TELANGANA UNIVERSITY

NIZAMABAD - 503 322

(Established under Act No.28 of 2006, A.P.)

UG CBCS MATHEMATICS SYLLABUS

(with effect from 2016-17 academic year)

DEPARTMENT OF MATHEMATICS

Telangana State
Council of Higher Education

Government of Telangana

Mathematics Course Structure

(B.Sc. Common Core Syllabus for All Universities in Telangana with effect from 2016-17)

Contents

B.Sc. Course Structure Template

B.Sc. PROGRAMME

FIRST YEAR SEMESTER-I				
Code	Course Title	Course Type	HPW	Credits
BS101	Communication	AECC-1	2	2
BS102	English	CC-1A	5	5
BS103	Second Language	CC –2A	5	5
BS104	Optional - I **Differential Calculus**	DSC-1A	4 T + 2P = 6	4+1=5
BS105	Optional - II	DSC-2A	4 T + 2P = 6	4+1=5
BS106	Optional – III	DSC-3A	4 T + 2P = 6	4+1=5
			30	27
SEMESTER-II				
BS201	Environmental Studies	AECC-2	2	2
BS202	English	CC-1B	5	5
BS203	Second Language	CC –2B	5	5
BS204	Optional - I **Differential Equations**	DSC-1B	4 T + 2P = 6	4+1=5
BS205	Optional - II	DSC-2B	4 T + 2P = 6	4+1=5
BS206	Optional – III	DSC-3B	4 T + 2P = 6	4+1=5
			30	27

B.Sc. PROGRAMME

SECOND YEAR SEMESTER-III				
BS301	A/B **Logic& Sets/Theory of Equations**	SEC-1	2	2
BS302	English	CC-1C	5	5
BS303	Second Language	CC-2C	5	5
BS304	Optional - I **Real Analysis**	DSC-1C	4 T + 2P = 6	4+1=5
BS305	Optional - II	DSC-2C	4 T + 2P = 6	4+1=5
BS306	Optional – III	DSC-3C	4 T + 2P = 6	4+1=5
			30	27
SEMESTER-IV				
BS401	C/D **Transportation & Game Theory/ Number Theory**	SEC-2	2	2
BS402	English	CC -1D	5	5
BS403	Second Language	CC-2D	5	5
BS404	Optional - I **Algebra**	DSC-1D	4 T + 2P = 6	4+1=5
BS405	Optional - II	DSC-2D	4 T + 2P = 6	4+1=5
BS406	Optional – III	DSC-3D	4 T + 2P = 6	4+1=5
			30	27

B.Sc. Course Structure Template

B.Sc. PROGRAMME

THIRD YEAR SEMESTER-V				
Code	Course Title	Course Type	HPW	Credits
BS501	E/F Probability and Statistics/Mathematical Modelling	SEC-3	2	2
BS502	Lattice Theory	GE-1	2 T	2
BS503	Optional - I Linear Algebra	DSC-1E	3 T + 2P = 5	3+1=4
BS504	Optional –II	DSC-2E	3 T + 2P = 5	3+1=4
BS505	Optional –III	DSC-3E	3 T + 2P = 5	3+1=4
BS506	Optional –I A/B/C Slid Geometry/ Integral Calculus	DSE- 1E	3 T + 2P = 5	3+1=4
BS507	Optional – II A/B/C	DSE-2E	3 T + 2P = 5	3+1=4
BS508	Optional – III A/B/C	DSE-3E	3 T + 2P = 5	3+1=4
			34	28
SEMESTER-VI				
BS601	G/H Boolean Algebra/Graph Theory	SEC-4	2	2
BS602	Elements of Number Theory	GE-2	2 T	2
BS603	Optional - I Numerical Analysis	DSC-1F	3 T + 2P = 5	3+1=4
BS604	Optional –II	DSC-2F	3 T + 2P = 5	3+1=4
BS605	Optional –III	DSC-3F	3 T + 2P = 5	3+1=4
BS606	Optional –I A/B/C Complex Analysis/ Vector Calculus	DSE- 1F	3 T + 2P = 5	3+1=4
BS607	Optional – II A/B/C	DSE-2F	3 T + 2P = 5	3+1=4
BS608	Optional – III A/B/C	DSE-3F	3 T + 2P = 5	3+1=4
			34	28
	TOTAL Credits			164

SUMMARY OF CREDITS

B.Sc. PROGRAMME

Sl. No.	Course Category	No. of Courses	Credits Per Course	Credits
1	AECC	2	2	4
2	SEC	4	2	8
3	CC	8	5	40
	Language	12	5	60
	DSC	6	4	24
	DSC			
4	DSE	6	4	24
5	GE	2	2	4
	TOTAL	**40**		**164**
	Optionals Total	24		108

Syllabus

DSC-1A DIFFERENTIAL CALCULUS BS:104

Theory: 4 credits and Practicals: 1 credits
Theory: 4 hours /week and Practicals: 2 hours /week

Objective: The course is aimed at exposing the students to some basic notions in differential calculus .

Outcome: By the time students completes the course they realize wide ranging applications of the subject.

Unit- I

Successive differentiation- Expansions of Functions- Mean value theorems

Unit – II

Indeterminate forms – Curvature and Evolutes

Unit – III

Partial differentiation – Homogeneous functions- Total derivative

Unit – IV

Maxima and Minima of functions of two variables – Lagrange's Method of multipliers –Asymptotes- Envelopes

Text : Shanti Narayan and Mittal, *Differential Calculus*

References: William Anthony Granville, Percey F Smith and William Raymond Longley; *Elements of the differential and integral calculus*

Joseph Edwards , *Differential calculus for beginners*

Smith and Minton, *Calculus*

Elis Pine, *How to Enjoy Calculus*

Hari Kishan ,*Differential Calculus*

Differential Calculus

Practicals Question Bank

UNIT-I

1. If $u = \tan^{-1} x$, prove that

$$(1+x^2)\frac{d^2u}{dx^2} + 2x\frac{du}{dx} = 0$$

and hence determine the values of the derivatives of u when $x=0$

2. If

$$y = \sin(m \sin^{-1} x), \text{ show that}$$

$$(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2 - m^2)y_n$$

and find $y_n(0)$.

