prove.

Let $aH \cap bH \neq \phi$

$\Rightarrow$ There exists atleast one element $c \in aH \cap bH$.

$\Rightarrow c \in aH$ and $c \in bH$

$\Rightarrow c = ah_1$ and $c = bh_2$ for some $h_1, h_2 \in H$.

Now $ah_1 = bh_2$

$\Rightarrow ah_1h_1^{-1} = bh_2h_1^{-1}$

$\Rightarrow a = b(h_3)$, for some $h_3 = h_2h_1^{-1}$ in $H \Rightarrow a = bh_3 \in bH$.

But $a \in aH$, from ①

Thus $aH \subseteq bH$

Similarly we can prove that $bH \subseteq aH$.

Hence $aH = bH$.

Thus any two left cosets are either equal or disjoint.

5. Let $aH = bH$.

Since $b \in bH$ we have $b \in aH$.

$$\Rightarrow a^{-1}b \in a^{-1}aH$$

$$\Rightarrow a^{-1}b \in eH$$

$$\Rightarrow a^{-1}b \in H.$$

Conversely let. $a^{-1}b \in H$

Then from (2) $a^{-1}bH = H$

$$\Rightarrow aa^{-1}bH = aH$$

$$\Rightarrow ebH = aH$$

$$\Rightarrow bH = aH$$

6. To prove that $|aH| = |bH|$ we will establish a one-one and onto mapping from aH onto bH.

Define a mapping $\phi : aH \rightarrow bH$ by $\phi(ah) = bh$ for

$a, b \in G$ $h \in H$.

Let $ah_1 = ah_2$ for $h_1, h_2 \in H$

$$\Rightarrow h_1 = h_2$$

$$\Rightarrow bh_1 = bh_2$$

$$\Rightarrow \phi(ah_1) = \phi(ah_2)$$

Thus ϕ is well defined.

similarly if $\phi(ah_1) = \phi(ah_2)$, for $h_1, h_2 \in H$

$$\Rightarrow bh_1 = bh_2 \Rightarrow h_1 = h_2 \Rightarrow ah_1 = ah_2$$

$\therefore \phi$ is one-one.

Also for $bh' \in bH$, we have $h' \in H$

$$\Rightarrow ah' \in aH \text{ such that } bh' = \phi(ah')$$

Thus ϕ is onto.

Hence the mapping $\phi : aH \to bH$ is one-one and onto

$$\therefore |aH| = |bH|$$

7. Let aH be a subgroup of G.

$\Rightarrow e \in aH$. But $e \in eH$.

Therefore we have $aH \cap eH \neq \phi$.

Then by (2), $aH = eH = H$

conversely let $a \in H$

Then by (2), $aH = H$.

8. Let $a_1 H, a_2 H \cdots\cdots a_r H$ be the distinct left cosets of H in G.

Then for each a in G we have

$aH = a_i H$ where $1 \le i \le r$.

Also from ① $a \in aH$.

Thus each $a \in G$ must belong to one of the cosets $a_i H$, for $1 \le i \le r$.

That is $a_1 H \cup a_2 H \ldots \cup a_r H = G$.

Lagrange's theorem :=

If G is a finite group and H is a subgroup of G. then $|H|$ divides $|G|$.

Proof :-

Let $|G| = n$. Since H is a subgroup of G. H is also finite.

Let $|H| = m$.

To prove that $|H|$ divides $|G|$, that is m divides n, we will prove ~~that~~ $\frac{n}{m}$ is an integer.

Let $a_1 H, a_2 H, \ldots a_k H$ be distinct left cosets of H in G.

Then $a_1 H \cup a_2 H \cup \ldots \cup a_k H = G$.

$\Rightarrow |a_1 H \cup a_2 H \cup \ldots \cup a_k H| = |G|$

Then Remark (3) of, we have

$$r^{|U(P)|} = 1$$

That is $r^{P-1} \equiv 1 \mod p$

$$\Rightarrow r^{p} \equiv r \mod p.$$

Hence $a^{p} \equiv a \mod p.$

Theorem :-

A group of prime order is cyclic.

proof :- Let G be a group of prime order p.

That is $|G| = p$.

Let $a \in G$ and $a \neq e$.

Let H be a subgroup of G given by $H = \langle a \rangle$.

Then $|H| \neq 1$

But by Lagranges theorem $|H|$ must divide $|G|$.

That is $|H| = p$. Since p is prime.

$$= |G|$$

Thus $H = G \Rightarrow G = \langle a \rangle$

Hence G is cyclic.

Definition :-

Let G be a group of permutations of a set S.

for each i in S,

let $Stab_G(i) = \{\phi \in G : \phi(i) = i\}$.

we call $stab_G(i)$ as the stabilizer of i in G.

Definition :=

Let G be a group of Permutations of a set S. For each s in S, let $orb_G(s) = \{\phi(s) \mid \phi \in G\}$. The set $orb_G(s)$ is a subset of S called the orbit of s under G.

Example :=

(i) Let $G = \{(1), (1\,3\,2), (4\,6\,5)(7\,8), (1\,3\,2)(4\,6\,5), (1\,2\,3)(4\,5\,6), (1\,2\,3)(4\,5\,6)(7\,8), (7\,8)\}$

Then

$orb_G(1) = \{1, 3, 2\}$, $\quad stab_G(1) = \{(1), (7\,8)\}$

$orb_G(2) = \{2, 1, 3\}$, $\quad stab_G(2) = \{(1), (7\,8)\}$

$orb_G(4) = \{4, 6, 5\}$, $\quad stab_G(4) = \{(1), (7\,8)\}$

$orb_G(7) = \{7, 8\}$, $\quad stab_G(7) = \{(1), (1\,3\,2)(4\,5\,6), (1\,2\,3)(4\,5\,6)\}$.

Theorem :=

Let G be a group of permutations of a set S. For each i in S, $stab_G(i)$ is a subgroup of G.

Proof := $stab_G(i) = \{\phi \in G : \phi(i) = i\}$.

The identity Permutation $I \in stab_G(i)$, since $I(i) = i$.

$\therefore$ $stab_G(i)$ is non empty.

Let $\alpha \in stab_G(i) \Rightarrow \alpha(i) = i$, for $i \in S$.

$\Rightarrow \alpha^{-1}(i) = i$, for $i \in S$.

Thus $\alpha^{-1} \in stab_G(i)$, for each $\alpha \in stab_G(i) \rightarrow (1)$

Let $\alpha, \beta \in stab_G(i) \Rightarrow \alpha(i) = i$ and

$\beta(i) = i$, $i \in S$

Then $\alpha\beta(i) = \alpha(\beta(i)) = \alpha(i) = i$

$\Rightarrow \alpha\beta \in stab_G(i) \rightarrow (2)$

From (1) and (2), $stab_G(i)$ is a subgroup of G.

Normal subgroups :-

A subgroup H of a group G is said to be a normal subgroup of G if $xhx^{-1} \in H$, for all $x \in G$ and $h \in H$.

Theorem :-

A subgroup H of a group G is a normal subgroup of G if and only if $xHx^{-1} = H$, for every $x \in G$.

Proof :-

If H is a normal subgroup of G, then we have $xhx^{-1} \in H$ $\forall x \in G$ and $h \in H$.

Consider $xHx^{-1} = \{xhx^{-1} : x \in G, h \in H\} \subseteq H \rightarrow (1)$

Also $x^{-1}Hx = \{x^{-1}hx : x \in G, h \in H\} \subseteq H$

Therefore, $x^{-1}Hx \subseteq H$

$\Rightarrow xx^{-1}Hx \subseteq xH$

$\Rightarrow (xx^{-1})Hx \subseteq xH$

$\Rightarrow eHx \subseteq xH \Rightarrow Hxx^{-1} \subseteq xHx^{-1}$

$\Rightarrow He \subseteq xHx^{-1}$

$\Rightarrow H \subseteq xHx^{-1} \rightarrow (2)$

from (1) & (2), we have

$xHx^{-1} = H$.

Theorem :-

Every quotient group of an abelian group is abelian.

Proof :- Let G be an abelian group.

If H is any subgroup of G then H is a normal sub group of G.

Let $\frac{G}{H}$ be the quotient group of G by H.

That is $\frac{G}{H} = \{aH : a \in G\}$

For $a, b \in G$, we have $ab = ba$

For $aH, bH \in G$, $(aH)(bH) = (ab)H$

$= (ba)H$

$= (bH)(aH)$

Thus, $\frac{G}{H}$ is abelian.

Applications of Factor Groups :-

If a group G is finite and a subgroup $H \neq \{e\}$, then the Factor group $\frac{G}{H}$ is smaller then G. we can observe:

* G and $\frac{G}{H}$ will have the same structure. $\frac{G}{H}$ is smaller than G. we can obtain less complicated approximation of G by the approximation of $\frac{G}{H}$.

$\frac{D_4}{K}$:

	K	$R_{90}K$	HK	DK
K	K	$R_{90}K$	HK	DK
$R_{90}K$	$R_{90}K$	K	DK	HH
HK	HK	DK	K	$R_{90}K$
DK	DK	HK	$R_{90}K$	K

* Some coset of K are in adjacent columns. multiplication table can be blocked off into boxes which are cosets of K in D_4.

2020-21

M.A.L.D.GOVT. DEGREE COLLEGE GADWAL.

Affiliated to Palamuru University, Mahabubnagar.

Department of Mathematics

STUDENT STUDY PROJECT

NAME OF TITLE: A Note on vector space and subspace

NAME OF THE STUDENTS:

1. B. Renuka - III B.Sc(M.P.Cs)
2. Naveena - III B.Sc(M.P.Cs)
3. B. Kaveri - III B.Sc(MPCs)
4. T. Shailaja - III B.Sc (MPCs)
5. J. Hemalatha - III B.Sc(MPCs)

In charge
Department of Mathematics

PRINCIPAL
M.A.L.D. Govt. Arts & Science College
GADWAL - 509 125

M.A.L.D Govt. Degree College (Gadwal - 509125)

Student Study Project

(A NOTE ON VECTOR SPACES AND SUBSPACES)

Research Guide

* Google
* Sri. Manoj kumar Sir Notes
* Linear Algebra Textbook

By

1. B. RENUKA B.Sc (MPCS)-III
2. NAVEENA B.Sc (MPCS)-III
3. KAVERI.B B.Sc (MPCS)-III
4. T. SHAILAJA B.Sc (MPCS)-III
5. J. HEMALATHA B.Sc (MPCS)-III

A NOTE ON VECTOR SPACES AND SUBSPACES

Contents

-: Introduction to Vector Spaces :-

This Section introduces the concept of vector space. In reality, linear algebra is the study of vector spaces and the functions of vector spaces (linear transformations). They form the fundamental objects which we will be study-ing throughout the remaining course. Once we define a vector space, we will go on to study the properties of vector spaces. Their importance lies in the fact that many mathe-matical questions can be rephrased as a question about vector spaces.

