GOVERNMENT DEGREE COLLEGE, LUXETTIPET

DEPARTMENT OF MATHEMATICS

MATHEMATICS

SYLLABUS

Principal
Govt. Degree Colleg
Luxettipet- 504 215

2.1 Differential and Integral Calculus

DSC-1A **BS:101**

Theory: 5 credits and Tutorials: 0 credits
Theory: 5 hours /week and Tutorials: 1 hours /week

Objective: The course is aimed at exposing the students to some basic notions in differential calculus.
Outcome: By the time students complete the course they realize wide ranging applications of the subject.

Unit- I
Partial Differentiation: Introduction - Functions of two variables - Neighbourhood of a point (a, b) - Continuity of a Function of two variables, Continuity at a point - Limit of a Function of two variables - Partial Derivatives - Geometrical representation of a Function of two Variables - Homogeneous Functions.

Unit- II
Theorem on Total Differentials - Composite Functions - Differentiation of Composite Functions - Implicit Functions - Equality of $f_{xy}(a, b)$ and $f_{yz}(a, b)$ - Taylor's theorem for a function of two Variables - Maxima and Minima of functions of two variables – Lagrange's Method of undetermined multipliers.

Unit- III
Curvature and Evolutes: Introduction - Definition of Curvature - Radius of Curvature - Length of Arc as a Function, Derivative of arc - Radius of Curvature - Cartesian Equations - Newtonian Method - Centre of Curvature - Chord of Curvature.
Evolutes: Evolutes and Involutes - Properties of the evolute.
Envelopes: One Parameter Family of Curves - Consider the family of straight lines - Definition - Determination of Envelope.

Unit- IV
Lengths of Plane Curves: Introduction - Expression for the lengths of curves $y = f(x)$ - Expressions for the length of arcs $x = f(y)$; $x = f(t)$, $y = \varphi(t)$; $r = f(\theta)$
Volumes and Surfaces of Revolution: Introduction - Expression for the volume obtained by revolving about either axis - Expression for the volume obtained by revolving about any line - Area of the surface of the frustum of a cone - Expression for the surface of revolution - Pappus Theorems - Surface of revolution.

Text:

- Shanti Narayan, P.K. Mittal *Differential Calculus, S.CHAND, NEW DELHI*
- Shanti Narayan *Integral Calculus, S.CHAND, NEW DELHI*

References:

- William Anthony Granville, Percey F Smith and William Raymond Longley; *Elements of the differential and integral calculus*
- Joseph Edwards , *Differential calculus for beginners*
- Smith and Minton, *Calculus*
- Elis Pine, *How to Enjoy Calculus*
- Hari Kishan, *Differential Calculus*

SEMESTER-II

2.2 Differential Equations

DSC-1B **BS:201**

Theory: 5 credits and Tutorials: 0 credits
Theory: 5 hours /week and Tutorials: 1 hours /week

Objective: The main aim of this course is to introduce the students to the techniques of solving differential equations and to train to apply their skills in solving some of the problems of engineering and science.
Outcome: After learning the course the students will be equipped with the various tools to solve few types differential equations that arise in several branches of science.

Unit- I
Differential Equations of first order and first degree: Introduction - Equations in which Variables are Separable - Homogeneous Differential Equations - Differential Equations Reducible to Homogeneous Form - Linear Differential Equations - Differential Equations Reducible to Linear Form - Exact differential equations - Integrating Factors - Change in variables - Total Differential Equations - Simultaneous Total Differential Equations - Equations of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

Unit- II
Differential Equations first order but not of first degree: Equations Solvable for p - Equations Solvable for y - Equations Solvable for x - Equations that do not contain x (or y)- Equations Homogeneous in x and y - Equations of the First Degree in x and y - Clairaut's equation.
Applications of First Order Differential Equations : Growth and Decay - Dynamics of Tumour Growth - Radioactivity and Carbon Dating - Compound Interest - Orthogonal Trajectories
.

Unit- III
Higher order Linear Differential Equations: Solution of homogeneous linear differential equations with constant coefficients - Solution of non-homogeneous differential equations $P(D)y = Q(x)$ with constant coefficients by means of polynomial operators when $Q(x) = be^{ax}, b\sin ax/b\cos ax, bx^k, Ve^{ax}$ - Method of undetermined coefficients.

Unit- IV
Method of variation of parameters - Linear differential equations with non constant coefficients - The Cauchy - Euler Equation - Legendre's Linear Equations - Miscellaneous Differential Equations.
Partial Differential Equations: Formation and solution- Equations easily integrable - Linear equations of first order.

Text:

- Zafar Ahsan, *Differential Equations and Their Applications*

References:

- Frank Ayres Jr, *Theory and Problems of Differential Equations.*

- Ford, L.R ; *Differential Equations.*
- Daniel Murray, *Differential Equations.*
- S. Balachandra Rao, *Differential Equations with Applications and Programs.*
- Stuart P Hastings, J Bryce McLead; *Classical Methods in Ordinary Differential Equations.*

KAKATIYA UNIVERSITY - WARANGAL - TELANGANA

Under Graduate Courses (w.e.f. academic year 2019-20 batch onwards)

B.Sc. MATHEMATICS II Year

SEMESTER – III

REAL ANALYSIS

Theory: 5 credits and Tutorials: 0 credits Theory: 5 hours /week and Tutorials: 1 hours /week

Objective: The course is aimed at exposing the students to the foundations of analysis which will be useful in understanding various physical phenomena.

Outcome: After the completion of the course students will be in a position to appreciate beauty and applicability of the course.

UNIT- I

Sequences: Limits of Sequences- A Discussion about Proofs-Limit Theorems for Sequences-Monotone Sequences and Cauchy Sequences -Subsequences-Limit sup's and Limit inf's - Series-Alternating Series and Integral Tests.

UNIT- II

Continuity: Continuous Functions -Properties of Continuous Functions -Uniform Continuity -Limits of Functions

UNIT- III

Differentiation: Basic Properties of the Derivative - The Mean Value Theorem - L'Hospital Rule - Taylor's Theorem.

UNIT- IV

Integration: The Riemann Integral - Properties of Riemann Integral-Fundamental Theorem of Calculus.

Text:

Kenneth A Ross, Elementary Analysis-The Theory of Calculus

References:

1] S.C. Malik and Savita Arora, Mathematical Analysis, Second Edition, Wiley Eastern Limited, New Age International (P) Limited, New Delhi, 1994.

2] William F. Trench, Introduction to Real Analysis

3] Lee Larson , Introduction to Real Analysis I

4] Shanti Narayan and Mittal, Mathematical Analysis

5] Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner; Elementary Real analysis

6] Sudhir R., Ghorpade, Balmohan V., Limaye; A Course in Calculus and Real Analysi

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B.Sc. MATHEMATICS II Year

SEMESTER – IV

ALGEBRA

Theory: 5 credits and Tutorials: 0 credits Theory: 5 hours /week and Tutorials: 1 hours /week

Objective: The course is aimed at exposing the students to learn some basic algebraic structures like groups, rings etc.

Outcome: On successful completion of the course students will be able to recognize algebraic structures that arise in matrix algebra, linear algebra and will be able to apply the skills learnt in understanding various such subjects.

UNIT- I

Groups: Definition and Examples of Groups- Elementary Properties of Groups-Finite Groups - Subgroups -Terminology and Notation -Subgroup Tests - Examples of Subgroups.
Cyclic Groups: Properties of Cyclic Groups - Classification of Subgroups Cyclic Groups.

UNIT- II

Permutation Groups: Definition and Notation -Cycle Notation-Properties of Permutations -A Check Digit Scheme Based on D5. Isomorphisms ; Motivation- Definition and Examples - Cayley's Theorem Properties of Isomorphisms -Automorphisms-Cosets and Lagrange's Theorem Properties of Cosets 138 - Lagrange's Theorem and Consequences-An Application of Cosets to Permutation Groups -The Rotation Group of a Cube and a Soccer Ball.

UNIT- III

Normal Subgroups and Factor Groups: Normal Subgroups-Factor Groups -Applications of Factor Groups -Group Homomorphisms - Definition and Examples -Properties of Homomorphisms -The Fifirst Isomorphism Theorem.

Introduction to Rings: Motivation and Definition -Examples of Rings -Properties of Rings - Subrings.

Integral Domains: Definition and Examples - Fields Characteristics of a Ring.

UNIT- IV

Ideals and Factor Rings: Ideals -Factor Rings -Prime Ideals and Maximal Ideals.
Ring Homomorphisms: Definition and Examples-Properties of Ring-
Homomorphisms.