3. If U_n denotes the nth derivative of $(Lx+M)/(x^2-2Bx+C)$., prove

$$\frac{x^2-2Bx+C}{(n+1)(n+2)}U_{n+2} + \frac{2(x-B)}{n+1}U_{n+1} + U_n = 0.$$

4. If $y = x^2e^x$, then

$$\frac{d^ny}{dx^n} = \frac{1}{2}n(n-1)\frac{d^2y}{dx^2} - n(n-2)\frac{dy}{dx} + \frac{1}{2}(n-1)(n-2)y.$$

5. Determine the intervals in which the function

$$(x^4+6x^3+17x^2+32x+32)e^{-x}$$

is increasing or decreasing.

6. Separate the intervals in which the function

$$(x^2+x+1)/(x^2-x+1)$$

is increasing or decreasing.

7. Show that if $x > 0$,

(i) $x - \dfrac{x^2}{2} < \log(1+x) < x - \dfrac{x^2}{2(1+x)}$.

(ii) $x - \dfrac{x^2}{2} + \dfrac{x^3}{3(1+x)} < \log(1+x) < x - \dfrac{x^2}{2} + \dfrac{x^3}{3}$.

8. Prove that

$$e^{ax}\sin bx = bx + abx^2 + \frac{3a^2b-b^3}{3!}x^3 + \dots$$

$$+ \frac{(a^2+b^2)^{\frac{1}{2}n}}{n!}x^n \sin\left(n\tan^{-1}\frac{b}{a}\right) + \dots$$

9. Show that $\cos^2 x = 1 - x^2 + \frac{1}{3}x^4 - \frac{2}{45}x^6 \dots\dots$

10. Show that

$$e^{m\tan^{-1}x} = 1 + mx + \frac{m^2}{2!}x^2 + \frac{m(m^2-2)}{3!}x^3 + \frac{m^2(m^2-8)}{4!}x^4 + \dots.$$

UNIT-II

1. Find the radius of curvature at any point on the curves

(i) $y = c\cosh(x/c)$ (*Catenary*).

(ii) $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$.

(iii) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. (Astroid)

(iv) $x = (a\cos t)/t$, $y = (a\sin t)/t$.

2. Show that for the curve

$$x = a\cos\theta\,(1+\sin\theta),\ y = a\sin\theta\,(1+\cos\theta),$$

the radius of curvature is, a, at the point for which the value of the parameter is $-\pi/4$.

3. Prove that the radius of curvature at the point $(-2a, 2a)$ on the curve $x^2y = a(x^2+y^2)$ is, $-2a$.

4. Show that the radii of curvature of the curve

$$x=ae^{\theta}\ (\sin\theta-\cos\theta),\ y=ae^{\theta}\ (\sin\theta+\cos\theta),$$

and its evolute at corresponding points are equal.

5. Show that the whole length of the evolute of the ellipse

$$x^2/a^2+y^2/b^2=1$$

is $4(a^2/b-b^2/a)$.

6. Show that the whole length of the evolute of the astroid

$$x=a\cos^3\theta,\ y=a\sin^3\theta$$

is $12a$.

7. Evaluate the following :

(*i*) $\lim\limits_{x\to 0}\dfrac{xe^x-\log(1+x)}{x^2}$. (*D.U. 1952*) (*ii*) $\lim\limits_{x\to 0}\dfrac{x\cos x-\log(1+x)}{x^2}$.

(*D. U. Hons. 1951, P.U. 1957*)

(*iii*) $\lim\limits_{x\to 0}\dfrac{e^x\sin x-x-x^2}{x^2+x\log(1-x)}$. (*D.U. 1953*) (*iv*) $\lim\limits_{x\to 0}\left\{\dfrac{1}{x}-\dfrac{1}{x^2}\log(1+x)\right\}$.

(*D.U. 1955*)

8. If the limit of

$$\frac{\sin 2x+a\sin x}{x^3},$$

as x tends to zero, be finite, find the value of a and the limit.

9. Determine the limits of the following functions :

(*i*) $x\log\tan x$, $(x\to 0)$. (*ii*) $x\tan(\pi/2-x)$, $(x\to 0)$.

(*iii*) $(a-x)\tan(\pi x/2a)$, $(x\to 0)$.

10. Determine the limits of the following functions :

i. $\dfrac{e^x-e^{-x}-x}{x^2\sin x}$, $(x\to 0)$. ii. $\dfrac{\log x}{x^3}$, $(x\to\infty)$.

iii. $\dfrac{1+x\cos x-\cosh x-\log(1+x)}{\tan x-x}$, $(x\to 0)$.

iv. $\dfrac{\log(1+x)\log(1-x)-\log(1-x^2)}{x^4}$, $(x\to 0)$.