Many concepts concerning vectors in $\mathbb{R}^n$ can be expressed extended to other mathematical systems. we think of a vector space in general, as a collection of objects that behave as vectors do in $\mathbb{R}^n$. The objects of such a set are called vectors.

A vector Space is a non-empty set V of object, called vectors are which are ~~defined~~ two operations, addition and Multiplication by Scalars (real numbers). Subject to the axioms below the axioms must hold for all u, v and w in V and for all scalars c and d

1. $u+v$ is in V
2. $u+v = v+u$
3. $(u+v)+w = u+(v+w)$
4. There is a vector (called the zero vector) 0 in V such that $u+0=u$

5. For each u in V there is vector $-u$ in V satisfying $u+(-u)=0$

6. cu is in V.

7. $c(u+v) = cu+cv$

8. $(c+d)u = cu+dv$

9. $(cd)u = c(du)$

10. $1u = u$.

Examples of vector Space :-

* $\mathbb{R}^n$
* $\mathbb{R}^{m\times n}$
* $\mathbb{P}_n$
* $D(a, b)$.

Prove that $\mathbb{R}^n$ is a vector Space.

$\mathbb{R}^n = \{x_1, x_2, \dots x_n \,//\, x_i \in \mathbb{R}\}$

Let $\alpha = (x_1, x_2, \dots x_n)$

$\beta = (y_1, y_2, \dots y_n)$

$\gamma = (z_1, z_2 \dots z_n)$

1st condition

$\alpha+\beta$

$=(x_1, x_2 \dots x_n) + (y_1, y_2 \dots y_n)$

$=(x_1+y_1, x_2+y_2 \dots x_n+y_n) \in \mathbb{R}^n$

$\alpha+\beta \in \mathbb{R}^n$

2^{nd} condition

$$\alpha+(\beta+\gamma) = (x_1, x_2 \dots x_n) + (y_1, y_2 \dots y_n) + (z_1, z_2 \dots z_n)$$

$$= (x_1, x_2 \dots x_n) + (y_1+z_1, y_2+z_2 \dots y_n+z_n)$$

$$= x_1+(y_1+z_1), x_2+(y_2+z_2) \dots x_n(y_n+z_n)$$

$$= (x_1+y_1)+z_1, (x_2+y_2)+z_2 \dots (x_n+y_n)+z_n$$

$$= (x_1+y_1, x_2+y_2 \dots x_n+y_n) + (z_1, z_2 \dots z_n)$$

$$= (\alpha+\beta)+\gamma$$

$$\therefore \alpha+(\beta+\gamma) = (\alpha+\beta)+\gamma$$

V satisfies the associative law.

3^{rd} condition :-

Let $\bar{0} = (0, 0, \dots 0)$

$$\alpha+\bar{0} = (x_1, x_2 \dots x_n) + (0, 0, \dots 0)$$

$$= (x_1+0, x_2+0, \dots x_n+0)$$

$$= \alpha$$

$$\bar{0}+\alpha = (0, 0, \dots 0) + (x_1, x_2, \dots x_n)$$

$$= (x_1, x_2 \dots x_n)$$

$$= \alpha$$

$$\therefore \alpha+\bar{0} = \bar{0}+\alpha = \alpha$$

$\therefore$ V has a identity element

4^{th} condition

Let $\hat{\alpha} = (-x_1, -x_2 \cdots -x_n)$ $(\because \hat{\alpha} \in R^n$

$$\alpha + \hat{\alpha} = (x_1, x_2 \cdots x_n) + (-x_1, -x_2 \cdots -x_n)$$
$$= (x_1 - x_1, x_2 - x_2, \cdots x_n - x_n)$$
$$= (0, 0, \cdots 0)$$
$$= \bar{0}$$

$$\hat{\alpha} + \alpha = (-x_1, -x_2 \cdots -x_n) + (x_1, x_2 \cdots x_n)$$
$$= (0, 0 \cdots 0)$$
$$= \bar{0}$$

$\therefore \alpha + \hat{\alpha} = \hat{\alpha} + \alpha = \bar{0}$

$\therefore$ V satisfies Inverse law.

5^{th} condition :-

$$\alpha + \beta = (x_1, x_2 \cdots x_n) + (y_1, y_2 \cdots y_n)$$
$$= x_1 + y_1, x_2 + y_2, \cdots x_n + y_n$$
$$= (y_1 + x_1, y_2 + x_2, \cdots y_n + x_n)$$
$$= \beta + \alpha$$

$\therefore \alpha + \beta = \beta + \alpha$

$\therefore$ V has commutative law

$\therefore$ $(R^n, +)$ is commutative group.

6^{th} Condition

$\forall a \in F, \alpha \in \mathbb{R}^n$ then

$a.\alpha = a(x_1, x_2 \dots x_n)$

$= ax_1, ax_2 \dots ax_n \in \mathbb{R}^n$

7^{th} condition

$\exists 1 \in F$

$1.\alpha = 1(x_1, x_2 \dots x_n)$

$= (x_1, x_2, \dots x_n)$

$= \alpha$

8^{th} condition

$(a+b)\alpha = (a+b)(x_1, x_2 \dots x_n)$

$= a(x_1, x_2 \dots x_n) + b(x_1, x_2 \dots x_n)$

$= (ax_1, ax_2 \dots ax_n) + (bx_1, bx_2 \dots bx_n)$

$= a.\alpha + b.\alpha = a(x_1, x_2 \dots x_n) + b(x_1, x_2 \dots x_n)$

$\therefore (a+b)\alpha = a.\alpha + b.\alpha$

9^{th} Condition:-

$a(\alpha+\beta) = a(x_1+y_1, x_2+y_2 \dots x_n+y_n)$

$= a(x_1+y_1), a(x_2+y_2) \dots a(x_n+y_n)$

$= ax_1 + ay_1, ax_2 + ay_2 \dots ax_n + ay_n$

$a\alpha + a.\beta = a(x_1, x_2 \dots x_n) + a(y_1, y_2 \dots y_n)$

$$= (ax_1, ax_2 \cdots ax_n) + (ay_1, ay_2 \cdots ay_n)$$

$$= ax_1 + ay_1, ax_2 + ay_2 \cdots ax_n + ay_n$$

$$\therefore a(\alpha+\beta) = a.\alpha + a.\beta$$

10th condition

$$a(b\alpha) = a(bx_1, bx_2 \cdots bx_n)\ (abx_1, abx_2 \cdots abx_n)$$

$$= ab(x_1, x_2, \cdots x_n)$$

$$= ab.\alpha$$

$$\therefore a(b\alpha) = (ab)\alpha.$$

:- Properties of vector Space :-

If $V(R)$ is a vector space then

1) Zero vector $\bar{0}$ is unique

2) If $u \in V$ then the negative $-\bar{u}$ is unique in V

3) $0.\bar{u} = \bar{0}\ \forall\ \bar{u} \in V$

4) $c.\bar{0} = \bar{0}\ \forall\ c \in R$

5) If $\bar{u} \in V$ then $(-1)\bar{u} = -\bar{u}$

Proof

1) Suppose $\exists\, \bar{w} \in V \ni \bar{u} + \bar{w} = \bar{u}\ \ \forall\ \bar{u} \in V$

for $\bar{0} \in V$ we have $\bar{u} + \bar{0} = \bar{u}\ \ \forall\ \bar{u} \in V$

Now, $\bar{u} + \bar{w} = \bar{u} + \bar{0}$

Adding both sides $-\bar{u} \in V$

$(-\bar{u}) + [\bar{u} + \bar{w}] = (-\bar{u}) + [\bar{u} + \bar{0}]$

$(-\bar{u} + \bar{u}) + \bar{w} = (-\bar{u} + \bar{u}) + \bar{0}$

$\bar{0} + \bar{w} = \bar{0} + \bar{0}$

$\bar{w} = \bar{0}$

$\therefore \bar{0}$ is unique element in V.

2) Suppose $\exists \bar{w} \in V, \bar{u} + \bar{w} = \bar{0} \ \forall \bar{u} \in V$

for $\bar{u} + (-\bar{u}) = 0$

Now $\bar{u} + \bar{w} = \bar{u} + (-\bar{u})$

adding with $-\bar{u}$ on both sides

$-\bar{u} + \bar{u} + \bar{w} = -\bar{u} + \bar{u} + (-\bar{u})$

$\bar{w} = -\bar{u}$

$\therefore -\bar{u}$ is a unique element in V.

3) $0.\bar{u} = (0+0)\bar{u}$

$0.\bar{u} = 0.\bar{u} + 0.\bar{u}$

adding on both sides
with $-0.\bar{u}$

$-0.\bar{u} + 0.\bar{u} = -0.\bar{u} + 0.\bar{u} + 0.\bar{u}$

$\bar{0} = 0.\bar{u}$

$0.\bar{u} = \bar{0}$,

4) $c.\bar{o} = c(\bar{o}+\bar{o})$

$c.\bar{o} = c.\bar{o} + c.\bar{o}$

Adding on both sides with $-c.\bar{o}$

$-c.\bar{o}+c.\bar{o} = -c.\bar{o}+c.\bar{o}+c.\bar{o}$

$\bar{0} = c.\bar{o}$

$c.\bar{o} = \bar{o}$

5) Take $(1+(-1))\bar{u} = 0.\bar{u}$

$1.\bar{u} + (-1)\bar{u} = 0$

Adding on both sides with $-\bar{u} \in V$

$-\bar{u} + (\bar{u} + (-1)\bar{u}) = -\bar{u} + \bar{0}$

$(-\bar{u}+\bar{u}) + (-1)\bar{u} = -\bar{u}$

$\bar{0} + (-1)\bar{u} = -\bar{u}$

$(-1)\bar{u} = -\bar{u}$

$\therefore$ Hence proved

Subspaces :-

vector Spaces may be formed from Subsets of other vector Spaces. These are called Subspaces.

A Subspace of vector space V is a Subset H of V that has three properties:

a) The zero vector of V is in H.

b) For each u and v are in H, u+v is in H. (In this case we say H is closed under vector addition)

c) For each u in H and each scalar c, cu is in H. (In this case we say H is closed under Scalar Multiplication).

If the Subset H satisfies these three properties, then H itself is a vector space.

Thus in this Section we define a Subspace of a vector space and find out various conditions under which a Subset of a vector space becomes a Subspace.

Theorem

A subset W of vector space V is a subspace of V iff and only if $\alpha, \beta \in W$ and $a, b \in F$ implies $a\alpha + b\beta \in W$

proof:

let W be a subspace of V

then for $\alpha, \beta \in W \Rightarrow \alpha + \beta \in W$

$a \in F, \alpha \in W \Rightarrow a\alpha \in W$

- Ily $b \in F, \beta \in W \Rightarrow b\beta \in W$

Now $a\alpha \in W, b\beta \in W \Rightarrow a\alpha + b\beta \in W$

Conversely,

let W be a Subset of V

which satisfies $a, b \in F, \alpha, \beta \in W$

$\Rightarrow a\alpha + b\beta \in W$

Take $a=0, b=0$ then $0\alpha + 0\beta = \bar{0} + \bar{0} = 0 \in W$

$a=1, b=1$ then $1\alpha + 1\beta = \alpha + \beta \in W$

$a=a, b=0$ then $a\alpha + 0\beta = a\alpha \in W$

W satisfies three conditions defined as Subspace.