Text:

Joseph A Gallian, Contemporary Abstract algebra (9th edition)

References:

1] Bhattacharya, P.B Jain, S.K.; and Nagpaul, S.R,Basic Abstract Algebra 2]
Fraleigh, J.B, A Fifirst Course in Abstract Algebra.
3] Herstein, I.N, Topics in Algebra
4] Robert B. Ash, Basic Abstract Algebra
5] I Martin Isaacs, Finite Group Theory
6] Joseph J Rotman, Advanced Modern Algebra

CURRICULUM FOR MATHEMATICS
IN UNDER GRADUATE DEGREE PROGRAMME

CBCS SYLLABUS SCHEDULE 2016 - 2017
SEMESTER - V

By

Chairperson

Board of Studies

Department of Mathematics

Kakatiya University, Warangal.

Skill Enhancement Course - III
B.Sc., III Year, V Semester
For All Science Faculty Departments
Verbal Reasoning For Aptitude Test
Credits: 2

Theory: 2 hours/week **Marks - 50**

UNIT - I - Numbers And Diagrams

1.1. **Series Completion:** Number series, Alphabet Series.
1.2. **Series Completion:** Alpha Numeric Series, Continuous Pattern Series.
1.3. **Logical Venn Diagrams**.
1.4. **Mathematical Operations:** Problem solving by substitution, Interchange of signs and numbers.

UNIT - II - Arithematical Reasoning

2.1. Mathematical Operations: Deriving the appropriate conclusions.
2.2. Arithmetical Reasoning: Calculation based problems, Data based problems .
2.3. Arithmetical Reasoning: Problems on ages, Venn diagram based problems.
2.4. Cause and Effect Reasoning.

TEXT: *A Modern Approach to Verbal and Non-Verbal Reasoning* by Dr.R.S. Aggarwal

Kakatiya University
B.Sc. Mathematics, V Semester
LINEAR ALGEBRA

DSC-1E
BS:503

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours/week and Practicals: 2 hours/week

Objective: The students are exposed to various concepts like vector spaces, bases, dimension, Eigen values etc.

Outcome: After completion this course students appreciate its interdisciplinary nature.

UNIT-I

Vector Spaces : Vector Spaces and Subspaces -Null Spaces, Column Spaces, and Linear Transformations -Linearly Independent Sets; Bases -Coordinate Systems

UNIT-II

The Dimension of a Vector Space, Rank-Change of Basis - Eigenvalues and Eigenvectors .

UNIT-III

The Characteristic Equation, Diagonalization -Eigenvectors and Linear Transformations -Complex Eigenvalues - Applications to Differential Equations .

UNIT-IV

Orthogonality and Least Squares : Inner Product, Length, and Orthogonality -Orthogonal Sets.

TEXT: David C Lay,*Linear Algebra and its Applications 4e*
References:

- S Lang, *Introduction to Linear Algebra*
- Gilbert Strang ,*Linear Algebra and its Applications*
- Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence; *Linear Algebra*
- Kuldeep Singh; *Linear Algebra*
- Sheldon Axler;*Linear Algebra Done Right*

Practical Question Bank

UNIT-I

(1) Let H be the set of all vectors of the form $\begin{bmatrix} -2t \\ 5t \\ 3t \end{bmatrix}$. Find a vector v in R^3 such that $H = Span\{v\}$. Why does this show that H is a subspace of R^3?.

(2) Let V be the first quadrant in the xy-plane; that is let $V = \{\begin{bmatrix} x \\ y \end{bmatrix} x \geq 0, y \geq 0\}$

(a). If u and v are in V is $u + v$ in V ? why?

(b) Find a specific vector u in V and a specific scalar c such that

(3) Let $v_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -2 \\ 7 \\ -9 \end{bmatrix}$. Determine if $\{v_1, v_2\}$ is a basic for R^3. Is $\{v_1, v_2\}$ a basis for R^2.

(4) The set $B = \{1 + t^2, t + t^2, 1 + 2t + t^2\}$ is a basis for P_2. Find the coordinate vector of $p(t) = 1 + 4t + 7t^2$ relative to B.

(5) set $B = \{1-t^2, t-t^2, 2-t+t^2\}$ is a basis for P_2. Find the coordinate vector of $p(t) = 1+3t-6t^2$ relative to B.

(6) The vector $v_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ -8 \end{bmatrix}, v_3 = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$ $spanR^2$ but do not form a basis . Find two different ways to express $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as a linear combination of v_1, v_2, v_3

(7) Let V be the set of all real - valued functions defined on a set D Then $f + g$ is the function whose value at t in the domain D is $f(t) + g(t)$ and for any scalar$c$ and for any f in V, the scalar multiple cf is the function whose value at t is $cf(t)$.

(8) The vector space $\mathbf{R}^2$ is not a subspace of $\mathbf{R}^3$ because $\mathbf{R}^2$ is not even a subset of $\mathbf{R}^3$.(The vectors in $\mathbf{R}^3$ all have three entries,where as the vectors in $\mathbf{R}^2$ have only two.) The set $\mathbf{H} = \{ \begin{pmatrix} s \\ t \\ 0 \end{pmatrix}$:is s and t are real$\}$ is a subset of $\mathbf{R}^3$ that "looks" and " acts" like $\mathbf{R}^2$,although it is logically distinct from $\mathbf{R}^2$. Show that H is a subspace of $\mathbf{R}^3$.

(9) The differential equation $y'' + \omega^2 y = 0$ where ω is a constant,is used to describe a variety of physical system, such as the vibration of a weighted spring,the movement of a pendulum,and the voltage in an inductance - capacitance electrical ciruit. Then show that set of solutions of given differential equation is precisely kernel of the linear transformation that maps a function $y = f(t)$ into the function $y'' + \omega^2 y = 0$.

(10) Let $V_1 = \begin{pmatrix} 3 \\ 0 \\ -6 \end{pmatrix}, v_2 = \begin{pmatrix} -4 \\ 1 \\ 7 \end{pmatrix}, v_3 = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix}$ Determine if $\{v_1, v_2, v_3\}$ is a basis for $\mathbf{R}^3$

UNIT-II

(11) Find the dimension of the subspace of all vectors in R^3 whose first and third entries are equal

(12) Find the dimension of the subspace H of R^2 spanned by $\begin{bmatrix}1\\-5\end{bmatrix}\begin{bmatrix}-2\\10\end{bmatrix}\begin{bmatrix}-3\\15\end{bmatrix}$

(13) Let H be an n dimensional subspace of an n dimensional vectorspace V. Show that H=V.

(14) Explain why the space P of all polynomials is an infinite dimensional space

(15) If a 4x7 matrix A has rank 3 ,find dim Null$A$,dim Row A and rank A^T

(16) If a 7X5 matrix A has rank 2 ,find dim Null$A$,dim Row A and rank A^T

(17) If the null space of an 8x5 matrix A is 3 dimensional,what is the dimension of the row space of A?

(18) If A is a 3x7 matrix what is the smallest possible dimension of Null A ?

(19) Let $U = \begin{bmatrix}1\\2\end{bmatrix}$ find V in R^3 such that $\begin{bmatrix}1 & -3 & 4\\2 & -6 & 8\end{bmatrix} = UV^T$

(20) If A is a 7x5 matrix,what is the largest possible rank of A ? If A is a 5x7 matrix,what is the largest possible rank of A ? Explain your answers.

UNIT-III

(21) Without calculations list $rank(A)$ and $dim(A)$, $Nul(A)$

if A= $\begin{bmatrix}2 & 6 & -6 & -6 & 3 & 6\\-2 & -3 & 6 & -3 & 0 & -6\\4 & 9 & 12 & 9 & 3 & 12\\-2 & 3 & 6 & 3 & 3 & -6\end{bmatrix}$

(22) Use a property of determinants to show A and A^T have same characteristic polynomial.

(23) Find the characteristic equation of

A= $\begin{bmatrix}5 & -2 & 6 & -1\\0 & 3 & -8 & 0\\0 & 0 & 5 & 4\\0 & 0 & 0 & 1\end{bmatrix}$

(24) Find characteristic polynomial and the real eigen values of

$\begin{bmatrix}4 & 0 & -1\\0 & 4 & -1\\1 & 0 & 2\end{bmatrix}\begin{bmatrix}-1 & 0 & 2\\3 & 1 & 0\\0 & 1 & 2\end{bmatrix}$

(25) Let A=PDP^{-1} and compute A^4 where P=$\begin{bmatrix}5 & 7\\2 & 3\end{bmatrix}$ D=$\begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}$

(26) Let B $=\{b_1, b_2, b_3\}$ and D=$\{d_1, d_2\}$ be bases for vector spaces V and W respectively. Let $T : V \longrightarrow W$ be a linear transformation with the property that $T(b_1) = 3d_1 - 5d_2$, $T(b_2) = -d_1 + 6d_2$, $T(b_3) = 4d_2$ Find the matrix T relative to B and D.