UNIT-III

1. If $z=xy\,f(x/y)$, show that

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2z.$$

2. If $z(x+y)=x^2+y^2$, show that

$$\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right).$$

3. If $z=3xy-y^3+(y^2-2x)^{\frac{3}{2}}$, verify that

$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x}\text{ and }\frac{\partial^2 z}{\partial x^2}\frac{\partial^2 z}{\partial y^2}=\left(\frac{\partial^2 z}{\partial x\partial y}\right)^2.$$

4. If $z=f(x+ay)+\varphi(x-ay)$, prove that

$$\frac{\partial^2 z}{\partial y^2}=a^2\frac{\partial^2 z}{\partial x^2}.$$

5. If $u=\tan^{-1}\dfrac{x^3+y^3}{x-y}$, find

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}.$$

6. If $f(x,y)=0$, $\varphi(y,z)=0$, show that

$$\frac{\partial f}{\partial y}\cdot\frac{\partial\varphi}{\partial z}\cdot\frac{dz}{dx}=\frac{\partial f}{\partial x}\cdot\frac{\partial\varphi}{\partial y}.$$

7. If $x\sqrt{(1-y^2)}+y\sqrt{(1-x^2)}=a$, show that

$$\frac{d^2y}{dx^2}=\frac{a}{(1-x^2)^{\frac{3}{2}}}.$$

8. Given that

$$f(x, y)\equiv x^3+y^3-3axy=0, \text{ show that}$$

$$\frac{d^2y}{dx^2}\cdot\frac{d^2x}{dy^2}=\frac{4a^6}{xy(xy-2a^2)^3}.$$

9. If u and v are functions of x and y defined by

$$x=u+e^{-v}\sin u,\ y=v+e^{-v}\cos u,$$

prove that

$$\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}.$$

10. *If $H=f(y-z,\ z-x,\ x-y)$; prove that,*

$$\frac{\partial H}{\partial x}+\frac{\partial H}{\partial y}-\frac{\partial H}{\partial z}=0.$$

UNIT-IV

1. Find the minimum value of $x^2+y^2+z^2$ when
   (*i*) $x+y+z=3a$.
   (*ii*) $xy+yz+zx=3a^2$.
   (*iii*) $xyz=a^3$.
2. Find the extreme value of xy when
   $$x^2+xy+y^2=a^2.$$
3. In a plane triangle, find the maximum value of
   $$\cos A\cos B\cos C.$$

4. ***Find the envelope of the family of semi-cubical parabolas***
   $$y^2-(x+a)^3=0.$$
5. *Find the envelope of the family of ellipses*
   $$x^2/a^2+y^2/b^2=1,$$
   where the two parameter a, b, are connected by the relation
   $$a+b=c\ ;$$
   c, being a constant.
6. *Show that the envelope of a circle whose centre lies on the parabola $y^2=4ax$ and which passes through its vertex is the cissoid*
   $$y^2(2a+x)+x^3=0.$$
7. Find the envelope of the family of straight lines $x/a+y/b=1$ where a, b are connected by the relation
   (*i*) $a+b=c$. (*ii*) $a^2+b^2=c^2$. (*iii*) $ab=c^2$,
   c is a constant.
8. *Find the asymptotes of*
   $$x^3+4x^2y+4xy^2+5x^2+15xy+10y^2-2y+1=0.$$
9. *Find the asymptotes of*
   $$x^3+4x^2y+4xy^2+5x^2+15xy+10y^2-2y+1=0.$$
10. Find the asymptotes of the following curves
    i. $xy(x+y)=a(x^2-a^2)$.
    ii. $(x-1)(x-2)(x+y)+x^2+x+1=0$.
    iii. $y^3-x^3+y^2+x^2+y-x+1=0$.

DSC- 1B | DIFFERENTIAL EQUATIONS | BS:204

DIFFERENTIAL EQUATIONS

Theory: 4 Credits and Practicals: 1 credits
Theory: 4 hours /week and Practicals: 2 hours /week

Objective: The main aim of this course is to introduce the students to the techniques of solving differential equations and to train to apply their skills in solving some of the problems of engineering and science.

Outcomes: After learning the course the students will be equipped with the various tools to solve few types differential equations that arise in several branches of science.

Unit – I

Differential Equations of first order and first degree:
Exact differential equations – Integrating Factors – Change in variables – Total Differential Equations – Simultaneous Total Differential Equations – Equations of the form $dx/P = dy/Q = dz/R$
Differential Equations first order but not of first degree: Equations Solvable for y – Equations Solvable for x – Equations that do not contain x (or y) – Clairaut's equation

Unit – II

Higher order linear differential equations: Solution of homogeneous linear differential equations with constant coefficients – Solution of non-homogeneous differential equations P(D)y= Q(x) with constant coefficients by means of polynomial operators when $Q(x)=bx^k, be^{ax}, e^{ax}V, b\cos(ax), b\sin(ax)$

Unit – III

Method of undetermined coefficients – Method of variation of parameters – Linear differential equations with non constant coefficients – The Cauchy – Euler Equation

Unit – IV

Partial Differential equations- Formation and solution- Equations easily integrable – Linear equations of first order – Non linear equations of first order – Charpit's method – Non homogeneous linear partial differential equations – Separation of variables

Text: Zafar Ahsan, *Differential Equations and Their Applications*

References: Frank Ayres Jr, *Theory and Problems of Differential Equations*

Ford, L.R, *Differential Equations.*

Daniel Murray, *Differential Equations*

S. Balachandra Rao, *Differential Equations with Applications and Programs*

Stuart P Hastings, J Bryce McLead; *Classical Methods in Ordinary Differential Equations*

Differential Equations
Practicals Question Bank

Unit-I

Solve the following differential equations:

1. $y' = \sin(x+y) + \cos(x+y)$

2. $xdy - ydx = a(x^2 + y^2)dy$

3. $x^2 ydx - (x^3 + y^3)dy = 0$

4. $(y+z)dx + (x+z)dy + (x+y)dz = 0$

5. $y \sin 2x dx - (1 + y^2 + \cos^2 x)dy = 0$

6. $y + px = p^2 x^4$

7. $yp^2 + (x-y)p - x = 0$

8. $\dfrac{dx}{y - zx} = \dfrac{dy}{yz + x} = \dfrac{dz}{x^2 + y^2}$

9. $\dfrac{dx}{x(y^2 - z^2)} = \dfrac{dy}{y(z^2 - x^2)} = \dfrac{dz}{z(x^2 - y^2)}$

10. Use the transformation $x^2 = u$ and $y^2 = v$ to solve the equation

$axyp^2 + (x^2 - ay^2 - b)p - xy = 0$.