$\therefore$ W is a subspace of V.

Theorem:-

If H and K are two Subspaces of vector space V(F) then $H \cap K$ is a subspace of V(F)

proof:-

$\bar{0} \in H, \bar{0} \in K \Rightarrow \bar{0} \in H \cap K$

$H \cap K \neq \phi$

$\alpha, \beta \in H, \alpha, \beta \in K$

$a, b \in F$

$a\alpha + b\beta \in H, a\alpha + b\beta \in K$

$\therefore a\alpha + b\beta \in H \cap K$

Note:- If H and K are two Subspaces of the V(F) then $H \cup K$ need not be Subspace of V(F)

Example:- Let $H = \{(x, -x) \mid x \in R\}$ and $K = \{(x, 2x) \mid x \in R\}$ are Subspaces of $\mathbb{R}^2$

$H \cup K = \{(x, -x), (x, 2x) \mid x \in R\}$

Now $(1, -1) \in H \cup K, (2, 4) \in H \cup K$

but $(1, -1) + (2, 4) = (3, 3) \notin H \cup K$

$\therefore H \cup K$ is not a subspace.

2020-21

M.A.L.D.GOVT. DEGREE COLLEGE GADWAL.

Affiliated to Palamuru University, Mahabubnagar.

Department of Mathematics

STUDENT STUDY PROJECT

NAME OF TITLE: Applications of number Theory

NAME OF THE STUDENTS:

D. Prathyusha	III B.Sc M.P.Cs
V. Vaishnavi	III B.Sc M.P.Cs
Sheema Sultana	III B.Sc M.P.Cs
Rasheeda	III B.Sc M.P.Cs
Shireen	III B.Sc M.P.Cs

In charge
Department of Mathematics

PRINCIPAL
M.A.L.D. Govt. Arts & Science College
GADWAL

MALD Govt Degree College

Department of Mathematics

Topic : Applications of Number Theory

Team :

D. Prathyusha
C. Vaishnavi
Sheema Sultana
Rasheeda
Shireen

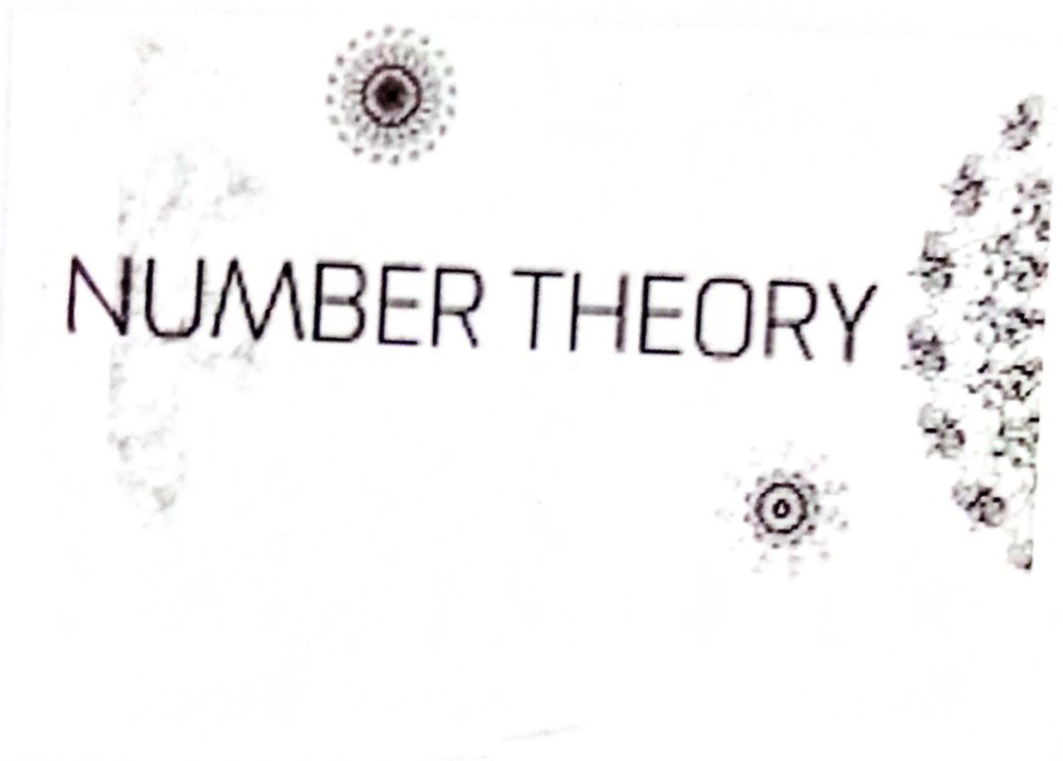
NUMBER THEORY

Objective: The main aim of this course is that we students can identify certain number theoretic functions and their properties. We can solve certain types of Diophantine equations. And also we can identify how number theory is related to and used in cryptography.

Outcome: After learning the course we will be equipped with the various tools regarding various types of number theory that arise in several branches of science.

Definition

Number theory (or arithmetic or higher arithmetic in older usage) is a branch of pure mathematics devoted primarily to the study of the integers and integer-valued functions. German mathematician Carl Friedrich Gauss (1777-1855) said, "Mathematics is the queen of the sciences and number theory is the queen of mathematics." Number theorists study prime numbers as well as the properties of mathematical objects made out of integers (for example, rational numbers) or defined as generalizations of the integers (for example, algebraic integers).



The older term for number theory is arithmetic. By the early twentieth century, it had been superseded by "number theory" (The word 'arithmetic' is used by the general public to mean 'elementary calculations'; it has also acquired other meanings in mathematical logic, as in Peano arithmetic, and computer science, as in floating point arithmetic). The use of the term arithmetic for number theory regained some ground in the second half of the 20th century, arguably in part due to French influence. In particular, arithmetical is commonly preferred as an adjective to number theoretic.

Example :

Prime Numbers - 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ...

Composite Numbers - 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, ...

1 (modulo 4) Numbers - 1, 5, 9, 13, 17, 21, 25, ...

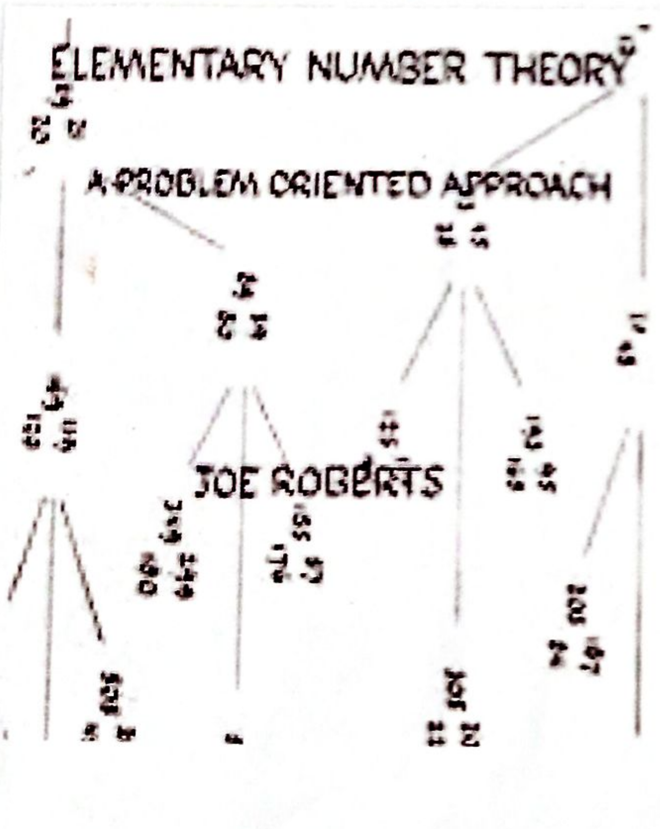
Triangular Numbers - 3, 6, 10, 15, 21, 28, 36, 45, ...

Perfect Numbers - 6, 28, 496, 8128, ...

Fibonacci Numbers - 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

Subdivisions of Number Theory

1. Elementary number theory



The term elementary generally denotes a method that does not use complex analysis. For example, the prime number theorem was first proven using complex analysis in 1896, but an elementary proof was found only in 1949 by Erdos and Selberg. The term is somewhat ambiguous: for example, proofs based on complex Tauberian theorems (for example, Wiener-Ikehara) are often seen as quite enlightening but not elementary, in spite of using Fourier analysis, rather than complex analysis as such. Here as elsewhere, an elementary proof may be longer and more difficult for most readers than a non-elementary one.

Number theory has the reputation of being a field many of whose results can be stated to the layperson. At the same time, the proofs of these results are not particularly accessible, in part because the range of tools they use is, if anything, unusually broad within mathematics.

Example:

If x is a number with $5x+3=33$, then $x=6$

Proof:

If $5x+3=33$, then $5x+3-3=33-3$ since subtracting the same number from two equal quantities gives equal results.

$5x+3-3=5x$ because adding 3 to $5x$ and then subtracting

3 just leaves $5x$, and also, $33-3=30$

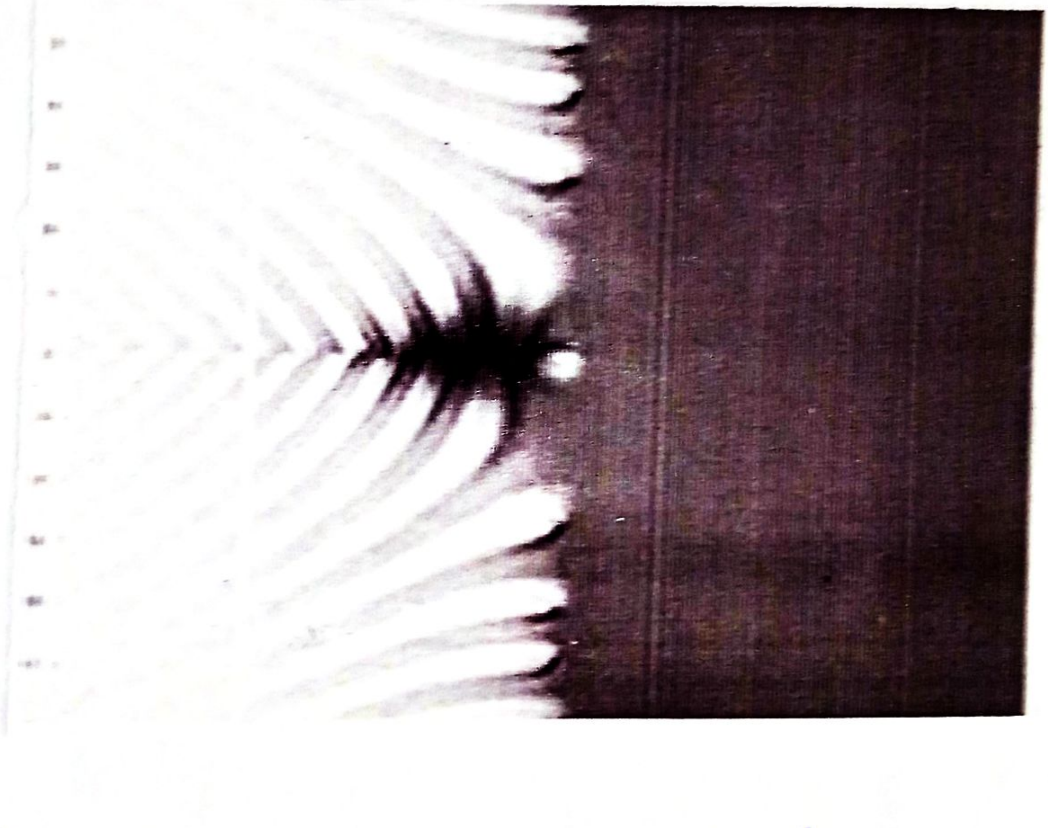
Hence $5x=30$

That is, x is a number which when multiplied by 5 equals 30.