(27) Let D=$\{d_1, d_2\}$ and B =$\{b_1, b_2\}$ be bases for vector spaces V and W respectively. Let $T : V \longrightarrow W$ be a linear transformation with the property that $T(d_1) = 3b_1 - 3b_2$, $T(d_2) = -2b_1 + 5b_2$. Find the matrix for T relative to B and D.

(28) Let $B = \{b_1, b_2, b_3\}$ be a basis for a vector space$V$and let $T : V \longrightarrow R^2$ be a linear transformation with the property that
$T(x_1b_1 + x_2b_2 + x_3b_3) = \begin{bmatrix} 2x_1 - 3x_2 + x_3 \\ -2x_1 + 5x_3 \end{bmatrix}$
find the matrix for T relative to B and the standard basis for $\mathbf{R}^2$.

(29) Let $T : P_2 \longrightarrow P_3$ be the transformation that maps a polynomial p(t) into the polynomial (t+3)p(t)
(a). Find the image of $p(t) = 3 - 2t + t^2$
(b). Show that T is a linear transformation
(c). Find the matrix for T relative to the basis $\{1, t, t^2\}$ and $\{1, t, t^2, t^3\}$

(30) Assume the mapping $T : P_2 \longrightarrow P_2$ defined by
$T(a_0 + a_1t + a_2t^2) = 3a_0 + (5a_0 - 2a_1)t + (4a_1 + a_2)t^2$ is linear.Find the matrix representation of T relative to the basis B=$\{1, t, t^2\}$

UNIT-IV

(31) Define $T : P_3 \longrightarrow R^4 by T(P) = \begin{bmatrix} P(-2) \\ P(3) \\ P(1) \\ P(0) \end{bmatrix}$
(a. Show that T is a linear transformation
(b. Find the matrix for T relative to the basis $\{1, t, t^2, t^3\}$ for P_3 and standard basis for R^4

(32) Let A be 2×2 matrix with eigen values -3 and -1 corresponding eigen vectors $V_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $V_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Let $X(t)$ be the position of a particle at time t solve the initial value problem $X' = AX$, X(0)=$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

(33) Construct the general solution of $X' = AX$,A=$\begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$, $\begin{bmatrix} -7 & 10 \\ -4 & 5 \end{bmatrix}$

(34) Compute the orthogonal projection of $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ onto the line through $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ and the origin.

(35) Let W be the subspace of R^2 spanned by $X = (\frac{2}{3}, 1)$. Find a unit vector in z that is a basis for W.

(36) Show that $\{u_1, u_2, u_3\}$ is an orthogonal set, where $u_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$, $u_2 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, $u_3 = \begin{pmatrix} -\frac{1}{2} \\ -2 \\ \frac{7}{2} \end{pmatrix}$.

(37) The set $S = \{u_1, u_2, u_3\}$ where $u_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, u_3 = \begin{pmatrix} -\frac{1}{2} \\ -2 \\ \frac{7}{2} \end{pmatrix}$ is an orthogonal basis for R^3. Express the vector $y = \begin{pmatrix} 6 \\ 1 \\ -8 \end{pmatrix}$ as a linear combination of the vectors in S.

(38) Show that $S = \{v_1, v_2, v_3\}$ is an orthonormal basis of R, where $v_1 = \begin{pmatrix} \frac{3}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} \end{pmatrix}, v_2 = \begin{pmatrix} -\frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix},$ $v_3 = \begin{pmatrix} -\frac{1}{\sqrt{66}} \\ -\frac{4}{\sqrt{66}} \\ \frac{7}{\sqrt{66}} \end{pmatrix}$

(39) Determine given set of vectors are orthogonal or not. $\begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}, \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -4 \\ -7 \end{pmatrix}$

(40) Let $U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{2}{3} \\ \frac{1}{\sqrt{2}} & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$ and $x = \begin{bmatrix} \sqrt{2} \\ 3 \end{bmatrix}$. Notice that U has orthonormal columns and $U^T U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{2}{3} \\ \frac{1}{\sqrt{2}} & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ verify that $\|Ux\| = \|x\|$.

Kakatiya University
B.Sc. Mathematics, V Semester
SOLID GEOMETRY

DSE-1E/A
BS:506

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours /week and Practicals: 2 hours/week

Objective: Students learn to describe some of the surfaces by using analytical geometry.

Outcome: Students understand the beautiful interplay between algebra and geometry.

UNIT- I

Sphere: Definition-The Sphere Through Four Given Points - Equations of a Circle - Intersection of a Sphere and a Line - Equation of a Tangent Plane - Angle of Intersection of Two Spheres - Radical Plane.

UNIT- II

Cones : Definition-Condition that the General Equation of second degree Represents a Cone - Cone and a Plane through its Vertex - Intersection of a Line with a Cone. The Right Circular Cone.

UNIT- III

Cylinder: Definition-Equation of a Cylinder-Enveloping Cylinder - The Cylinder - The Right Circular Cylinder.

UNIT- IV

The Conicoid: The General Equation of the Second Degree-Intersection of Line with a Conicoid- Plane of contact-Enveloping Cone and Cylinder.

TEXT: Shanti Narayan and P K Mittal, *Analytical Solid Geometry* (17e)
References:

- Khaleel Ahmed, *Analytical Solid Geometry*
- S L Loney , *Solid Geometry*
- Smith and Minton, *Calculus*

Practical Question Bank

UNIT-I

(1) Find the equation of the sphere through the four points (4,-1,2), (0,-2,3), (1,-5,-1), (2,0,1).

(2) Find the equation of the sphere through the four points (0,0,0,), (-a,b,c), (a,-b,c), (a,b,-c).

(3) Find the centre and radius of the circle $x + 2y + 2 = 15,\ x^2 + y^2 + z^2 - 2y - 4z = 11$.

(4) Show that the following points are concyclic:
(i) (5,0,2), (2,-6,0), (7,-3,8), (4,-9,6).
(ii) (-8,5,2), (-5,2,2), (-7,6,6), (-4,3,6).

(5) Find the centres of the two spheres which touch the plane $4x+3y = 47$ at the points (8.5,4) and which touch the sphere $x^2 + y^2 + z^2 = 1$

(6) Show that the spheres $x^2+y^2+z^2 = 25 \& x^2+y^2+z^2-24x-40y-18z+225 = 0$ touch externally and find the point of contact.

(7) Find the equation of the sphere that passes through the two points (0,3,0), (-2,-1,-4) and cuts orthogonally the two spheres
$x^2 + y^2 + z^2 - x - 3z - 2 = 0, \quad 2(x^2 + y^2 + z^2) + x + 3y + 4 = 0.$

(8) Find the limiting points of the co-axal system of spheres $x^2 + y^2 + z^2 - 20x + 30y - 40z + 29 + \lambda(2x - 3y + 4z) = 0$.

(9) Find the equation of the two spheres of the co-axal systems $x^2+y^2+z^2-5+\lambda(2x+y+3z-3) = 0$. which touch the plane $3x + 4y = 15$.

(10) Show that the radical planes of the spheres of a co-axal system and of any given sphere pass through a line.

UNIT-II

(11) Find the equation of cone whose vertex is (α, β, γ) and base $ax^2 + by^2 = 1,\ z = 0$

(12) The section of a cone whose vertex is P and guiding curve the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$ by the plane $x = 0$ is a rectangular hyperbola . show that the locus of P is $\frac{x^2}{a^2} + \frac{y^2+z^2}{b^2} = 1$

(13) Find the equation of the cone whose vertex is the point (1,1,0)and whose guiding curve is $y = 0, x^2 + y^2 = 4$

(14) Find the equation of the cone whose vertex is the point (1,2,3) and guiding curve the circle $x^2 + y^2 + z^2 = 4, x + y + z = 1$

(15) Find the enveloping cone of the sphere $x^2 + y^2 + z^2 - 2x + 4z = 1$ with vertex at (1,1,1).

(16) Show that the plane $z = 0$ cuts the envolping cone of the sphere $x^2 + y^2 + z^2 = 11$ which has its vertex at (2,4,1) in a rectangular hyperbola.

(17) Find the equation of the quadric cone whose vertex is at the origin and which passes through the curve given by the equations $ax^2 + by^2 + cz^2 = 1,\ lx + my + nz = p$.

(18) Find the equations to the cones with vertex at origin and which pass through the curve given by the equations $ax^2 + by^2 = 2z, lx + my + nz = p$.