Unit-II

Solve the following differential equations:

1. $D^2 y + (a+b)Dy + aby = 0$

2. $D^3 y - D^2 y - Dy - 2y = 0$

3. $D^3 y + Dy = x^2 + 2x$

4. $y'' + 3y' + 2y = 2(e^{-2x} + x^2)$

5. $y^{(5)} + 2y''' + y' = 2x + \sin x + \cos x$

6. $(D^2 + 1)(D^2 + 4)y = \cos\frac{x}{2}\cos\frac{3x}{2}$

7. $(D^2 + 1)y = \cos x + xe^{2x} + e^x \sin x$

8. $y'' + 3y' + 2y = 12e^x$

9. $y'' - y = \cos x$

10. $4y'' - 5y' = x^2 e^x$

Unit-III

Solve the following differential equations:

1. $y'' + 3y' + 2y = xe^x$

2. $y'' + 3y' + 2y = \sin x$

3. $y'' + y' + y = x^2$

4. $y'' + 2y' + y = x^2 e^{-x}$

5. $x^2 y'' - xy' + y = 2\log x$

6. $x^4 y''' + 2x^3 y'' - x^2 y' + xy = 1$

7. $x^2 y'' - xy' + 2y = x\log x$

8. $x^2 y'' - xy' + 2y = x$

Use the reduction of order method to solve the following homogeneous equation whose one of the solutions is given:

9. $y'' - \frac{2}{x}y' + \frac{2}{x^2}y = 0$, $y_1 = x$

10. $(2x^2 + 1)y'' - 4xy' + 4y = 0$, $y_1 = x$

Unit-IV

1. Form the partial differential equation , by eliminating the arbitrary constants from $z=(x^2+a)(y^2+b)$.

2. Find the differential equation of the family of all planes whose members are all at a constant distance r from the origin.

3. Form the differential equation by eliminating arbitrary function F from

$F(x^2+y^2, z-xy)=0$.

Solve the following differential equations:

4. $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$

5. $x(z^2-y^2)p+y(x^2-z^2)q=z(y^2-x^2)$

6. $\left(p^2-q^2\right)z=x-y$

7. $z=px+qy+p^2q^2$

8. $z^2=pqxy$

9. $z^2(p^2+q^2)=x^2+y^2$

10. $r+s-6t=\cos(2x+y)$

SEC-1A **LOGIC AND SETS** BS: 301

Credits: 2
Theory : 2 hours /week

Objective: Students learn some concepts in set theory and logic.
Outcome: After the completion of the course students appreciate its importance in the development of computer science.

Unit – I

Basic Connectives and truth tables – Logical equivalence : Laws of Logic – Logical Implication : Rules Inference : The Use of Quantifiers – Quantifiers, Definitions, and proofs of Theorems

Unit – II

Sets and Subsets – Set Operations and the Laws of Set Theory – Counting and Venn Diagrams – A First Word on Probability – The axioms of Probability – Conditional Probability: Independence – Discrete Random variables

Text : Ralph P Grimaldi, *Discrete and Combinatorial Mathematics (5e)*

References: P R Halmos, *Naïve Set Theory*

E Kamke, *Theory of Sets*

SEC-1B THEORY OF EQUATIONS BS: 301

Credits: 2

Theory: 2 hours /week

Objective: Students learn the relation between roots and coefficients of a polynomial equation, ,Descartes's rule of signs in finding the number of positive and negative roots if any of a polynomial equation bsides some other concepts.

Outcome: By using the concepts learnt the students are expected to solve some of the polynomial equations.

Unit I

Graphic representation of a polynomial-Maxima and minima values of polynomials-Theorems relating to the real roots of equations-Existence of a root in the general equation –Imaginary roots-Theorem determining the number of roots of an equation-Equal roots-Imaginary roots enter equations in pairs-Descartes' rule of signs for positive roots- Descartes' rule of signs for negative roots-

Unit II

Relations between the roots and coefficients-Theorem-Applications of the theorem-Depression of an equation when a relation exists between two of its roots-The cube roots of unity-Symmetric functions of the roots-examples.

Text: W.S. Burnside and A.W. Panton, *The Theory of Equations*

References: C. C. Mac Duffee, Theory *of Equations*

Hall and Knight, *Higher Algebra*

DSC- 1C REAL ANALYSIS BS: 304

Theory: 4 credits and Practicals: 1 credits
Theory: 4 hours /week and Practicals: 2 hours /week

Objective: The course is aimed at exposing the students to the foundations of analysis which will be useful in understanding various physical phenomena.

Outcome: After the completion of the course students will be in a position to appreciate beauty and applicability of the course.

Unit – I

Sequences: Limits of Sequences- A Discussion about Proofs-Limit Theorems for Sequences-Monotone Sequences and Cauchy Sequences

Unit – II

Subsequences-Lim sup's and Lim inf's-Series-Alternating Series and Integral Tests

Unit – III

Sequences and Series of Functions: Power Series-Uniform Convergence-More on Uniform Convergence-Differentiation and Integration of Power Series (Theorems in this section without Proofs)

Unit – IV

Integration : The Riemann Integral – Properties of Riemann Integral-Fundamental Theorem of Calculus

Text: Kenneth A Ross, *Elementary Analysis-The Theory of Calculus*

References: William F. Trench, *Introduction to Real Analysis*
Lee Larson, *Introduction to Real Analysis I*

Shanti Narayan and Mittal, *Mathematical Analysis*

Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner; *Elementary Real analysis*

Sudhir R. Ghorpade Balmohan V. Limaye ,*A Course in Calculus and Real Analysis*

Real Analysis

Practicals Question Bank

UNIT-I

1

For each sequence below, determine whether it converges and, if it converges, give its limit. No proofs are required.