The only number with this property is 6.

Therefore, if $5x+3=33$ then $x=6$.

2. Analytic number theory



Analytic number theory may be defined

→ in terms of its tools, as the study of the integers by means of tools from real and complex analysis.

(or)

→ in terms of its concerns, as the study within number theory of estimates on size and density, as opposed to identities.

Some subjects generally considered to be part of analytic number theory, for example, sieve theory, are better covered by the second rather than the first definition: some of sieve theory, for instance, uses little analysis, yet it does belong to analytic number theory.

The following are examples of problems in analytic number theory: the prime number theorem, the Goldbach Conjecture (or the twin prime conjecture, or the Hardy-Littlewood conjectures), the Waring problem and the Riemann hypothesis. Some of the most important tools of analytic number theory are the circle method, sieve method and L-functions (or, rather, the study of their properties). The theory of modular forms (and, more generally, automorphic forms) also occupies an increasingly central place in the toolbox of analytic number theory.

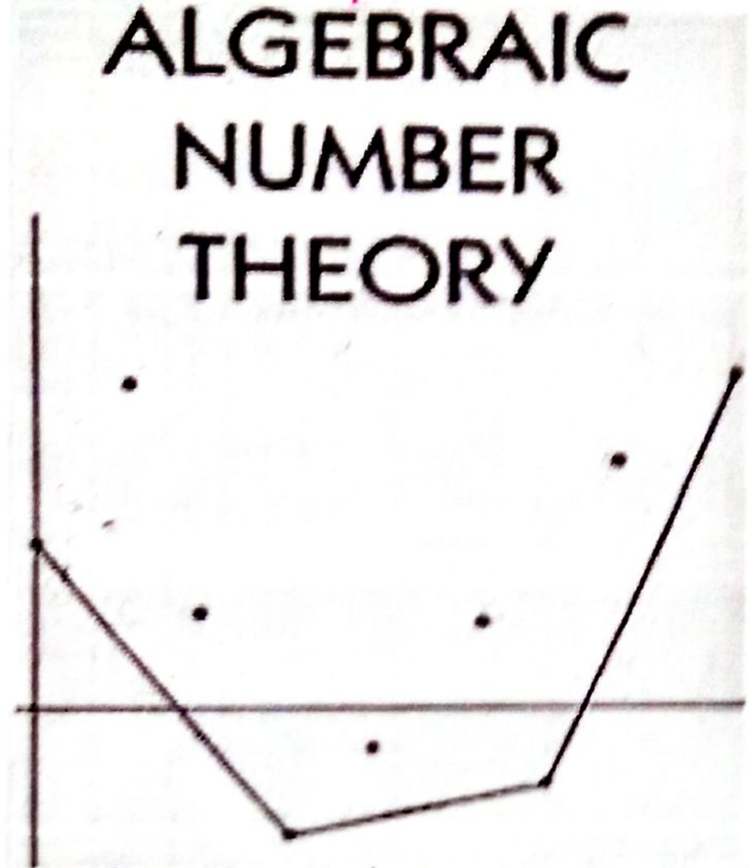
Example:

The prime number theorem can be generalised to this problem; letting

$\pi(x,a,q)$ = number of primes $\leq x$ such then if a and q are coprime,

$$\lim_{x\to\infty} \frac{\pi(x,a,q)\,\phi(q)}{x/\log x} = 1.$$

3. Algebraic number theory



An algebraic number is any complex number that is a solution to some polynomial equation $f(x)=0$ with rational coefficients; for example, every solution x of $x^5+(11/2)x^3-7x^2+9=0$ is an algebraic number. Fields of algebraic numbers are also called algebraic number fields, or shortly number fields. Algebraic number theory studies algebraic number fields. Thus, analytic and algebraic number theory can and do overlap: the former is defined by its methods, the latter by its objects of study.

Number fields are often studied as extensions of smaller number fields: a field L is said to be an extension of a field K if L contains K. (For example, the complex numbers C are an extension of the reals R, and the real R are an extension of the rationals Q). Classifying the possible extensions of a given number field is a difficult and partially open problem. Abelian extensions- that is, extensions L of K such that the Galois group $Gal(L|K)$ of L over K is an abelian group- are relatively well understood. Their classification was the object of the programme of class field theory, which was initiated in the late 19th century (partly by Kronecker and Eisenstein) and carried out largely in 1900-1950.

An example of an active area of research in algebraic number theory is Iwasawa theory. The Langlands program, one of the main current large scale research plans in mathematics, is sometimes described as an attempt to generalise class field theory to non-abelian extensions of number fields.

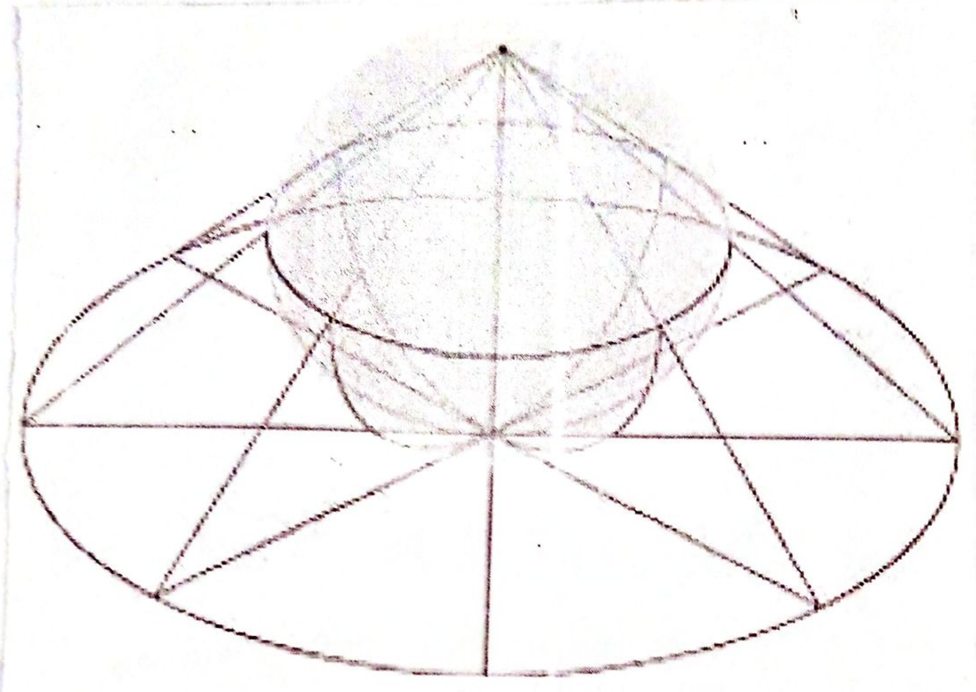
Example :

For example, in the Gaussian integers $Z[i]$, the numbers $1+2i$ and $-2+i$ are associate because the latter is the product of the former by i, but there is no way to single out one as being more canonical than the other. This leads to equations such as

$$5 = (1+2i)(1-2i) = (2+i)(2-i),$$

which prove that in $Z[i]$, it is not true that factorizations are unique up to the order of the factors.

4. Diophantine geometry



The central problem of Diophantine geometry is to determine a Diophantine equation has solutions, and if it does, how many. The approach taken is to think of the solutions of an equation as a geometric object.

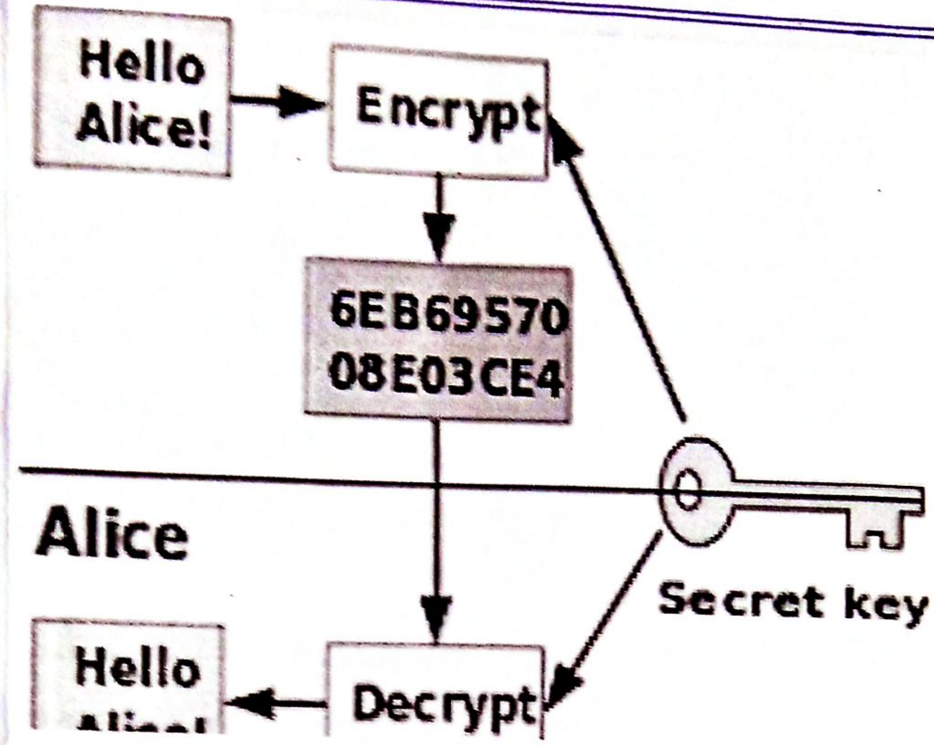
In Diophantine geometry, one asks whether there are any rational points (points all of whose coordinates are integers) or integral points (points all of whose coordinates are integers) on the curve of surface. If there are any such points, the next step is to ask how many there are and how they are distributed. A basic question in this direction is if there are finitely or infinitely many rational points on a given curve (or surface).