(19) Find the equation of the cone with vertex at the origin and direction cosines of its generators satisfying the relation $3l^2 - 4m^2 + 5n^2 = 0$

(20) Find the equations to the cones with vertex at origin and which pass through the curve given by the equations $z = 2, x^2 + y^2 = 4$

UNIT-III

(21) Find the equation of a cylinder whose generating line have the direction cosines (l, m, n) and which passes through the circle $x^2 + z^2 = a^2, y = 0$.

(22) Find the equation of the cylinder whose generators are parallel to
$x = -\frac{1}{2}y = \frac{1}{3}z$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 3$.

(23) Find the envoloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y = 1$ having the generators paerallel to the line $x = y = z$.

(24) The axis equation of a right circular cylinder of radius 2 is $\frac{(x-1)}{2} = \frac{y}{3} = \frac{(z-3)}{1}$;
Show that its equation is $10x^2 + 5y^2 + 13z^2 - 12xy - 6yz - 4zx - 8x - 30y - 74z + 59 = 0$.

(25) Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-2}{2}$.

(26) Find the equation of the right circular cylinder of radius 2 whose axis passes through the point $(1, 2, 3)$ and has direction cosines proportional to $(2, -3, 6)$.

(27) Find the right circular cylinder of radius 4 and axis the line $x = 2y = -z$. Also prove that the area of cross - section of the cylinder by the plane $z = 0$ is 24π

(28) Obtain the equation of the right circular cylinder described on the circle through the three points $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ as guiding circle.

(29) Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{(x-1)}{2} = (y-2) = \frac{(z-3)}{2}$

(30) Find the equation, if the cylinder whose generator touch the sphere $x^2+y^2+z^2 = a^2$ and parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

UNIT-IV

(31) Find the points of intersection of the line $-\frac{1}{3}(x + 5) = (y - 4) = \frac{1}{7}(z - 11)$, with the conicoid $12x^2 - 17y^2 + 7z^2 = 7$.

(32) Find the equations to the tangent planes to $7x^2 - 3y^2 - z^2 + 21 = 0$, which passes through the line, $7x - 6y + 9 = 3, z = 3$.

(33) Obtain the tangent planes to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, which are parallel to the plane $lx + my + nz = 0$.

(34) Show that the plane $3x + 12y - 6z - 17 = 0$ touches the conicoid
$3x^2 - 6y^2 + 9z^2 + 17 = 0$, and find point of contact.

(35) Find the equations to the tangent planes to the surface
$4x^2 - 5y^2 + 7z^2 + 13 = 0$ parallel to the plane $4x + 20y - 21z = 0$.
Find their points of contact also.

(36) Find the locus of the perpendiculars from the origin to the tangent planes to the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which cut off from its axes intercepts the sum of whose reciprocals is equal to a constant $\frac{1}{k}$.

(37) If the section of the enveloping one of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, whose vertex is P by the plane $z = 0$ is a rectangular hyperbola, show that the locus of P is $\frac{x^2+y^2}{a^2+b^2} + \frac{z^2}{c^2} = 1$.

(38) Find the locus of points from which three mutually perpendicular tangent lines can be drawn to the conicoid $ax^2 + by^2 + cz^2 = 1$.

(39) $P(1, 3, 2)$ is a point on the conicoid $x^2 - 2y^2 + 3z^2 + 5 = 0$. Find the locus of the mid-points of chords drawn parallel to OP.

Kakatiya University
B.Sc. Mathematics, V Semester
INTEGRAL CALCULUS

DSE-1E/B
BS:506

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours/week and Practicals: 2 hours/week

Objective: Techniques of multiple integrals will be taught.

Outcome: Students will come to know about its applications in finding areas and volumes of some solids.

UNIT-I

Areas and Volumes: Double Integrals-Double Integrals over a Rectangle-Double Integrals over General Regions in the Plane.

UNIT-II

Double integrals,Changing the order of Integrationm,Triple Integrals: The Integrals over a Box.

UNIT-III

Elementary Regions in Space-Triple Integrals in General,Triple Integral.

UNIT-IV

Change of Variables: Coordinate Transformations-Change of Variables in Triple Integrals.

TEXT: Susan Jane Colley, *Vector Calculus(4e)*
References

- Smith and Minton,*Calculus*
- Shanti Narayan and Mittal, *Integral calculus*
- Ulrich L. Rohde , G. C. Jain, Ajay K. Poddar and A. K. Ghosh;*Introduction to Integral Calculus*

2.14.1 Practicals Question Bank
UNIT-I

(1) Let $R = [-3,3]\times[-2,2]$.Without explicitly evaluating any iterated integrals,determine the value of $\iint_R (x^5 + 2y)dA$.

(2) Integrate the function $f(x,y) = 3xy$ over the region bounded by $y = 32x^3$ and $y = \sqrt{x}$.

(3) Integral the function $f(x,y) = x + y$ over the region bounded by $x + y = 2$ and $y^2 - 2y - x = 0$.

(4) Evaluate $\iint_D xydA$,where D is the region bounded by $x = y^3$ and $y = x^2$.

(5) Evaluate $\iint_D e^{x^2} dA$, where D is the triangular region with vertices $(0,0),(1,0)$ and $(1,1)$.

(6) Evaluate $\iint_D 3ydA$,where D is the region bounded by $xy^2 = 1, y = x, x = 0$ and $y = 3$.

(7) Evaluate $\iint_D (x - 2y)dA$,where D is the region bounded by $y = x^2 + 2$ and $y = 2x^2 - 2$.

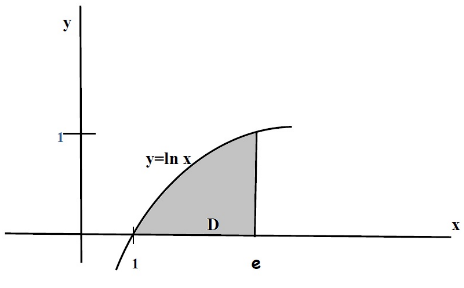
(8) Evaluate $\iint_D (x^2 + y^2)dA$,where D is the region in the first quadrant bounded by $y = x, y = 3x$ and $xy = 3$.

(9) Let D be the region bounded by the parabolas $y = 3x^2, y = 4 - x^2$and the y-axis (Note that parabolas intersect at the point (1,3)).Since D is the type1 elementary region,with $f(x,y) = x^2y$ then find $\iint_D x^2ydA = \int_0^1 \int_{3x^2}^{4-x^2} x^2ydydx$

(10) Find the volume of the region under the graph of
$f(x,y) = 2 - |x| - |y|$.
and above the xy-plane.

Unit-II

(11) Calculate area of shaded region from the given figure. Consider D as type-I region



(12) Use change of order of the integration find integral $\int_0^2 \int_{y^2}^4 ycos(x^2)dxdy$.

(13) consider the integral $\int_0^2 \int_{x^2}^{2x} (2x + 1)dydx$ (a)Evaluate this integral. (b)Sketch the region of integration. (c)Write an equivalent iterated integral with the order of integration reversed.Evaluate this new integral and check that your answer agrees with part(a).

(14) Evaluate $\iiint_{[-2,3]\times[0,1]\times[0,5]} (x^2e^y + xyz)dV$

(15) Evaluate $\iiint_{[-1,1]\times[0,2]\times[1,3]} xyzdV$

(16) Evaluate $\iiint_{[0,1]\times[0,2]\times[0,3]}(x^2+y^2+z^2)dV$

(17) Evaluate $\iiint_{[1,e]\times[1,e]\times[1,e]} \frac{1}{xyz}dV$

(18) Find the value of $\iiint_W zdV, where W = [-1,2]\times[2,5]\times[-3,3]$, without resorting to explicit calulation.

(19) Evaluate the iterative integral. $\int_{-1}^{2}\int_{1}^{z^2}\int_{0}^{y+z} 3yz^2dxdydz.$

(20) Evaluate the iterative integral. $\int_{1}^{3}\int_{0}^{z}\int_{1}^{xz}(x+2y+z)dydxdz.$

Unit-III

(21) Let W be the solid region bounded by the hemisphere $x^2+y^2+z^2=4$ where $z\leq 0$ and the paraboloid $z=4-x^2-y^2$ put solid bounded by them in type1,type2,type3,type4 forms and discuss the same geometrically.

(22) Put the solid bounded by the ellipsoid $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1, a,b,c$ are positive constants in the in type1,type2,type3,type4 forms and discuss the same geometrically.

Integrate the following over the indicated W.

(23) $f(x,y,z)=2x-y+z$; W is the region bounded by the cylinder $z=y^2$,the xy-plane,the planes $x=0, x=1, y=-2, y=2$.

(24) $f(x,y,z)=y$; W is the region bounded by the plane $x+y+z=2$,the cylinder $x^2+z^2=1$ and $y=0$.