(a) $a_n = \frac{n}{n+1}$ **(b)** $b_n = \frac{n^2+3}{n^2-3}$

(c) $c_n = 2^{-n}$ **(d)** $t_n = 1 + \frac{2}{n}$

(e) $x_n = 73 + (-1)^n$ **(f)** $s_n = (2)^{1/n}$

2

Determine the limits of the following sequences, and then prove your claims.

(a) $a_n = \frac{n}{n^2+1}$ **(b)** $b_n = \frac{7n-19}{3n+7}$

(c) $c_n = \frac{4n+3}{7n-5}$ **(d)** $d_n = \frac{2n+4}{5n+2}$

(e) $s_n = \frac{1}{n}\sin n$

3

Suppose $\lim a_n = a$, $\lim b_n = b$, and $s_n = \frac{a_n^3+4a_n}{b_n^2+1}$. Prove $\lim s_n = \frac{a^3+4a}{b^2+1}$ carefully, using the limit theorems.

4

Let $x_1 = 1$ and $x_{n+1} = 3x_n^2$ for $n \geq 1$.

(a) Show if $a = \lim x_n$, then $a = \frac{1}{3}$ or $a = 0$.

(b) Does $\lim x_n$ exist? Explain.

(c) Discuss the apparent contradiction between parts (a) and (b).

5

Which of the following sequences are increasing? decreasing? bounded?

(a) $\frac{1}{n}$ **(b)** $\frac{(-1)^n}{n^2}$

(c) n^5 **(d)** $\sin(\frac{n\pi}{7})$

(e) $(-2)^n$ **(f)** $\frac{n}{3^n}$

6

Let (s_n) be a sequence such that

$$|s_{n+1} - s_n| < 2^{-n} \quad \text{for all} \quad n \in \mathbb{N}.$$

Prove (s_n) is a Cauchy sequence and hence a convergent sequence.

7

Let (s_n) be an increasing sequence of positive numbers and define $\sigma_n = \frac{1}{n}(s_1 + s_2 + \cdots + s_n)$. Prove (σ_n) is an increasing sequence.

8

Let $t_1 = 1$ and $t_{n+1} = [1 - \frac{1}{4n^2}] \cdot t_n$ for $n \geq 1$.

(a) Show $\lim t_n$ exists.

(b) What do you think $\lim t_n$ is?

9

Let $t_1 = 1$ and $t_{n+1} = [1 - \frac{1}{(n+1)^2}] \cdot t_n$ for $n \geq 1$.

(a) Show $\lim t_n$ exists.

(b) What do you think $\lim t_n$ is?

(c) Use induction to show $t_n = \frac{n+1}{2n}$.

(d) Repeat part (b).

10

Let $s_1 = 1$ and $s_{n+1} = \frac{1}{3}(s_n + 1)$ for $n \geq 1$.

(a) Find s_2, s_3 and s_4.

(b) Use induction to show $s_n > \frac{1}{2}$ for all n.

(c) Show (s_n) is a decreasing sequence.

(d) Show $\lim s_n$ exists and find $\lim s_n$.

UNIT-II

11

Let $a_n = 3 + 2(-1)^n$ for $n \in \mathbb{N}$.

(a) List the first eight terms of the sequence (a_n).

(b) Give a subsequence that is constant [takes a single value]. Specify the selection function σ.

12

Consider the sequences defined as follows:

$$a_n = (-1)^n, \qquad b_n = \frac{1}{n}, \qquad c_n = n^2, \qquad d_n = \frac{6n+4}{7n-3}.$$

(a) For each sequence, give an example of a monotone subsequence.

(b) For each sequence, give its set of subsequential limits.

(c) For each sequence, give its $\limsup$ and $\liminf$.

(d) Which of the sequences converges? diverges to $+\infty$? diverges to $-\infty$?

(e) Which of the sequences is bounded?

13

Prove $\limsup |s_n| = 0$ if and only if $\lim s_n = 0$.

14

Let (s_n) and (t_n) be the following sequences that repeat in cycles of four:

$$(s_n) = (0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, \ldots)$$
$$(t_n) = (2, 1, 1, 0, 2, 1, 1, 0, 2, 1, 1, 0, 2, 1, 1, 0, 2, \ldots)$$

Find

(a) $\liminf s_n + \liminf t_n$, **(b)** $\liminf(s_n + t_n)$,
(c) $\liminf s_n + \limsup t_n$, **(d)** $\limsup(s_n + t_n)$,
(e) $\limsup s_n + \limsup t_n$, **(f)** $\liminf(s_n t_n)$,
(g) $\limsup(s_n t_n)$

15

Determine which of the following series converge. Justify your answers.

(a) $\sum \frac{n^4}{2^n}$ **(b)** $\sum \frac{2^n}{n!}$

(c) $\sum \frac{n^2}{3^n}$ **(d)** $\sum \frac{n!}{n^4+3}$

(e) $\sum \frac{\cos^2 n}{n^2}$ **(f)** $\sum_{n=2}^{\infty} \frac{1}{\log n}$

16

Prove that if $\sum a_n$ is a convergent series of nonnegative numbers and $p > 1$, then $\sum a_n^p$ converges.

17

Show that if $\sum a_n$ and $\sum b_n$ are convergent series of nonnegative numbers, then $\sum \sqrt{a_n b_n}$ converges. *Hint*: Show $\sqrt{a_n b_n} \leq a_n + b_n$ for all n.

18

We have seen that it is often a lot harder to find the value of an infinite sum than to show it exists. Here are some sums that can be handled.

(a) Calculate $\sum_{n=1}^{\infty}(\frac{2}{3})^n$ and $\sum_{n=1}^{\infty}(-\frac{2}{3})^n$.