In the Pythagorean equation $x^2+y^2=1$, we would like to study its rational solutions, that is, its solutions (x, y) such that x and y are both rational. This is the same as aking for all integer solutions to $a^2+b^2=c^2$; any solution to the latter equation gives us a solution $x=a/c$, $y=b/c$ to the former. It is also the same as asking for all points with rational coordinates on the curve described by $x^2+y^2=1$.

Diophantine geometry should not be confused with the geometry of numbers, which is a collection of graphical methods for answering certain questions in algebraic number theory. Arithmetic geometry, however, is a contemporary term for much the same domain as that covered by the term Diophantine geometry. The term arithmetic geometry is arguably used most often when one wishes to emphasise the connections to modern algebraic geometry (as in, for instance, Falting's theorem) rather than to techniques in Diophantine approximations.

Applications of Number Theory

The best known application of Number theory is public key cryptography, such as the RSA algorithm. Public key cryptography in turn enables many technologies we take for granted, such as the ability to make secure online transactions.



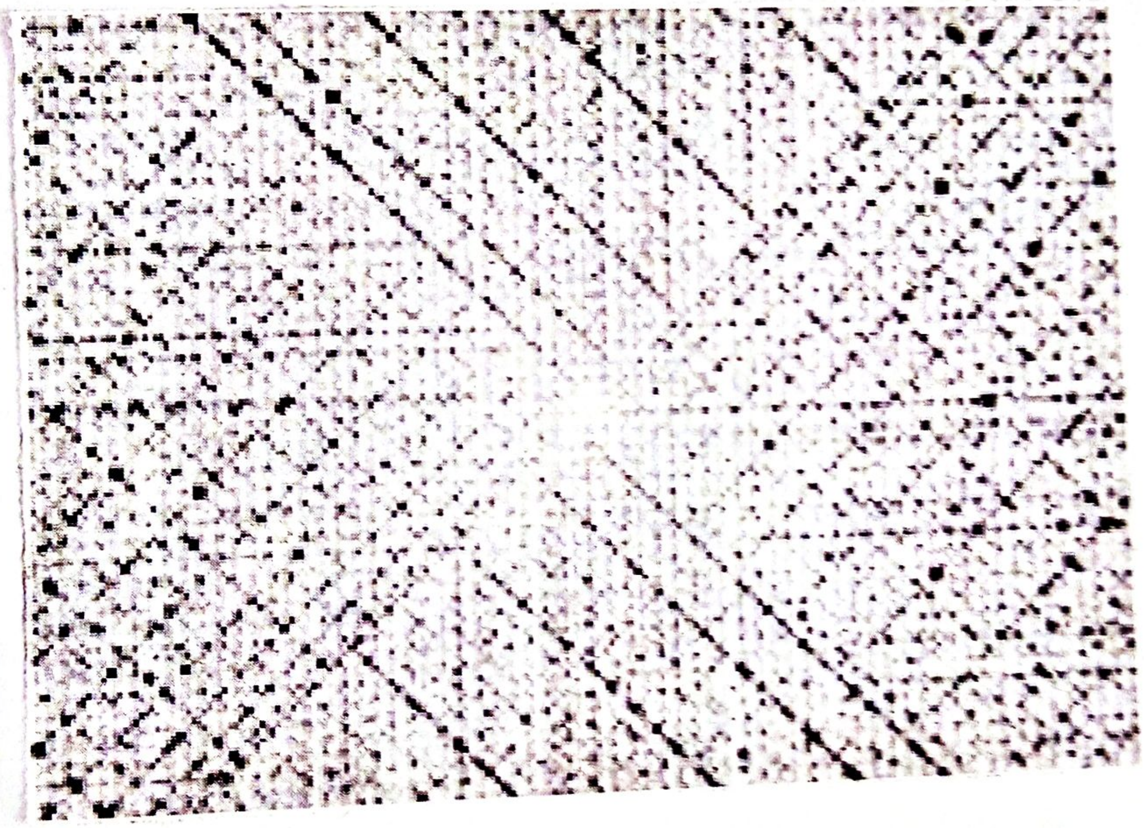
Cryptography is a method of protecting information and communications through the use of codes, so that only those for whom the information is intended can read and process it. The prefix 'crypt' means 'hidden' or 'vault' and the suffix 'graphy' stands for 'writing'.

Example :

An example of basic cryptography is a encrypted message in which letters are replaced with other characters. To decode the encrypted contents, you would need a grid or table that defines how the letters are transposed. For example, the translation grid below could be used to decode

"1234125678906" as "techterms.com"

1	t	6	m
2	e	7	s
3	c	8	.
4	h	9	c
5	r	0	o



2020-21

M.A.L.D.GOVT. DEGREE COLLEGE GADWAL.

Affiliated to Palamuru University, Mahabubnagar.

Department of Mathematics

STUDENT STUDY PROJECT

NAME OF TITLE: Crypography

NAME OF THE STUDENTS: C. Manasa II B.Sc (MSCs)
S. Shivani II B.Sc (MPCs)
Ashwini II B.Sc (MSCs)

PRINCIPAL
M.A.L.D. Govt. Arts & Science College
GADWAL - 509 125

In charge
Department of Mathematics

MALD Govt Degree College

Department of Mathematics

Topic : Cryptography

Presented by : C. Manasa

S. Shivani.

Ashwini.

cryptography

Cryptography is the study of secure communications techniques that allow only the sender and intended recipient of a message to view its contents.

The term is derived from the Greek word kryptos, which means hidden. It is closely associated to encryption.

When transmitting electronic data, the most common use of cryptography is to encrypt and decrypt email and other plain-text messages. The simplest method uses the symmetric or "secret key" system.

Cryptography Projects helps on transforming secure data across various channel.

"The cryptography Handbook" addresses the escalating need for cryptography in this ever-more connected world.

Cryptography is the stuff of spy novels and action comics.

The history of cryptography

The code Books:

The Evolution of Secrecy from Mary, Queen of Scots, to Quantum Cryptography, Simon Singh, Doubleday & Company, Inc., 1999, ISBN.

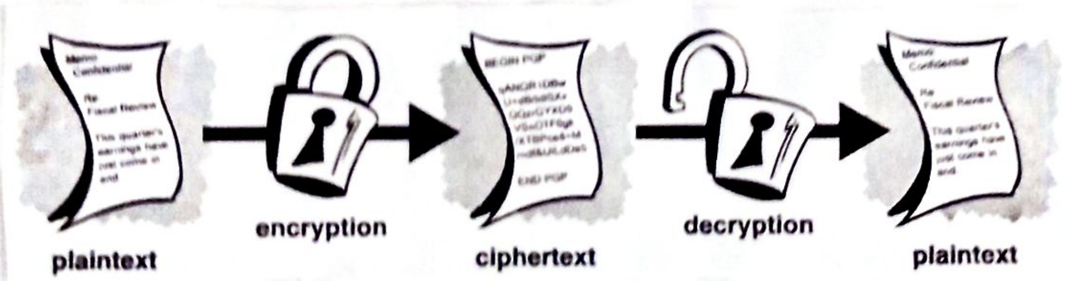
The Codebreakers:

The Story of Secret Writing, David kahn, Simon & Schuster Trade, 1996, ISBN 0-684-83130-9. this book is a history of codes and code breakers from the time of the Egyptians to the end of WWII. kahn first wrote it in the sixties; this is the revised edition. this book won't teach you anything about how cryptography is done, but it has been the inspiration of the whole modern generation of cryptographers.

* When Julius Caeser Sent messages to his generals, he didn't trust messengers. He replaced every A — D, B — E, and so on through the alphabet. Only someone who knew "the shift by 3" rule could decipher the messages.

Encryption and decryption

Data that can be read and understand without any special measures is called plaintext or clear text. The method of disguising plaintext in such a way as to hide its substance is called encryption. Encrypting plaintext results in unreadable gibberish called ciphertext. the process of reverting ciphertext to its original plaintext is called decryption.



What is cryptography?

Cryptography is the science of using mathematics to encrypt and decrypt data. Cryptography enables you to store sensitive information or transmit it accross insecure networks.

While cryptography is the science of

securing data. cryptanalysis is the science of analyzing and breaking secure communication. Classical cryptanalysis involves an interesting combination of analytical reasoning, application of mathematical tools, pattern finding, patience, determination, and luck. Cryptanalysts are also called attackers.

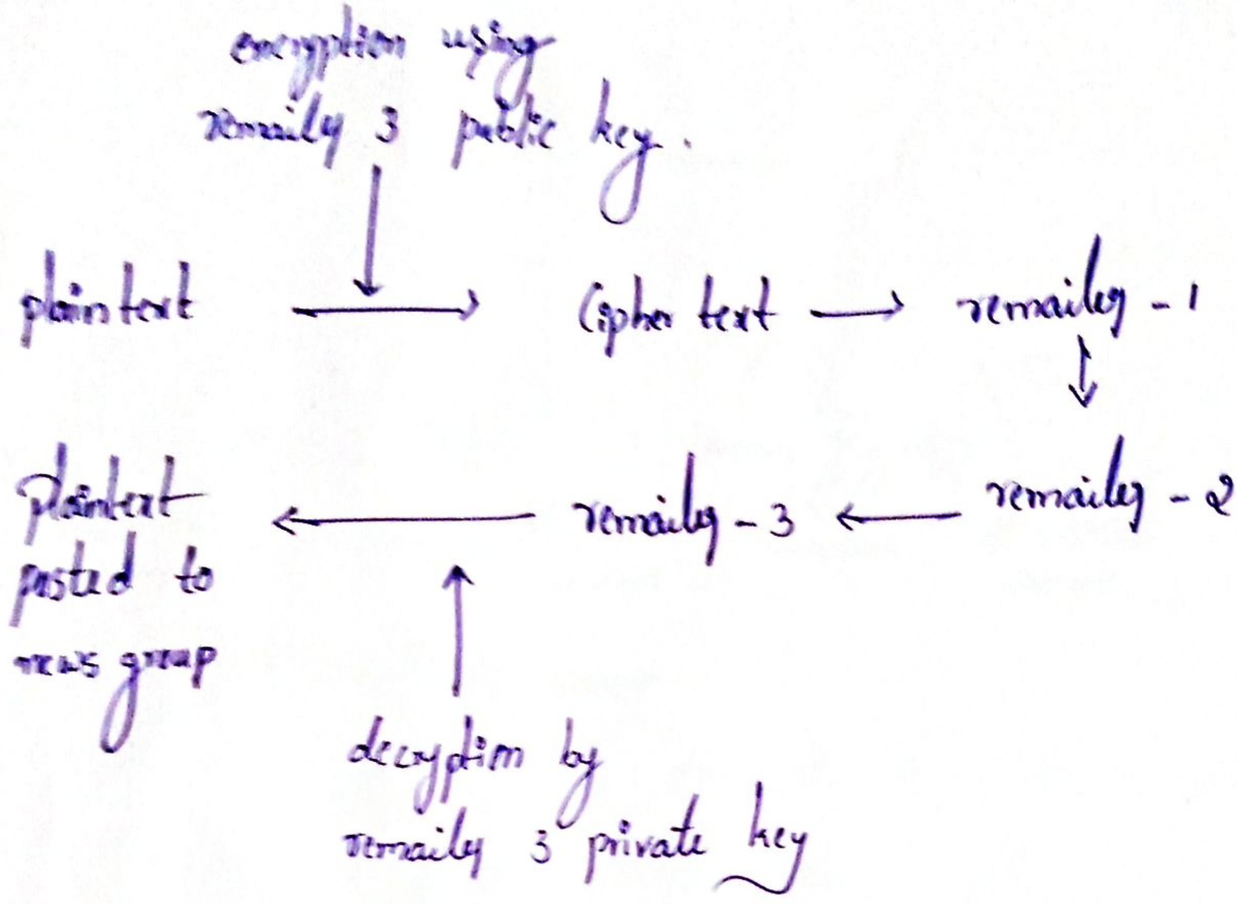
Cryptology embraces both cryptography and cryptanalysis.