(25) $f(x,y,z)=8xyz$; W is the region bounded by the cylinder $y=x^2$,the plane $y+z=9$ and the xy-plane.

(26) $f(x,y,z)=z$; W is the region in the first octant bounded by the cylinder $y^2+z^2=9$ and the planes $y=x, x=0$ and $z=0$.

(27) $f(x,y,z)=1-z^2$; W is the tetrahedron with vertices $(0,0,0),(1,0,0),(0,2,0)$ and $(0,0,3)$.

(28) $f(x,y,z)=3x$; W is the region in the octant bounded by $z=x^2+y^2, x=0, y=0$ and $z=4$.

(29) $f(x,y,z)=x+y$; W is the region bounded by the cylinder $x^2+3z^2=9$ and the plane $y=0, x+y=3$.

(30) $f(x,y,z)=z$;W is the region bounded by $z=0, x^2+4y^2=4$ and $z=x+2$.

Unit-IV

(31) Let $T: R^3\longrightarrow R^3$ be given by $T(u,v,w)=(2u, 2u+3v+w, 3w)$ write T by matrix multiplication.
Integrate the following over the indicated region W.

(32) $f(x,y,z)=4x+y$;W is the region bounded by $x=y^2, y=z, x=y$ and $z=0$.

(33) $f(x,y,z)=x$; W is the region in the first octant bounded by $z=x^2+2y^2,\ z=6-x^2-y^2,\ x=0$ and $y=0$.

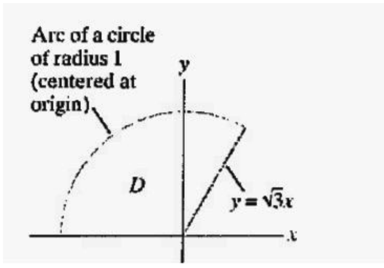
(34) Let $T(u, v) = (3u, -v)$. Write $T(u, v)$ as $A[y]$ for a suitable matrix A.

(35) Describe the image $D = T(D^*)$,where D^* is the unit square $[0, 1] \times [0, 1]$.

(36) Determine the value of $\iint_D \sqrt{\frac{x+y}{x-2y}} dA$, where D is the region in R^2 enclosed by the lines $y = x\frac{2}{,}y = 0$ and $x + y = 1$.

(37) Evaluate $\iint_D \sqrt{\frac{(2x+y-3)^2}{(2y-x+6)^2}} dx\ dy$, where D is the square with vertices $(0, 0), (2, 1), (3, -1)$ and $(1, -2)$. (Hint:First sketch D and find the equations of its sides).

(38) Evaluate $\iint_D \cos(x^2 + y^2) dA$ where D is the shaded region in the following figure.



(39) Evaluate $\iint_D \frac{1}{\sqrt{4-x^2-y^2}} dA$. where D is the disk of radius 1 with center at $(0, 1)$.(Be careful when you describe D.)

(40) Determine the value of $\iiint_W \frac{z}{\sqrt{x^2+y^2}} dV$. where W is the solid region bounded by the plane $z = 12$ and the paraboloid $z = 2x^2 + 2y^2 - 6$.

CURRICULUM FOR MATHEMATICS
IN UNDER GRADUATE DEGREE PROGRAMME

CBCS SYLLABUS SCHEDULE 2016 - 2017
SEMESTER - VI

By

Chairperson

Board of Studies

Department of Mathematics

Kakatiya University, Warangal.

Kakatiya University
B.Sc. Mathematics, VI Semester
Skill Enhancement Course - IV
B.Sc., III Year, VI Semester
Quantitative Aptitude Test

Credits: 2 Theory: 2 hours/week **Marks - 50**

Unit I : Arithematical Ability

1.1 Arithmetical Ability: Ratio and Proportion
1.2 Arithmetical Ability: Time and Work, Time and Distance
1.3 Arithmetical Ability: Simple Interest, Compound Interest
1.4 Arithmetical Ability: Stocks and Shares

Unit II : Data Interpretation

2.1 Data Interpretation: Tabulation
2.2 Data Interpretation: Bar Graphs
2.3 Data Interpretation: Pie Charts
2.4 Data Interpretation: Line Graphs
TEXT: *Quantitative Aptitude* by Dr.R.S.Aggarwal

Kakatiya University
B.Sc. Mathematics, VI Semester
NUMERICAL ANALYSIS

DSC-1F
BS:603

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours/week and Practicals: 2 hours/week

Objective: Students will be made to understand some methods of numerical analysis.

Outcome: Students realize the importance of the subject in solving some problems of algebra and calculus.

UNIT-I

Solutions of Equations in One Variable : The Bisection Method - Fixed-Point Iteration - Newtons Method and Its Extensions - Error Analysis for Iterative Methods - Accelerating Convergence - Zeros of Polynomials and Mullers Method - Survey of Methods and Software.

UNIT-II

Interpolation and Polynomial Approximation: Interpolation and the Lagrange Polynomial - Data Approximation and Nevilles Method - Divided Differences.

UNIT-III

Hermite Interpolation - Cubic Spline Interpolation. Numerical Differentiation and Integration: Numerical Differentiation - Richardsons Extrapolation

UNIT-IV

Elements of Numerical Integration- Composite Numerical Integration - Romberg Integration - Adaptive Quadrature Methods - Gaussian Quadrature.

TEXT: Richard L. Burden and J. Douglas Faires,*Numerical Analysis (9e)*
References

- M. K. Jain, S. R. K. Iyengar and R. K. Jain, *Numerical Methods for Scientific and Engineering computation*
- B. Bradie, *A Friendly introduction to Numerical Analysis*

UNIT-I

(1) Use the Bisection method to find P_3 for $f(x) = \sqrt{x} - cosx$ on [0,1].

(2) Let $f(x) = 3(x+1)(x-1/2)(x-1)$. Use the Bisection method on the following intervals to find P_3.
(a) [-2,1.5]
(b) [-1.25,2.5]

(3) Use the Bisection method to find solutions accurate with in 10^{-5} for the following problems.
(a) $x - 2^{-x} = 0$ for $0 \leq x \leq 1$
(b) $e^x - x^2 + 3x - 2 = 0$ for $0 \leq x \leq 1$
(c) $2xcos(2x) - (x+1)^2 = 0$ for $-3 \leq x \leq -2$ and$-1 \leq x \leq 0$.

(4) Use algebraic manipulation to show that each of the following functions has a fixed point at p precisely when $f(p) = 0$, where $f(x) = x^4 + 2x^2 - x - 3$.
(a) $g_1(x) = (3 + x - 2x^2)^{1/4}$
(b) $g_2(x) = (\frac{x+3-x^4}{2})^{\frac{1}{2}}$

(5) Use a fixed-point iteration method to determine a solution accurate to with in 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on [1,2]. Use $p_0 = 1$.

(6) Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^3-x-1 = 0$ on [1,2] .Use $p_0 = 1$.

(7) Use a fixed-point iteration method to find an approximation to $\sqrt{3}$ that is accurate to within 10^{-4}.

(8) The equation $x^2 - 10cosx = 0$ has two solutions,±1.3793646. Use Newton's method to approximate the solutions to within 10^{-5} with the following values of P_0.
(a) $P_0 = -100$
(b) $P_0 = -50$
(c) $P_0 = -25$
(d) $P_0 = 25$
(e) $P_0 = 50$
(f) $P_0 = 100$

(9) The equation $4x^2 - e^x - e^{-x} = 0$ has two positive solutions x_1 and x_2.Use Newton's method to approximate the solution to within 10^{-5} with the following values of p_0.
(a) $P_0 = -10$ (b) $P_0 = -5$ (c) $P_0 = -3$
(d) $P_0 = -1$ (e) $P_0 = 0$ (f) $P_0 = 1$
(g) $P_0 = 3$ (h) $P_0 = 5$ (i) $P_0 = 10$

(10) Use each of the following methods to find a solution in $[0.1, 1]$ accurate to within 10^{-4} for $600x^4 - 550x^3 + 200x^2 - 20x - 1 = 0$
(a) Bisection method
(b) Newton method
(c) Secant method
(d) Method of False position
(e) Muller's method

UNIT-II

(11) For the given function $f(x)$, let $x_0 = 0$, $x_1 = 0.6$, and $x_2 = 0.9$. Construct interpolation polynomial of degree at most one and at most two to approximate f(0.45),and find the absolute error

(a) $f(x) = cosx$ (b) $f(x) = ln(x+1)$

(12) For the given function $f(x)$, let $x_0 = 1$, $x_1 = 1.25$ and $x_2 = 1.6$. Construct interpolation polynomial degree at most one and at most two to approximate f(1.4), and find the absolute error.