(b) Prove $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$. *Hint*: Note that $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \sum_{k=1}^{n}[\frac{1}{k} - \frac{1}{k+1}]$.

(c) Prove $\sum_{n=1}^{\infty} \frac{n-1}{2^{n+1}} = \frac{1}{2}$. *Hint*: Note $\frac{k-1}{2^{k+1}} = \frac{k}{2^k} - \frac{k+1}{2^{k+1}}$.

(d) Use (c) to calculate $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

19

Determine which of the following series converge. Justify your answers.

(a) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\log n}$ **(b)** $\sum_{n=2}^{\infty} \frac{\log n}{n}$

(c) $\sum_{n=4}^{\infty} \frac{1}{n(\log n)(\log\log n)}$ **(d)** $\sum_{n=2}^{\infty} \frac{\log n}{n^2}$

20

Show $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges if and only if $p > 1$.

UNIT-III

21

For each of the following power series, find the radius of convergence and determine the exact interval of convergence.

(a) $\sum n^2 x^n$ **(b)** $\sum (\frac{x}{n})^n$

(c) $\sum (\frac{2^n}{n^2}) x^n$ **(d)** $\sum (\frac{n^3}{3^n}) x^n$

(e) $\sum (\frac{2^n}{n!}) x^n$ **(f)** $\sum (\frac{1}{(n+1)^2 2^n}) x^n$

(g) $\sum (\frac{3^n}{n \cdot 4^n}) x^n$ **(h)** $\sum (\frac{(-1)^n}{n^2 \cdot 4^n}) x^n$

22

For $n = 0, 1, 2, 3, \ldots$, let $a_n = [\frac{4+2(-1)^n}{5}]^n$.

(a) Find $\limsup(a_n)^{1/n}$, $\liminf(a_n)^{1/n}$, $\limsup|\frac{a_{n+1}}{a_n}|$ and $\liminf|\frac{a_{n+1}}{a_n}|$.

(b) Do the series $\sum a_n$ and $\sum(-1)^n a_n$ converge? Explain briefly.

23

Let $f_n(x) = \frac{1+2\cos^2 nx}{\sqrt{n}}$. Prove carefully that (f_n) converges uniformly to 0 on $\mathbb{R}$.

24

Prove that if $f_n \to f$ uniformly on a set S, and if $g_n \to g$ uniformly on S, then $f_n + g_n \to f + g$ uniformly on S.

25

Let $f_n(x) = \frac{x^n}{n}$. Show (f_n) is uniformly convergent on $[-1, 1]$ and specify the limit function.

26

Let $f_n(x) = \frac{n+\cos x}{2n+\sin^2 x}$ for all real numbers x.

(a) Show (f_n) converges uniformly on $\mathbb{R}$. *Hint*: First decide what the limit function is; then show (f_n) converges uniformly to it.

(b) Calculate $\lim_{n\to\infty} \int_2^7 f_n(x)\, dx$. *Hint*: Don't integrate f_n.

27

Show $\sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx$ converges uniformly on $\mathbb{R}$ to a continuous function.

28

Show $\sum_{n=1}^{\infty} \frac{x^n}{n^2 2^n}$ has radius of convergence 2 and the series converges uniformly to a continuous function on $[-2, 2]$.

29

(a) Show $\sum \frac{x^n}{1+x^n}$ converges for $x \in [0, 1)$.

(b) Show that the series converges uniformly on $[0, a]$ for each a, $0 < a < 1$.

30

Suppose $\sum_{k=1}^{\infty} g_k$ and $\sum_{k=1}^{\infty} h_k$ converge uniformly on a set S. Show $\sum_{k=1}^{\infty}(g_k + h_k)$ converges uniformly on S.

UNIT-IV

31

Let $f(x) = x$ for rational x and $f(x) = 0$ for irrational x.

(a) Calculate the upper and lower Darboux integrals for f on the interval $[0, b]$.

(b) Is f integrable on $[0, b]$?

32

Let f be a bounded function on $[a, b]$. Suppose there exist sequences (U_n) and (L_n) of upper and lower Darboux sums for f such that $\lim(U_n - L_n) = 0$. Show f is integrable and $\int_a^b f = \lim U_n = \lim L_n$.

33

A function f on $[a, b]$ is called a *step function* if there exists a partition $P = \{a = u_0 < u_1 < \cdots < c_m = b\}$ of $[a, b]$ such that f is constant on each interval (u_{j-1}, u_j), say $f(x) = c_j$ for x in (u_{j-1}, u_j).

(a) Show that a step function f is integrable and evaluate $\int_a^b f$.

(b) Evaluate the integral $\int_0^4 P(x)\, dx$ for the postage-stamp function

34

Show $|\int_{-2\pi}^{2\pi} x^2 \sin^8(e^x)\, dx| \leq \frac{16\pi^3}{3}$.

35

Let f be a bounded function on $[a, b]$, so that there exists $B > 0$ such that $|f(x)| \leq B$ for all $x \in [a, b]$.

(a) Show

$$U(f^2, P) - L(f^2, P) \leq 2B[U(f, P) - L(f, P)]$$

for all partitions P of $[a, b]$. *Hint*: $f(x)^2 - f(y)^2 = [f(x) + f(y)] \cdot [f(x) - f(y)]$.

(b) Show that if f is integrable on $[a, b]$, then f^2 also is integrable on $[a, b]$.

36

Calculate

(a) $\lim_{x \to 0} \frac{1}{x} \int_0^x e^{t^2} \, dt$ **(b)** $\lim_{h \to 0} \frac{1}{h} \int_3^{3+h} e^{t^2} \, dt$.

37

Show that if f is a continuous real-valued function on $[a, b]$ satisfying $\int_a^b f(x)g(x) \, dx = 0$ for every continuous function g on $[a, b]$, then $f(x) = 0$ for all x in $[a, b]$.