Strong cryptography

There are two kinds of cryptography in this world: Cryptography that will stop ur kid sister from reading ur files, and cryptography that will stop major governments from reading ur files.

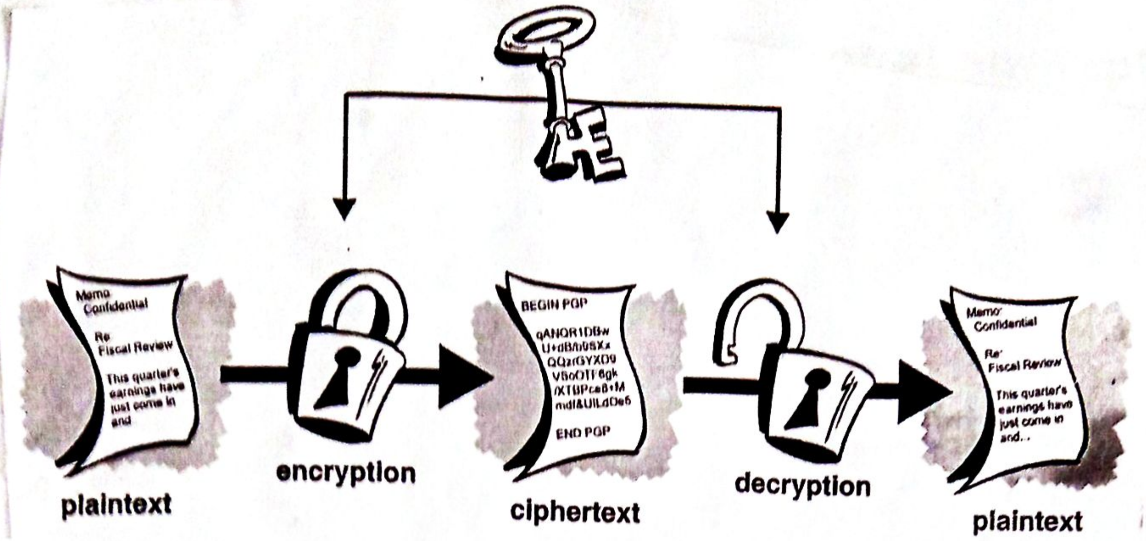
How does cryptography work?

A cryptography algorithm, or cipher, is a mathematical & decryption process. A cryptographic algorithm works in combination with a key - a word, number, or phrase - to encrypt the plaintext



Conventional cryptography

In conventional cryptography, also called secret-key or symmetric-key encryption, one key is used both for encryption and decryption. The Data Encryption Standard (DES) is an example of a conventional cryptosystem. which is widely employed by the U.S government.



Caesar's cipher

An extremely * simple example of conventional cryptography is a substitution cipher.

For Example :

If we encode the word "SECRET" using Caesar's key value of 3, we off set the alphabet so that the 3rd letter down (D) begins the alphabet.

Ex

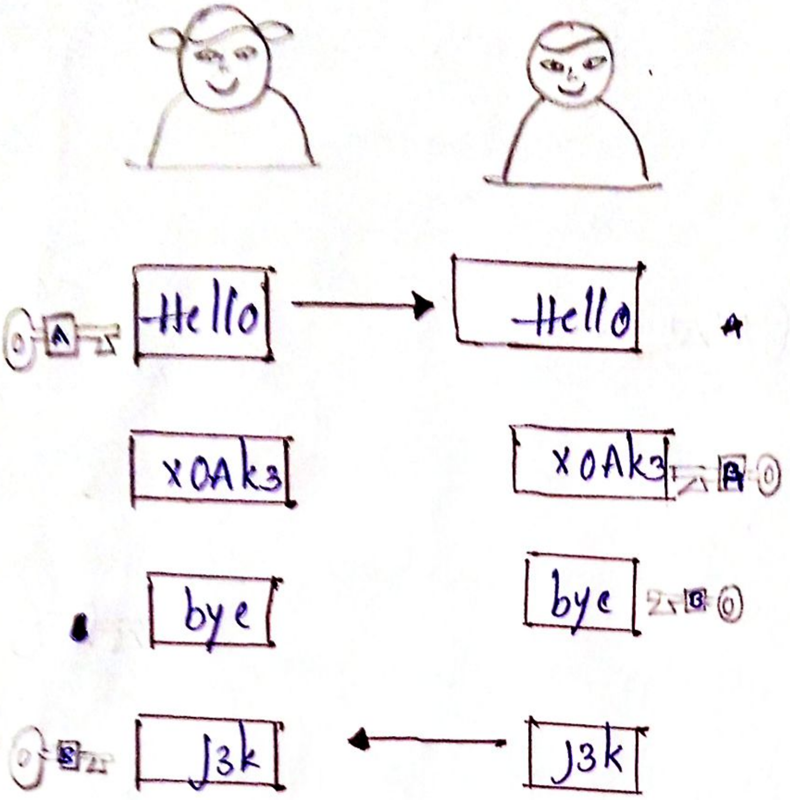
ABCDEFGHIJKLMNOPQRSTUVWXYZ.

& sliding everything up by 3, we get.

DEFGHIJKLMNOPQRSTUVWXYZABC.

Here, D=A, E=B, F=C & so on.

Importance of Cryptography :-



Cryptography in Every Day Life.

Electronic money.

It is also known as electronic cash or Digital Cash.

Digital Signatures

Bob → Bobs Private key → Create Digital Sign

Verify Digital Sign ← Bobs public key ← Send Over internet

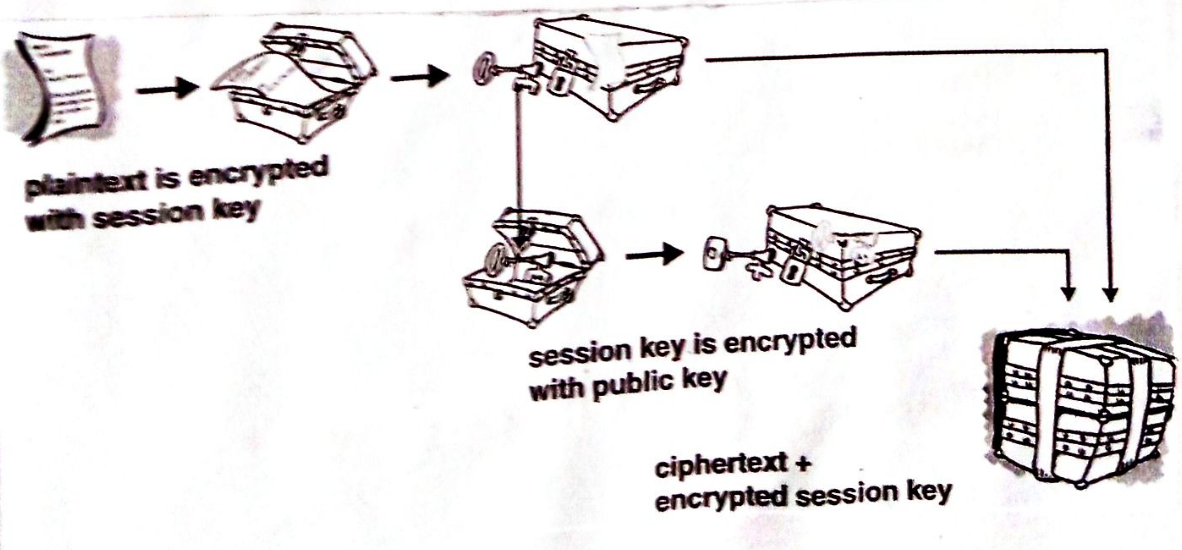
How PGP works

PGP Combines some of the best features of both Conventional and public key cryptography. PGP is a hybrid cryptosystem.

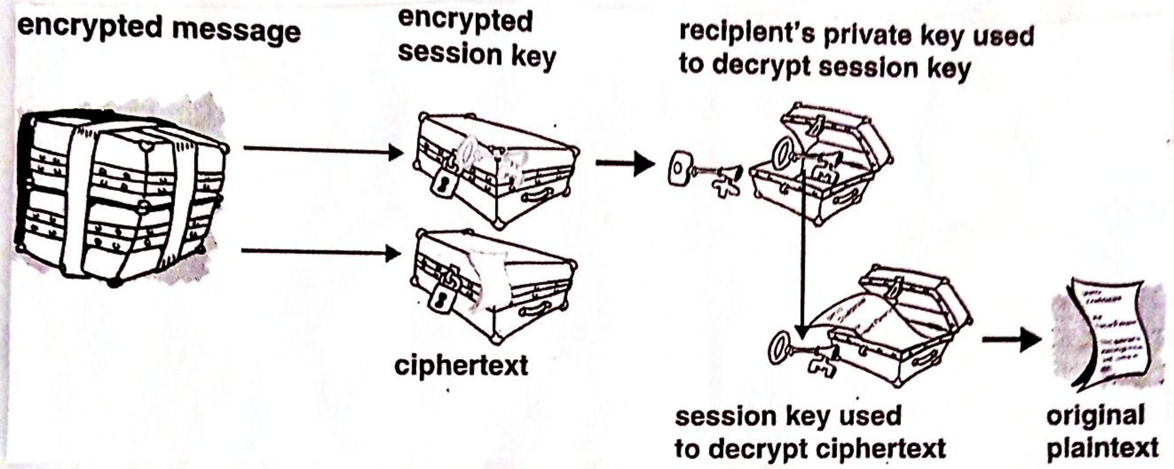
When a user encrypts plaintext with PGP, PGP first compresses the plaintext.

the session key works with a very secure, fast conventional encryption algorithm to encrypt the plaintext; the result is ciphertext.

this public key - encrypted session key is transmitted along with the ciphertext to the recipient.