(a) $f(x) = sin\pi x$ (b) $f(x) = log(3x-1)$

(13) Let $P_3(x)$ be the interpolating polynomials for the data $(0,0)$,$(0.5,y)$,$(1,3)$ and $(2,2)$. The coefficient of x^3 in $P_3(x)$ is 6. Find y

(14) Neville's method is used to approximate $f(0.4)$, giving the following table.

$x_0 = 0$	$P_0 = 1$			
$x_1 = 0.25$	$P_1 = 2$	$P_{0,1} = 2.6$		
$x_2 = 0.5$	P_2	$P_{1,2}$	$P_{0,1,2}$	
$x_3 = 0.75$	$P_3 = 8$	$P_{2,3} = 2.4$	$P_{1,2,3} = 2.96$	$P_{0,1,2,3} = 3.016$

Determine $P_2 = f(0.5)$.

(15) Neville's method is used to approximate $f(0.5)$, giving the following table.

$x_0 = 0$	$P_0 = 0$		
$x_1 = 0.4$	$P_1 = 2.8$	$P_{0,1} = 3.5$	
$x_2 = 0.7$	P_2	$P_{1,2}$	$P_{0,1,2} = \frac{27}{7}$

Determine $P_2 = f(0.7)$.

(16) Neville's Algorithm is used to approximate $f(0)$ using $f(-2)$,$f(-1)$,$f(1)$ and $f(2)$. Suppose $f(-1)$ was overstated by 2 and $f(1)$ was understated by 3. Determine the error in the original calculation of the value of the interpolating polynomial to approximate $f(0)$.

(17) Compute the divided difference table for the data

x	1.0	1.3	1.6	1.9	2.2
$f(x)$	0.7651977	0.6200860	0.4554022	0.2818186	0.1103623

(18) Use the Newton forward-difference formula to construct interpolating polynomaials of degree one,two,and three for the following data. Approximate the specified value using each of the polynomials.

(a) $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$

(b) $f(0.18)$ if $f(0.1) = -0.29004986$, $f(0.2) = -0.56079734$, $f(0.3) = -0.81401972$, $f(0.4) = -1.0526302$

(19) Use the Newton backward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

(a) $f(0.43)$ if $f(0) = 1$,$f(0.25) = 1.64872$,$f(0.5) = 2.71828$,$f(0.75) = 4.48169$

(b) $f(0.25)$ if $f(-1) = 0.86199480$, $f(-0.5) = 0.95802009$, $f(0) = 1.0986123$, $f(0.5) = 1.2943767$

(20) Use Stirling's formula to approximate $f(0.43)$ for the following data

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	1.0000	1.22140	1.49182	1.82212	2.22554

UNIT-III

(21) Use the Hermite Polynomial to find an approximation of $f(1.5)$ for the following data

k	x_k	$f(x_k)$	$f'(x_k)$
0	1.3	0.6200860	-0.5220232
1	1.6	0.4554022	-0.56989959
2	1.9	0.2818186	-0.5811571

(22) A car travelling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance is in feet, and the speed is in feet per second.

Time	0	3	5	8	13
Distance	0	225	383	623	993
Speed	75	77	80	74	72

Use Hermites polynomial to predict the position of the car and its speed when $t = 10second$

(23) Use the following values and five - digit - rounding arithematic to construct the Hermite interpolating polynomial to approximate $sin(0.34)$

x	sin$x$	$D_x sinx = cosx$
0.30	0.29552	0.95534
0.32	0.31457	0.94924
0.35	0.34290	0.93937

(24) Determine the natural cubic spline S that interpolates the data $f(0) = 0$,$f(1) = 1$,and $f(2) = 2$.

(25) Determine the clamped cubic spline S that interpolates the data $f(0) = 0$,$f(1) = 1$,$f(2) = 2$, and satisfies $s'(0) = s'(2) = 1$.

(26) Use the forward-difference formula and backward-difference formula to determine each missing entry in the following tables.

(a)

x	$f(x)$	$f'(x)$
0.5	0.4794	
0.6	0.5646	
0.7	0.6442	

(b)

x	$f(x)$	$f'(x)$
0.0	0.0000	
0.2	0.74140	
0.4	1.3718	

(27) Consider the following table of data

x	0.2	0.4	0.6	0.8	1.0
$f(x)$	0.9798652	0.9177710	0.808038	0.6386093	0.3843735

Use all the appropriate formulas given in this section to approxmate $f'(0.4)$ and $f''(0.4)$.

(28) Derive a method for approximating $f'''(x_0)$ whose error term is of order h^2 by expanding the function f in a fourth Taylor polynomial about x_0 and evaluating at $x_0 \pm h$ and $x_0 \pm 2h$.

(29) The forward-difference formula can be expressed as

$f'(x_0) = \frac{1}{h}[f(x_0+h) - f(x_0)] - \frac{h}{2}f''(x_0) - \frac{h^2}{6}f'''(x_0) + O(h^3)$. Use extrapolation to derive $O(h^3)$ formula for $f^{'}(x_0)$

(30) Show that $lim_{h \longrightarrow 0}(\frac{2+h}{2-h})^{\frac{1}{h}} = e$

UNIT-IV

(31) Approximation the following integrals using the Trapezoidal rule.
(a) $\int_{0.5}^{1} x^4 dx$
(b) $\int_{0}^{0.5} \frac{2}{x-4} dx$
(c) $\int_{1}^{1.5} x^2 lnx dx$
(d) $\int_{0}^{1} x^2 e^{-x} dx$

(32) Approximate the following integral using Trapezoidal Rule
(a) $\int_{-0.25}^{0.25} (cosx)^2 dx$
(b) $\int_{-0.5}^{0} xln(x+1) dx$

(33) The Trapezoidal rule applied to $\int_0^2 f(x)dx$ gives the value 5, and the midpoint rule gives the value 4. What value does Simpson's rule give?

(34) The quadrature formula $\int_0^2 f(x)dx = c_0 f(0) + c_1 f(1) + c_2 f(2)$ is exact for all polynomials of degree less than or equal to 2. Determine c_0,c_1, and c_2.

(35) Find the constants c_0, c_1 and x_1 so that quadrature formula $\int_0^1 f(x)dx = c_0 f(0) + c_1 f(x_1)$ has the highest possible degree of precision.

(36) Use the composite Trapezoidal Rule with the indicated values of n to approximate the following integrals
(a) $\int_1^2 xlnx dx$, n=4
(b) $\int_{-2}^{2} x^3 e^x dx$, n=4.

(37) Suppose that $f(0) = 1, f(0.5) = 2.5, f(1) = 2$ and $f(0.25) = f(0.75) = \infty$. Find ∞ if the Composite Trapezoidal rule with $n = 4$ gives the value 1.75 for $\int_0^1 f(x)dx$

(38) Romberg integration is used to approximate $\int_2^3 f(x)dx$.
If $f(2) = 0.51342$,$f(3) = 0.36788$ $R_{31} = 0.43687$, $R_{33} = 0.43662$, find $f(2.5)$

(39) Use Romberg integration to compute $R_{3,3}$ for the following integrals.
(a) $\int_{1}^{1.5} x^2 lnx dx$
(b) $\int_{0}^{1} x^2 e^{-x} dx$

(40) Use Romberg integration to compute $R_{3,3}$ for the following integrals.
(a) $\int_{-1}^{1} (cosx)^2 dx$
(b) $\int_{-0.75}^{0.75} xln(x+1) dx$

Kakatiya University
B.Sc. Mathematics, VI Semester
COMPLEX ANALYSIS

DSE-1F/A
BS:606

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours/week and Practicals: 2 hours/week

Objective: Analytic Functions, contour integration and calculus of residues will be introduced to the students.

Outcome: Students realize calculus of residues is one of the power tools in solvng some problems, like improper and definite integrals, effortlessly.

UNIT-I

Regions in the Complex Plane - Analytic Functions - Functions of a Complex Variable - Mappings - Mappings by the Exponential Function - Limits - Theorems on Limits - Limits Involving the Point at Infinity - Continuity - Derivatives - Differentiation Formulas - Cauchy-Riemann Equations - Sufficient Conditions for Differentiability - Polar Coordinates-Harmonic Functions.

UNIT-II

Elementary Functions: The Exponential Function - The Logarithmic Function - Branches and Derivatives of Logarithms - Some Identities Involving Logarithms Complex Exponents - Trigonometric Functions - Hyperbolic Functions.

UNIT-III

Integrals: Derivatives of Functions w(t) - Definite Integrals of Functions w(t) - Contours - Contour Integrals - Some Examples - Examples with Branch Cuts - Upper Bounds for Moduli of Contour Integrals -Antiderivatives.