SEC-2C TRANSPORTATION AND GAME THEORY BS: 401

Credits: 2
Theory: 2 hours /week

Objective: Students learn Transportation problem, assignment problem Games with mixed strategies.
Outcome: Students come to know about nice applications of Operations Research.

Unit I

The Transportation and Assignment Problems : The Transportation Problem - A Streamlined Simplex Method for the Transportation Problem - The Assignment Problem

Unit II

Game Theory: The Formulation of Two-Person, Zero-Sum Games - Solving Simple Games—A Prototype Example - Games with Mixed Strategies - Graphical Solution Procedure - Solving by Linear Programming - Extensions

Text : Frederick S Hillier and Gerald J Lieberman, *An Elementary Introduction to Operations Research (9e)*

References : Hamdy A Taha , *Operations Research :An introduction*

Gupta and Kapur ; *Operations Research*

SEC-2D **NUMBER THEORY** BS: 401

NUMBER THEORY

Credits: 2
Theory: 2 hours /week

Objective: Students will be exposed to some of the jewels like Fermat's theorem, Euler's theorem in the number theory.
Outcome: Student uses the knowledge acquired solving some divisor problems.

Unit I

The Goldbach conjecture – Basic properties of congruences- Binary and Decimal Representation of lntegers – Number Theoretic Functions; The Sum and Number of divisors- The Mobius Inversion Formula- The Greatest integer function

Unit II

Euler's generalization of Fermat's Theorem: Euler's Phi function- Euler's theorem- Some Properties of the Euler's Phi function

Text: David M Burton, *Elementary Number Theory (7e)*

References: Thomas Koshy, *Elementary Number Theory and its Applications*

Kenneth H Rosen, *Elementary Number Theory*

DSC- 1D ALGEBRA BS: 404

Theory: 4 credits and Practicals: 1 credits
Theory: 4 hours /week and Practicals: 2 hours /week

Objective: The course is aimed at exposing the students to learn some basic algebraic structures like groups, rings etc.

Outcome: On successful completion of the course students will be able to recognize algebraic structures that arise in matrix algebra, linear algebra and will be able to apply the skills learnt in understanding various such subjects.

Unit – I

Groups: Definition and Examples of Groups- Elementary Properties of Groups -Finite Groups; Subgroups -Terminology and Notation -Subgroup Tests - Examples of Subgroups Cyclic Groups: Properties of Cyclic Groups – Classification of Sub groups Cyclic Groups-Permutation Groups: Definition and Notation -Cycle Notation -Properties of Permutations -A Check Digit Scheme Based on D_5

Unit – II

Isomorphisms ; Motivation- Definition and Examples -Cayley's Theorem Properties of Isomorphisms -Automorphisms-Cosets and Lagrange's Theorem Properties of Cosets 138 | Lagrange's Theorem and Consequences-An Application of Cosets to Permutation Groups -The Rotation Group of a Cube and a Soccer Ball -Normal Subgroups and Factor Groups ; Normal Subgroups-Factor Groups -Applications of Factor Groups -Group Homomorphisms - Definition and Examples -Properties of Homomorphisms -The First Isomorphism Theorem

Unit – III

Introduction to Rings: Motivation and Definition -Examples of Rings -Properties of Rings -Subrings -Integral Domains : Definition and Examples –Characteristics of a

Ring -Ideals and Factor Rings; Ideals -Factor Rings -Prime Ideals and Maximal Ideals

Unit – IV

Ring Homomorphisms: Definition and Examples-Properties of Ring-Homomorphisms -The Field of Quotients Polynomial Rings: Notation and Terminology

Text: Joseph A Gallian, *Contemporary Abstract algebra (9th edition)*

References: Bhattacharya, P.B Jain, S.K.; and Nagpaul, S.R, *Basic Abstract Algebra*

Fraleigh, J.B. *A First Course in Abstract Algebra*.

Herstein, I.N. *Topics in Algebra*

Robert B. Ash, *Basic Abstract Algebra*

I Martin Isaacs, *Finite Group Theory*

Joseph J Rotman, *Advanced Modern Algebra*

Practicals Question Bank
ALGEBRA
Unit-I

1. Show that $\{1, 2, 3\}$ under multiplication modulo 4 is not a group but that $\{1, 2, 3, 4\}$ under multiplication modulo 5 is a group.

2. Let G be a group with the property that for any x, y, z in the group, $xy = zx$ implies $y = z$. Prove that G is Abelian.

3. Prove that the set of all 3×3 matrices with real entries of the form

$$\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$$

   is a group under multiplication.

4. Let G be the group of polynomials under addition with coefficients from Z_{10}. Find the orders of $f(x) = 7x^2 + 5x + 4, g(x) = 4x^2 + 8x + 6$, and $f(x) + g(x)$

5. If a is an element of a group G and $|a| = 7$, show that a is the cube of some element of G.

6. Suppose that $\langle a \rangle, \langle b \rangle$ and $\langle c \rangle$ are cyclic groups of orders 6, 8, and 20, respectively. Find all generators of $\langle a \rangle, \langle b \rangle$, and $\langle c \rangle$.

7. How many subgroups does Z_{20} have? List a generator for each of these subgroups.

8. Consider the set $\{4, 8, 12, 16\}$. Show that this set is a group under multiplication modulo 20 by constructing its Cayley table. What is the identity element? Is the group cyclic? If so, find all of its generators.

9. Prove that a group of order 4 cannot have a subgroup of order 3.

10. Determine whether the following permutations are even or odd.
    a. (135)
    b. (1356)
    c. (13567)
    d. (12)(134)(152)
    e. (1243)(3521).

Unit-II

1. Show that the mapping $a \longrightarrow \log_{10} a$ is an isomorphism from R^+ under multiplication to R under addition.