Decryption works in the reverse. The recipient's copy of PGP uses his or her privatate key to recover the session key,



* which PGP then uses to decrypt the conventionally encrypted ciphertext.

2020-21

M.A.L.D.GOVT. DEGREE COLLEGE GADWAL.

Affiliated to Palamuru University, Mahabubnagar.

Department of Mathematics

Maharani Adilakshmi Devamma Govt. Arts & Science College
GADWAL

STUDENT STUDY PROJECT

NAME OF TITLE: Applications of Geometry

NAME OF THE STUDENTS:

1	Saba	II year	MPC E/M	19033024441018
2	Seema	"	MPCs	19033024468035
3	Rashida	"	MPC E/M	19033024441016
4	K.Laxmi	"	MPCs	19033024468018
5	Naveena	"	MPC E/M	19033024441017

PRINCIPAL
M.A.L.D. Govt. Arts & Science College
GADWAL - 509 125

In charge
Department of Mathematics

M.A.L.D Govt Degree Clg.

Dept. of MATHS

* Saba - BSc (MPC) E/m - 19033024441018
* Seema - BSc (MPCs) E/m - 19033024468035
* Rashida - BSc (MPC) E/m - 19033024441016
* K. Laxmi - BSc (MPCs) E/m - 19033024468018
* Naveena - BSc (MPC) E/m - 19033024441017

Objective :- Model and (objective) Solve geometric Situations using algebraic properties. Recognize parallel or perpendicular lines by analysing the slope or angle relationships. Recognize and complete different styles of proof using deductive reasoning. Differentiate between parallel, perpendicular skew, and intersecting lines.

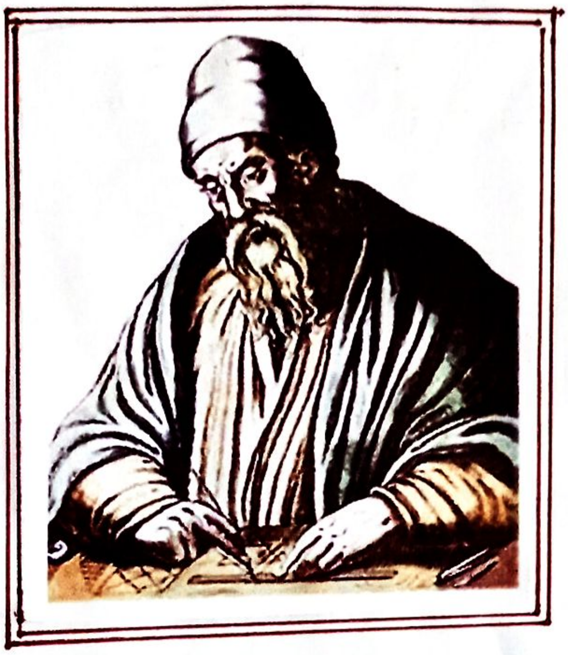
Outcomes of Geometry :- After studying this Courses, you should be able to : understand geometrical terminology for angles, triangles, quadrilaterals and circles. measure angles using a protractor. Use geometrical results to determine unknown angles.

Applications Of Geometry

The best use of geometry in daily life is the construction of the buildings, dams, rivers, roads, temples e.t.c... Smart phones, laptops, Computers etc are designed using geometrical consepts. In fact, the games we play also use geometry to find relevance between the distance and shape of objects designed.

What is Geometry ?

The word "Geometry" is derived from the Greek word "Geo" and "Metron" which mean Earth and Measurement respectively. Translating roughly to Earth's Measurement," geometry plays a great role in determining the areas, volumes, and lengths. Euclid is considered to be the "father of Geometry".



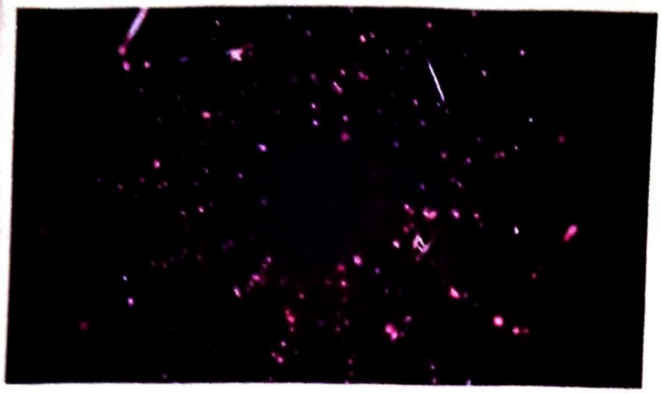
Examples Of Geometry In Everyday life :-

1) Nature :- The most important example of geometry in everydaylife is formed by the nature surrounding. humans. If one looks closely, one might find different geometrical shapes and patterns in leaves, flowers, stems, roots, bark and the goes on the organisation of the human digestive system as a tube

within a tube also ascertains the role of geometry. The leaves on the trees are of varying shapes, sizes and symmetries.

The next interesting example of the role of geometry in nature is formed by the pattern popularly known as "Six-Around-One" patterns, also called "closest packing of circles," ~~Heg~~ "Hexagonal packaging," and "Tessellating Hexagons."

2) Technology :-



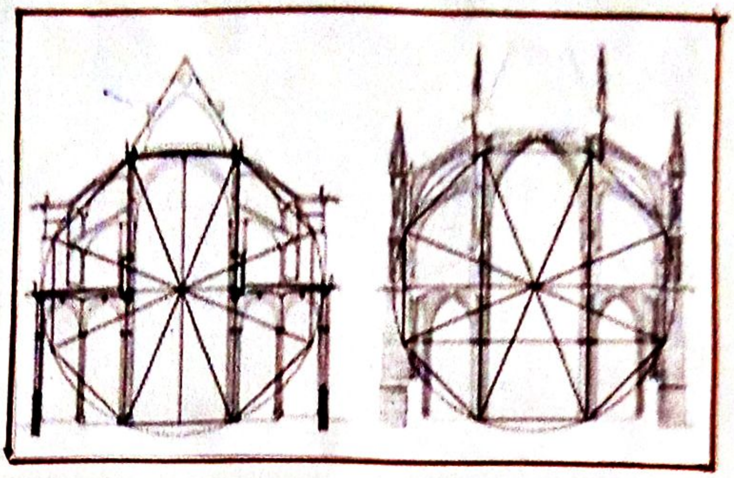
The most common example of geometry in everyday life is technology. Be it robotics or computers, geometry is applied at almost all the underlying concepts. The computer programmers are able to work because the concepts of geometry are always at their disposal ray casting, the process of shooting, employs as 2-D map for stimulating the 3-D world of the video games. Ray casting helps in increasing processing as the calculations are carried out for the vertical lines on the screen.

3) Homes :-

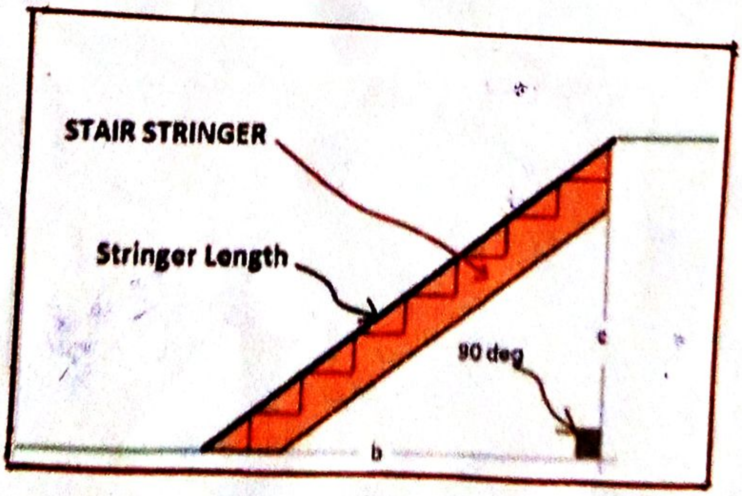
Geometry does not leave even a single chance to play a significant in homes as well as, the windows, doors, beds, chairs, tables, TV, mats, rugs, cushions. etc. have different shapes. A house is made to look more presentable by using vases, paintings, and various decorative pieces, which are of different geometrical shapes and have different patterns made on them.

4) Architecture :- The construction of various buildings or monuments has a close relations-hip with geometry. Before, help construction

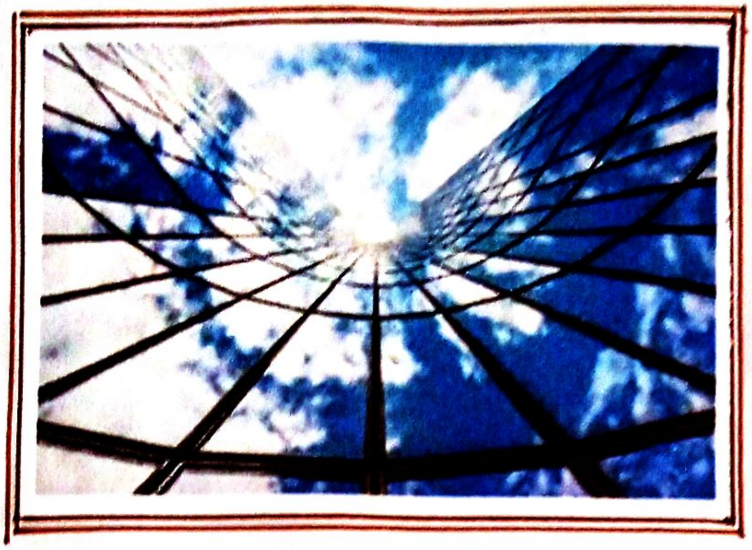
architectual forms, mathematics and geometry.



help put forth the structural blue print of the building pythagoras' "principles of Harmony" along with geometry were employed in the architectural designs of sixth Century BC. The staircase in all the buildings take into consider-ation the angles of geometry and are constructed at 90°.



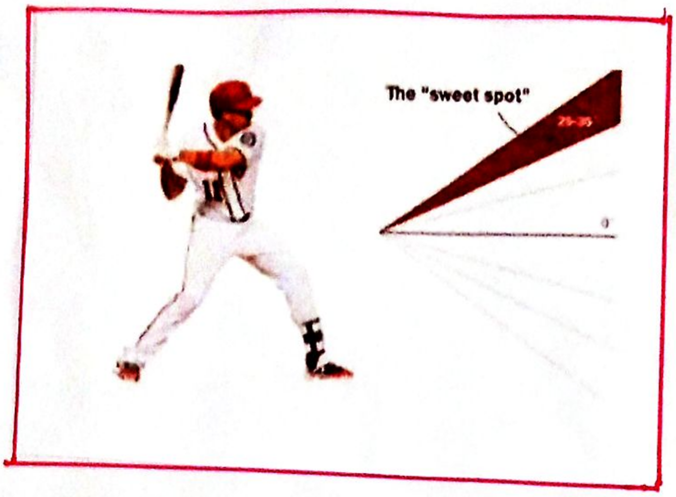
5) Art :-



What does art includes ? Art encompasses the formation of figures & shapes, a basic understanding of 2D & 3D, knowledge about spatial concepts, and contribution of estimation, patterns & measurement. From the aforesaid, it is evident that there is a close relationship between art and geometry. geometrical forms like circle, triangle, Square, mandala, or octagon.