UNIT-IV

Cauchy-Goursat Theorem - Proof of the Theorem - Simply Connected Domains - Multiply Connected Domains - Cauchy Integral Formula - An Extension of the Cauchy Integral Formula - Some Consequences of the Extension - Liouville's Theorem and the Fundamental Theorem of Algebra- Maximum Modulus Principle.

TEXT: James Ward Brown and Ruel V. Churchill, *Complex Variables and Applications* (8e)
References:

- Joseph Bak and Donald J Newman, *Complex analysis*
- Lars V Ahlfors , *Complex Analysis*
- S.Lang, *Complex Analysis*
- B Choudary, *The Elements Complex Analysis*

UNIT-I

(1) Sketch the following set and determine which are domains (a) $|z-2+i| \leq 1$
(b) $|2z+3| > 4$
(c) $Imz > 1$
(d) $Imz = 1$.

(2) Sketch the region onto which the sector $r \leq 1, 0 \leq \theta \leq \frac{\pi}{4}$ is mapped by the transformation

(a) $w = z^2$

(b) $w = z^3$

(c) $w = z^4$

(3) Find all roots of the equation
(a) $sinhz = i$ (b) $coshz = \frac{1}{2}$

(4) Find all values of z such that
(a) $e^z = -2$; (b) $e^z = 1 + \sqrt{3}i$; (c) $exp(2z-1) = 1$.

(5) Show that
$lim_{z \to z_0} f(z)g(z)$ if $lim_{z \to z_0} f(z) = 0$
and if there exists a positive number M such that $|g(z)| \leq M$ for all z in some neighborhood of z_0.

(6) Show that $f'(z)$ does not exist at any point if
(a) $f(z) = \overline{z}$ (b)$f(z) = z - \overline{z}$
(c)$f(z) = 2x + ixy^2$ (d)$f(z) = e^x e^{-iy}$

(7) Verify that each of these functions is entire
(a)$f(z) = 3x + y + i(3y - x)$ (b)$f(z) = sinxcoshy + icosxsinhy$
(c)$f(z) = e^{-y}sinx - ie^{-y}cosx$ (d)$f(z) = (z^2-2)e^{-x}e^{-iy}$.

(8) State why a composition of two entire functions is entire. Also, state why any linear combination $c_1 f_1(z) + c_2 f_2(z)$ of two entire functions, where c_1 and c_2 are complex constants, is entire.

(9) Show that $u(x, y)$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$ when
(a) $u(x, y) = 2x(1 - y)$ (b)$u(x, y) = 2x - x^3 + 3xy^2$
(c)$u(x, y) = sinhxsiny$ (d) $u(x, y) = \frac{y}{x^2+y^2}$

(10) Show that if v and V are harmonic conjugates of $u(x, y)$ in a domain D, then $v(x, y)$ and $V(x, y)$ can differ at most by an additive constant.

UNIT-II

(11) Show that $exp(z + \pi i) = -exp(z)$

(12) Find all values of z such that $e^z = -2$

(13) Show that $exp\overline{z} = \overline{expz}$ $\forall z$ and $exp(\overline{iz}) = \overline{exp(iz)}$

(14) Show that the function $exp\overline{z}$ is not analytic any where

(15) Show that $cos(i\overline{z}) = \overline{cos(iz)}\ \forall z$
$sin(i\overline{z}) = \overline{sin(iz)}$ if and only if $z = n\pi i\ (n = 0, \pm1, \pm2....)$

(16) Show that neither $sin\overline{z}$ nor $cos\overline{z}$ is an analytic function of z any where

(17) Show that $sin^{-1}(-i) = n\pi + i(-1)^{n+1}cos(1+\sqrt{2})\ (n = 0, \pm1, \pm2....)$

(18) Show that $cos(-ei) = 1 - \frac{\pi}{2}i$

(19) Find all the roots of the equation $coshz = \frac{1}{2}$

(20) Find all the root of the equation $sinhz = i$

UNIT-III

(21) Evaluate $\int_C f(z)dz$
where $f(z) = \frac{(z+2)}{z}$ and C is

- (a) the semicircle $z = 2e^{i\theta}(0 \leq \theta \leq \pi)$
- (b) the semicircle $z = 2e^{i\theta}(\pi \leq \theta \leq 2\pi)$
- (c) the circle $z = 2e^{i\theta}(0 \leq \theta \leq 2\pi)$

(22) $f(z)$ is defined by the means of the equations $f(z) = \begin{cases} 1 & when\, y < 0 \\ 4\text{y} & when\, y > 0 \end{cases}$ and C is the arc from $z = -1-i$ to $z = 1+i$ along the curve $y = x^3$, then find $\int_C f(z)dz$.

(23) Let C denote the line segment from $z = i$ to $z = 1$. By observing that of all the points on that line segment, the midpoint is the closest to the origin, show that
$|\int_C \frac{dz}{z^4}| \leq 4\sqrt{2}$
without evaluating the integral.

(24) Let C_R denote the upper half of the circle $|z| = R(R > 2)$, taken in the counter clockwise direction. Show that
$|\int_{C_R} \frac{2z^2-1}{z^4+5z^2+4}dz| \leq \frac{\pi R(2R^2+1)}{(R^2-1)(R^2-4)}$.
Then , by dividing the numerator on the right here by R^4 , show that the value of the integral tends to zero as R tends to infinity.

(25) By finding an antiderivative, evaluate each of these integrals, where the path is any contour between the indicated limits of integration:
(a)$\int_i^{i/2} e^{\pi z}dz$ (b)$\int_0^{\pi+2i} cos(\frac{z}{2})dz$ (c)$\int_1^3 (z-2)^3dz$

(26) Use an antiderivative to show that for every contour C extending from a point z_1 to a point z_2,
$\int_C z^n dz = \frac{1}{n+1}(z_2^{n+1} - z_1^{n+1})$ $(n = 0, 1, 2, ...)$

(27) Let C_0 and C denote the circle $z = z_0 + Re^{i\theta}(-\pi \leq \theta \leq \pi)$ and $z = Re^{i\theta}(-\pi \leq \theta \leq \pi)$ respectively.
(a) Use these parametric representations to show that
$\int_{C_0} f(z-z_0)dz = \int_C f(z)dz$

(28) Evaluate the integral $\int_C z^m z^{-n}dz$
where m and n are integers and C is the unit circle $|\ z\ |= 1$ taken counterclockwise.

(29) $f(z) = 1$ and C is an arbitary contour from any fixed point z_1 to any fixed point z_2 in the z plane .Evaluate
$\int_C f(z)dz$

(30) $f(z) = \pi exp(\pi \bar{z})$ and C is the boundary of the square with vertices at the points $0, 1, 1 + i$ and i the orientation of C being in the counterclockwise direction .Evaluate
$\int_c f(z)dz$

UNIT-IV

(31) Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate each of these integrals.
$a.\ \int_C \frac{e^{-z}}{z-(\frac{\pi i}{2})}dz$
$b.\ \int_C \frac{cosz}{z(z^2+8)}dz$
$c.\ \int_C \frac{z}{2z+1}dz$

(32) Find the value of the integral $g(z)$ around the circle$\mid z - i \mid = 2$ in the positive sense when
a.$g(z) = \frac{1}{z^2+4}$
b.$g(z) = \frac{1}{(z^2+4)^2}$

(33) C be the circle $\mid z \mid = 3$ described in the positive sense. Show that if
$g(z) = \int_C \frac{2s^2-s-2}{s-z}dz$, $(\mid z \mid \neq 3)$
then $g(2) = 8\pi i$. What is the value of $g(z)$ when $\mid z \mid > 3$?

(34) Let C be any simple closed contour ,described in the positive sense in z plane ,and write
$g(z) = \int_C \frac{s^3+2s}{(s-z)^3}dz$
Show that $g(z) = 6\pi iz$ when z is inside C and that $g(z) = 0$ when z is outside.

(35) Show that if f is analytic within and on a simple closed contour C and z_0 is not on C, then
$\int_C \frac{f'(z)}{z-z_0}dz = \int_C \frac{f(z)}{(z-z_0)^2}dz$

(36) Let C be the unit circle $z = e^{i\theta}$ $(-\pi \leq \theta \leq \pi)$. First show that for any real constant a
$\int_C \frac{e^{az}}{z}dz = 2\pi i$
Then write this integral in terms of θ to derive the integration formula
$\int_0^\pi e^{acos\theta}cos(asin\theta)d\theta = \pi$

(37) suppose that $f(z)$ is entire and that the harmonic function $u(x, y) = Re|f(z)|$ has an upper bound u_0; that is $u(x, y) \leq u_0$ in the xyplane. Show that $u(x, y)$ must be constant throughtout the plane.

