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DEPARTMENT OF MATHEMATICS

Title

"CHARACTERISTIC VALUES AND CHARACTERISTICS VECTORS AND IT'S APPLICATINS"

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SYNOPSIS

CHARACTERISTIC VALUES AND CHARACTERISTICS VECTORS AND ITS APPLICATIONS

Introduction: This is the special case for square matrices. In this notes we start with definition of characteristic values and characteristic vectors in the vector spaces.

We can easily find its determinant, trace and also we can define non-singular and singular matrices and also we decide the matrix has an inverse and determine the power of matrix.

This characteristic values and characteristic vectors concept used in various fields such as find the solution of the system of linear differential equations, find the matrix of a Quadratic form and use the principal axes theorem to perform a rotation of axes for a conic and a quadratic.

Aim and Objectives:

- Easily identify the eigen values of the some square matrices.
- Use the concept of eigen values and eigen vectors to find solutions of Linear Differential Equations and Equation of Rotation Conic of the Quadratic Forms.

Methodology:

We are gathering the information from books and internet pdf notes etc. for doing this project.

Analysis of Data:

Definition: Let A be a square matrix of order n. $|A - \lambda I|$ is known as **characteristic polynomial** of matrix A inderminant in λ, where I is an n-rowed unit matrix. The equation $|A - \lambda I| = 0$ is called **characteristic equation** of A and the roots (zeros) of the characteristic equation are called **characteristic roots** of the matrix A. If λ is a characteristic root of n-rowed matrix A, then there exist non-zero column (vector) matrix X such that $AX = \lambda X$ is called **characteristic vector** of A Corresponding to the characteristic root λ.

- If λ is a characteristic root of the matrix A , then matrix $[A - \lambda I]$ is a singular.
- The characteristic values of the lower triangular matrix, upper triangular matrix, diagonal matrix, unit matrix and scalar (constant) matrix are simply elements along their principal diagonal.
- Let $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots, \lambda_{n-1}, \lambda_n$ are eigen values of the square matrix A. Then

  (i) Sum of the eigen values equal to the trace of the matrix A. i.e., $Tr(A) = \sum_{i=1}^{n} \lambda_i$.

  (ii) Product of the eigen values is equal to the determinant of matrix A. i.e., $|A| = \prod_{i=1}^{n} \lambda i$.

Non-singular and singular: A square matrix A is said to be **non-singular** matrix if all the characteristic values of the matrix A are non-zeros. If atleast one of the characteristic value of the matrix A is zero then the matrix is called as **singular** matrix.

DIAGONALIZATBLE MATRIX:

Similar matrix: A square matrix B is said to be " similar" to a matrix A if there exist a non-singular matrix P such that $B = P^{-1}AP$. It is denoted by $B \sim A$.

Diagonalizable matrix: Let a square matrix A is similar to a diagonal matrix D then it (A) is known as "Diagonalizable matrix " if there exist a non- singular matrix P such that $D = P^{-1}AP$. It can be written as $D = P^{-1}AP = \text{diag}(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n)$. where $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ are eigen valves of the matrix A. Here, a non – singular matrix P is called as "Diagonalizing Matrix ".

Minimal Polynomial: A Monic polynomial of lowest degree that annihilates a matrix A is called the **Minimal Polynomial** of A. It is denoted by $m(\lambda)$. Which implies that $m(A)=0$.

- An n- rowed matrix A is similar to a diagonal matrix D iff A has n linearly independent characteristic vectors.
- The necessary and sufficient condition for a square matrix A to be diagonalizable is that Algebraic multiplicity of each characteristic value coincides with its Geometric multiplicity.
- A matrix A has characteristic values are distinct then A is diagonalizable.
- A Nilpotent matrix is not diagonalizable.

Applications of eigen values and eigen vectors:

1. System Of Homogeneous Linear Differential Equations Of First Order:

We use this concept of characteristic values and characteristic vectors we can find solution of the System of homogeneous linear differential equations of first order.

The system of ordinary differential equations in the matrix form $X^1 = AX$, where A is an $n \times n$ coefficient matrix of constants, X is the $n\times1$ column vector of unknown functions and X^1 is the $n\times1$ column vector containing the derivatives of the unknowns.

Example: Solve the System of linear differential equations $x^1(t) = 2x + 5y$; $y^1(t) = x - 2y$

write the differential equations in the matrix form $\begin{bmatrix} x^1(t) \\ y^1(t) \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$$X^1 = AX \quad \text{--------------------- (1)}$$

$\therefore$ Eigen values of the A are $\lambda = 3, -3$.

$X_1 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ are eigen vector corresponding $\lambda = 3, -3$ (respectively)

Solutions of system of equation (1) is

$$\begin{bmatrix} x \\ y \end{bmatrix} = c_1 e^{\lambda_1 t} X_1 + c_2 e^{\lambda_2 t} X_2 = c_1 e^{3t} \begin{bmatrix} 5 \\ 1 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

solution of equation (1) is $x(t) = 5c_1 e^{3t} + c_2 e^{-3t}$

$$y(t) = c_1 e^{3t} - c_2 e^{-3t}.$$

2. Quadratic Forms:

Eigen values and eigenvectors can be used to solve the rotation of axes problems classifying the graph of the quadratic equation

$$ax^2 + bxy + cy^2 + dx + ey + f = 0 \quad , \quad A = \begin{bmatrix} a & b/2 \\ b/2 & c \end{bmatrix}$$

If the equation has an – xy term, however, then the classification is accomplished most easily by first performing a rotation of axes that eliminates the – xy term.

Rotation of a Conic

A rotation of axes to eliminate the xy – term in the quadratic equation is

$$13x^2 - 10xy + 13y^2 - 72 = 0.$$

The matrix of the quadratic form associated with this equation is $A = \begin{bmatrix} 13 & -5 \\ -5 & 13 \end{bmatrix}$

Characteristic polynomial of A is $|A - \lambda I| = \lambda^2 - 26\lambda + 144 = (\lambda - 8)(\lambda - 18)$

$\therefore$ Eigen values of A are $\lambda = 8, 18$

So, the equation of the rotated conic is $8(x^1)^2 + 18(y^1)^2 - 72 = 0$

This can be written in the standard form $\frac{(x1)2}{9} + \frac{(y1)2}{4} = 1$ (ellipse)

$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ are the eigen vectors of A corresponding eigen values 8 and 18

Normalize eigen vectors to form the columns of P, as follows

$$P = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Moreover, $\cos 45° = \sin 45° = \frac{1}{\sqrt{2}}$ => The angle of rotation is $\theta = 45°$.

Hence, the equation of the rotated conic is $8(x^1)^2 + 18(y^1)^2 - 72 = 0$.

Conclusion:

Thus, we conclude that the characteristic values and characteristic vectors concept very useful topic. In this concept we write the characteristic polynomial by using the trace, determinant and sum of the co-factors of the diagonal elements and we can find the determinant, trace of the matrix and define the singular and non-singular matrices by using the characteristic values of the matrix.

This characteristic values and characteristic vectors concept used in various fields such as find the solution of the system of linear differential equations, find the matrix of a Quadratic form and use the principal axes theorem to perform a rotation of axes for a conic.

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