6) Sports :-

Sports often does not fail a sole chance to make use of geometrical concepts. The buildings of the sports stadiums and athletic fields take into consideration geometric shapes. The athletic fields also employ



geometry; hockey, soccer, basketball, and football fields are rectangular in shape. The corner ckick spots, goal posts, arcs, D-section, and centre circle are marked on the field.

7) Designing :-

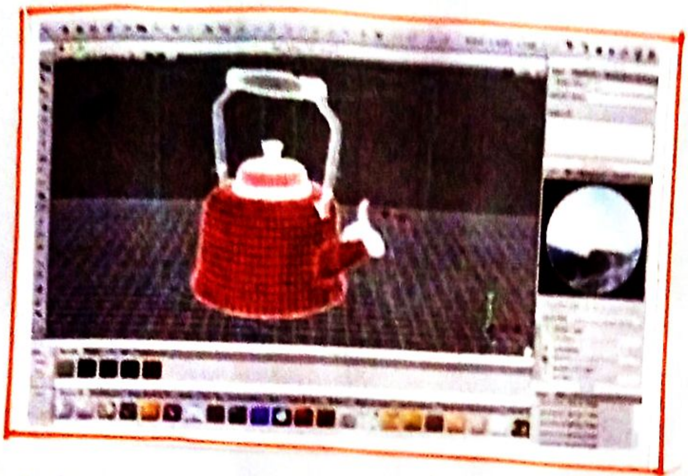
Geometry is widely applied in the field of designing; the creation of animated figures in the video games require geometry. In the case of art, almost every element of designing is entwined with geometric proportions, which is used to depict a story. Taking the examples of miniature paintings and manuscript illumination.

In designing, geometry has a symbolic role to play; as is evident from the carvings on the walls, roofs, and doors of various architectural marvels.

8) Computer Aided Design - CAD :-

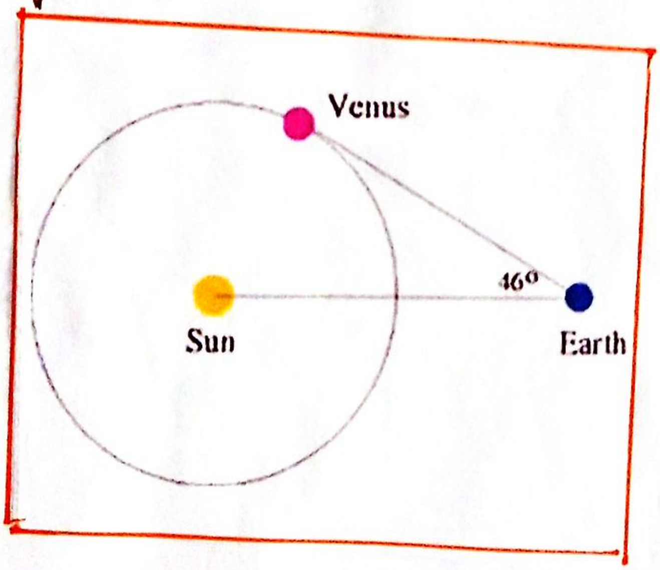
Geometry, one of the principle concepts of mathematics, entails lines, curves, shapes, and angles. Before any architectural design is made, a computer software helps in rendering visual images on the screen. CAD, a software, puts forth the blueprint of the design.

Moreover, it also aids in the simulation of the architectual forms which allows for the better understanding of the finished product.



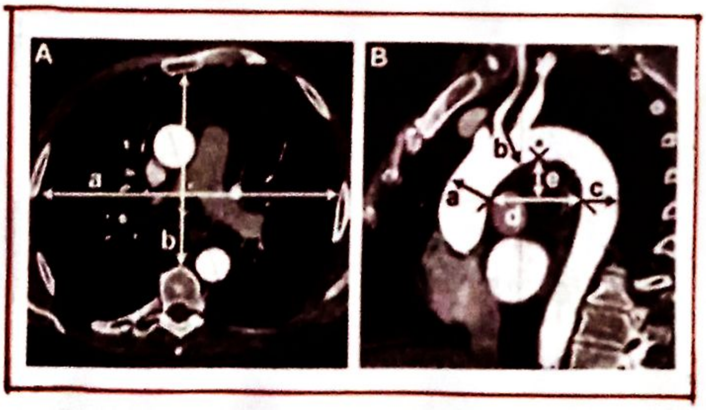
The principles of geometry are being used extensively in various industrial processes which allows the designing of graphics.

9> Mapping :-



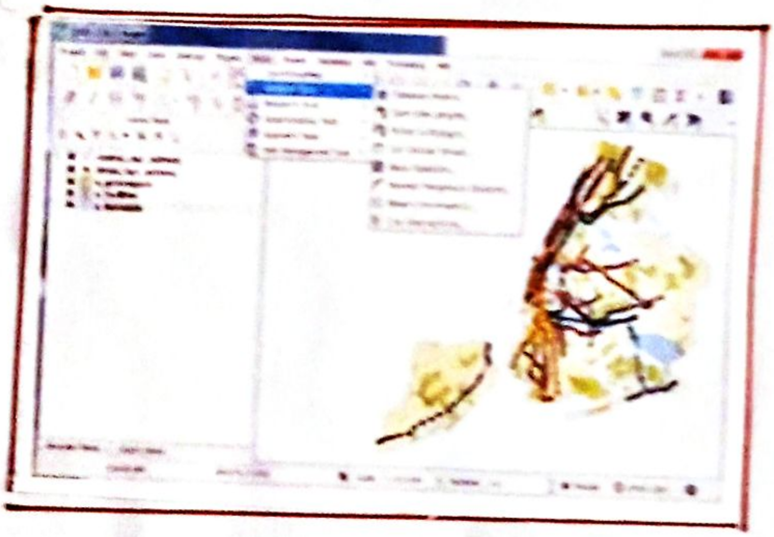
Geometry helps in the accurate calculation of the physical distances. It is employed in the field of astronomy to map the distances between different planets. It also aids in the determination of a relationship between the movements of different bodies in the celestial environment. Moreover, in navigation, the ships, watercraft, and air craft utilise angles and also depend on their mathematical concepts for carrying out basic operations.

10) Medicine :-



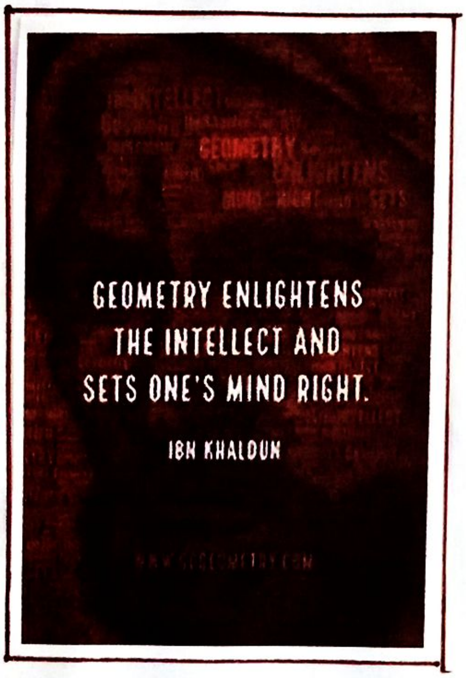
Techniques like x-rays, ultrasounds, MRIs, and nuclear imaging require the reconstruction of shapes of organs, bones, and tumors, which is based on geometry only. Physiotheraphy also employs geometry. Geometric properties and features help in defining the image in digital grids. The geometrical concept not only aid in visualization, manipulation, image segmentation, correction, and object representation. Bisecting angle techniques and parallel techniques are crucial in radiology.

11) Geographic Information Systems :-



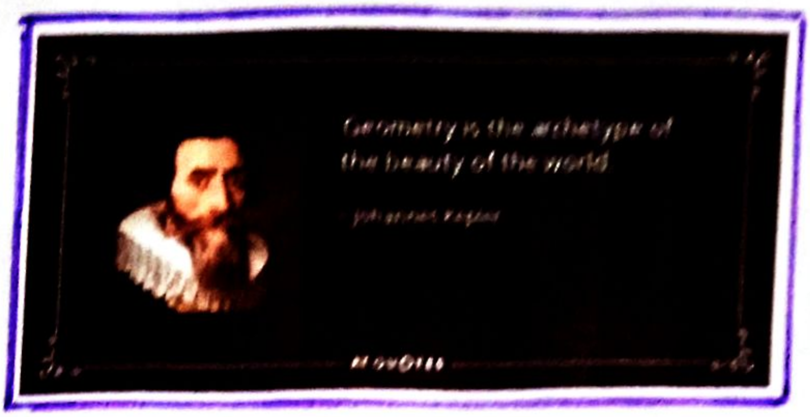
The GPS of the satellites use geometrical principles to calculate the positions of the satellites. The use of coordination geometry in the Global Positioning System (GPS) provides precise information about the location and time. GPS to track transportation accidents and carry out rescue operations. The coordination geometry also aids in enhancing flight Security, weather forecasting, earthquake monitoring, and environmental protection. Moreover, various facets of military operations are equipped with GPS.

Inspirational quotes Of Geometry :-

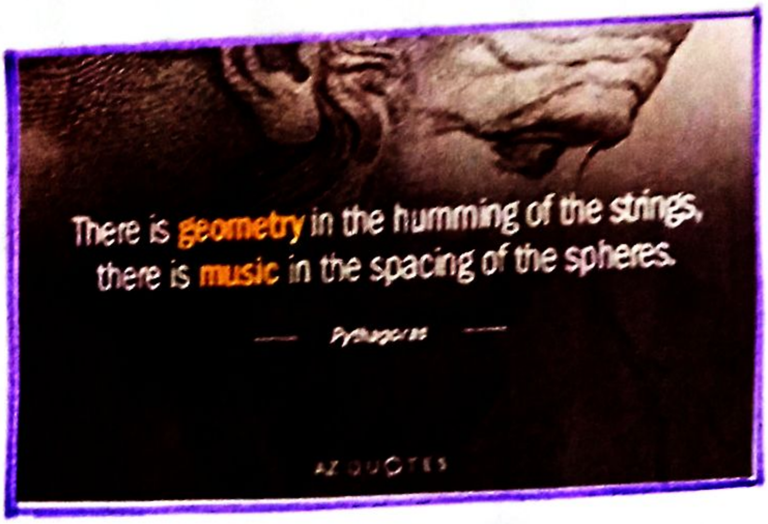


* Geometry Enlights The Intellect And Sets One's Mind Right.

* Equations are just the boring part of Mathematics. I attempt to see things in terms of Geometry.



*→Geometry is the archetype of the beauty of the world.



*→There is geometry in the humming of the strings, there is music in the spacing of the spheres.