(38) Let a function f be continuous on a closed bounded region R, and let it be analytic and not constant throughout the interior of R. Assuming that $f(z) \neq 0$ anywhere in R. Prove that $|f(z)|$ has a minimum value m in R which occur on the boundary of R and never in the interior. Do this by applying the corresponding result for maximum values to the function $g(z) = \frac{1}{f(z)}$

(39) Let the function $f(z) = u(x, y) + iv(x, y)$ be continuous on a closed bounded region R, and suppose that it is analytic and non constant in the interior of R. Show that the component function $v(x, y)$ has maximum and minimum values in R which are reached on the boundary of R and never in the interior, where it is harmonic

(40) Let f be the function $f(z) = e^z$ and R the rectangular region $0 \leq x \leq 1$, $0 \leq y \leq \pi$. Find points in R where the component function $u(x, y) = Re[f(z)]$ reaches its maximum and minimum values

Kakatiya University
B.Sc. Mathematics, VI Semester
VECTOR CALCULUS

DSE-1F/B
BS:606

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours/week and Practicals: 2 hours/week

Objective: Concepts like gradient, divergence, curl and their physical relevance will be taught.

Outcome: Students realize the way vector calculus is used to addresses some of the problems of physics.

UNIT- I

Line Integrals: Introductory Example : Work done against a Force-Evaluation of Line IntegralsConservative Vector Fields

UNIT- II

Surface Integrals: Introductory Example : Flow Through a PipeEvaluation of Surface Integrals. Volume Integrals: Evaluation of Volume integrals

UNIT- III

Gradient, Divergence and Curl: Partial differentiation and Taylor series in more than one variable-Gradient of a scalar field-Gradients, conservative fields and potentials-Physical applications of the gradient.

UNIT- IV

Divergence of a vector field -Physical interpretation of divergence-Laplacian of a scalar field- Curl of a vector field-Physical interpretation of curl-Relation between curl and rotation-Curl and conservative vector fields.
TEXT: P.C. Matthews, *Vector Calculus*
References:

- G.B. Thomas and R.L. Finney, *Calculus*
- H. Anton, I. Bivens and S. Davis ; *Calculus*
- Smith and Minton, *Calculus*

UNIT-I

(1) Evaluate the line integral $\int_c F \times dr$, where F is the vector field $(y, x, 0)$ and C is the curve $y = sinx$, $z = 0$, between $x = 0$ and $x = \pi$.

(2) Evaluate the line integral $\int_c x + y^2 dr$, where c is the parabola $y = x^2$ in the plane $z = 0$ connecting the points $(0, 0, 0)$ and $(1, 1, 0)$.

(3) Evaluate the line integral $\int_c f.dr$, where F=$(5z^2, 2x, x + 2y)$ and the curve C is given by $x = t, y = t^2, z = t^2$, $0 \leq t \leq 1$

(4) Find the line integral of the vector field $u = (y^2, x, z)$ along the curve given by $z = y = e^x$ from $x = 0$ and $x = 1$.

(5) Evaluate the line integral of the vector field $u = (xy, z^2, x)$ along the curve given by $x = 1+t, y = 0, z = t^2$,$0 \leq t \leq 3$.

(6) Find the line integral of $F = (y, -x, 0)$ along the curve consisting of the two straight line segments $y = 1$, $0 \leq x \leq 1$.

(7) Find the circulation of the vector $F = (y, -x, 0)$ around the unit circle $x^2 + y^2 = 1$, $z = 0$, taken in anticlockwise direction.

(8) Find the line integral $\oint r * dr$,where the curve C is the ellipse $x^2/a^2 + y^2/b^2 = 1$ taken in an anticlockwise direction.What do you notice about the magnitude of the answer?

(9) By considering the line integral of $F = (y, x^2 - x, 0)$ around the square in the x,y plane connecting the four points $(0, 0)$,$(1, 0)$,$(1, 1)$ and $(0, 1)$,show that F cannot be a conservative vector field.

(10) Evaluate the line integral of the vector field $u = (xy, z^2, x)$ along the curve given by $x = 1+t, y = 0, z = t^2$,$0 \leq t \leq 3$.

UNIT-II

(11) Evaluate the surface integral of $u = (y, x^2, z^2)$,over the surface S,where S is the triangular surface on $x = 0$ with $y \geq 0$,$z \geq 0$,$y + x \leq 1$,with the normal n directed in the positive x direction

(12) Find the surface integral of $u = r$ over the part of the paraboloid $z = 1 - x^2 - y^2$ with $z > 0$, with the normal pointing upwards.

(13) If S is the entire x,y plane,evaluate the integral $I = \int_s e^{-x^2-y^2} ds$, by transforming the integral into polar coordinates.

(14) A cube $0 \leq x, y, z \leq 1$ has a variable density given by $\rho = 1 + x + y + z$. what is the total mass of the cube?

(15) Find the volume of the tetrahedron with vertices (0, 0, 0), (a, 0, 0), (0, b, 0) (0, 0, c).

(16) Evaluate the surface integral of $\mathbf{u} = (xy, x, x + y)$ over the surface S defined by $z = 0$ with $0 \leq x \leq 1$, $0 \leq y \leq 2$, with the normal $\mathbf{n}$ directed in the positive z direction.

(17) The surface S is defined to be that part of the plane $z = 0$ lying between the curve $y = x^2$ and $x = y^2$. Find the surface integral of $\mathbf{u.n}$ over S where $u = (z, xy, x^2)$ and $\mathbf{n} = (0, 0, 1)$.

(18) Find the surface integral of **u.n** over S where S is the part of the surface $z = x + y^2$ with $z < 0$ and$x > -1$, u is the vector field $\mathbf{u} = (2y + x, -1, 0)$ and **n** has a negative z component.

(19) Find the volume integral of the scalar field $\phi = x^2 + y^2 + z^2$ over the region V specified by $0 \le x \le 1, 1 \le y \le 2, 0 \le z \le 3$.

(20) Find the volume of the section of the cylinder $x^2 + y^2 = 1$ that lies between the planes $z = x + 1$ and $z = -x - 1$.

(21) Find the unit normal **n** to the surface $x^2 + y^2 - z = 0$ at the point $(1, 1, 2)$.

(22) find the gradient of the scalar field $f = xyz$ and evaluate it at the point $(1, 2, 3)$. Hence find the diraction derivative of f at this point in the direction of the vector $(1, 1, 0)$.

UNIT-III

(23) Find the divergence of the vector field **u=r**.

(24) The vector field **u** is defined by $\mathbf{u} = (xy, z + x, y)$. Calculate $\bigtriangledown \times u$ and find the point where $\bigtriangledown \times u = 0$.

(25) Find the gradient $\bigtriangledown\phi$ and the Laplacian $\bigtriangledown^2\phi$ for the scalar field $\phi = x^2 + xy + yz^2$.

(26) Find the gradient and the Laplacian of $\phi = \sin(kx)\sin(ly)e^{\sqrt{k^2+l^2}z}$.

(27) Find the unit normal to the surface $xy^2 + 2yz = 4$ at the point (-2, 2, 3).

(28) For $\phi(x, y, z) = x^2 + y^2 + z^2 + xy - 3x$, find $\bigtriangledown\phi$ and find the minimum value of ϕ.

(29) Find the equation of the plane which is tangent to the surface $x^2 + y^2 - 2z^3 = 0$ at the point (1, 1, 1).

(30) Prove that $\nabla^2(\frac{1}{r}) = 0$

UNIT-IV

(31) Find both the divergence and the curl of the vector fields
(a) $\mathbf{u} = (y, z, x)$;
(b) $V = (xyz, z^2, x - y)$.

(32) For what values, if any, of the constants a and b is the vector field $\mathbf{u} = (y\cos x + axz, b\sin x + z, x^2 + y)$ irrotational?

(33) (a) Show that $\mathbf{u} = (y^2 z, -z^2\sin y + 2xyz, 2z\cos y + y^2 x)$ is irrotational.
(b) Find the corresponding potential function.
(c) Hence find the value of the line integral of **u** along the curve
$x = \sin\frac{\pi t}{2}, y = t^2 - t, z = t^4, 0 \le t \le 1$.

(34) Find the divergence of the vector field $u = \overrightarrow{r}$.

(35) The vector field u is defined by $u = (xy, x+z, y)$, then calculate $\nabla \times u$ and find the points where $\nabla \times u = 0$.

(36) Show that both the divergence and the curl are linear operators.

(37) Find $\nabla . \nabla \phi$ if $\phi = 2x^3y^2z^4$.

(38) If $A = x^2yi - 2xzj + 2yzk$ then find $curl\ curl\ A$.

(39) Show that $div\ curl\ A = 0$.

(40) If $A = xz^3i - 2x^2yzj + 2yz^4k$ then find $\nabla \ \times \ A$ at the point $(1, -1, 1)$.