2. Show that the mapping $f(a + bi) = a - bi$ is an automorphism of the group of complex numbers under addition.

3. Find all of the left cosets of $\{1, 11\}$ in $U(30)$.

4. Let C^* be the group of nonzero complex numbers under multiplication and let $H = \{a + bi \in C^*/a^2 + b^2 = 1\}$. Give a geometric description of the coset $(3+4i)H$. Give a geometric description of the coset $(c + di)H$.

5. Let $H = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} /a, b, d \in R, ad \neq 0 \right\}$. Is H a normal subgroup of $GL(2, R)$?

6. What is the order of the factor group $\frac{Z_{60}}{\langle 5 \rangle}$?

7. Let $G = U(16), H = \{1, 15\}$, and $K = \{1, 9\}$. Are H and K isomorphic? Are G/H and G/K isomorphic?

8. Prove that the mapping from R under addition to $GL(2, R)$ that takes x to

$$\begin{bmatrix} cosx & sinx \\ -sinx & cosx \end{bmatrix}$$

is a group homomorphism. What is the kernel of the homomorphism?

9. Suppose that f is a homomorphism from $Z30$ to $Z30$ and $Kerf = \{0, 10, 20\}$. If $f(23) = 9$, determine all elements that map to 9.

10. How many Abelian groups (up to isomorphism) are there
    a. of order 6?
    b. of order 15?
    c. of order 42?
    d. of order pq, where p and q are distinct primes?
    e. of order pqr, where p, q, and r are distinct primes?

Unit-III

1. Let $M_2(Z)$ be the ring of all 2×2 matrices over the integers and let $R = \left\{ \begin{pmatrix} a & a \\ b & b \end{pmatrix} /a, b \in Z \right\}$ Prove or disprove that R is a subring of $M_2(Z)$.

2. Suppose that a and b belong to a commutative ring R with unity. If a is a unit of R and $b^2 = 0$, show that $a + b$ is a unit of R.

3. Let n be an integer greater than 1. In a ring in which $x^n = x$ for all x, show that $ab = 0$ implies $ba = 0$.

4. List all zero-divisors in Z_{20}. Can you see a relationship between the zero-divisors of Z_{20} and the units of Z_{20}?

5. Let a belong to a ring R with unity and suppose that $a^n = 0$ for some positive integer n. (Such an element is called nilpotent.) Prove that $1 - a$ has a multiplicative inverse in R.

6. Let d be an integer. Prove that $Z[\sqrt{d}] = \{a + b\sqrt{d}/a, b \in Z\}$ is an integral domain.

7. Show that Z_n has a nonzero nilpotent element if and only if n is divisible by the square of some prime.

8. Find all units, zero-divisors, idempotents, and nilpotent elements in $Z_3 \bigoplus Z_6$.

9. Find all maximal ideals in
   a. Z_8.
   b. Z_{10}.
   c. Z_{12}.
   d. Z_n.

10. Show that $R[x]/\langle x^2+1\rangle$ is a field.

Unit-IV

1. Prove that every ring homomorphism f from Z_n to itself has the form $f(x) = ax$, where $a^2 = a$.

2. Prove that a ring homomorphism carries an idempotent to an idempotent.

3. In Z, let $A = \langle 2\rangle$ and $B = \langle 8\rangle$. Show that the group A/B is isomorphic to the group Z_4 but that the ring A/B is not ring-isomorphic to the ring Z_4.

4. Show that the number $9,897,654,527,609,805$ is divisible by 99.

5. Show that no integer of the form $111,111,111,...,111$ is prime.

6. Let $f(x) = 4x^3+2x^2+x+3$ and $g(x) = 3x^4+3x^3+3x^2+x+4$, where $f(x), g(x) \in Z_5[x]$. Compute $f(x)+g(x)$ and $f(x).g(x)$.

7. Let $f(x) = 5x^4+3x^3+1$ and $g(x) = 3x^2+2x+1$ in $Z_7[x]$.Determine the quotient and remainder upon dividing $f(x)$ by $g(x)$.

8. Let $f(x)$ belong to $Z_p[x]$. Prove that if $f(b) = 0$, then $f(b^p) = 0$.

9. Determine which of the polynomials below is (are) irreducible over Q.
   a. $x^5+9x^4+12x^2+6$
   b. x^4+x+1
   c. x^4+3x^2+3
   d. x^5+5x^2+1
   e. $(5/2)x^5+(9/2)x^4+15x^3+(3/7)x^2+6x+3/14$.

10. Show that x^2+x+4 is irreducible over Z_{11}.

SEC-3E PROBABITY AND STATISTICS BS: 501

Credits: 2
Theory: 2 hours /week

Objective: Students are exposed some basic ideas like random variables and its related concepts.
Outcome: Students will be able to their knowledge to solve some real world problems.

Unit I

Random Variables; Continuous Random Variables - Expectation of a Random Variable - Jointly Distributed Random Variables - Moment Generating Functions

Unit II

Conditional Probability and Conditional Expectation Introduction ; The Discrete Case -The Continuous Case - Computing Expectations by Conditioning - Computing Variances by Conditioning - Computing Probabilities by Conditioning

Text: Sheldon M Ross, *Introduction to Probability Models (9e)*

References: Miller and Miller, *Mathematical Statistics with Applications*

Hogg, McKeanand Craig, *Introduction to Mathematical Statistics*

Gupta and Kapur, *Mathematical Statistics*

SEC -3F MATHEMATICAL MODELING BS: 501

Credits: 2

Theory: 2 hours /week

Objective: Some of the Physics problems will be solved using Differential Equations.

Outcome: Student realizes some beautiful problems can be modeled by using differential equations.

Unit I

Linear Models-Nonlinear Models-Modeling with Systems of First-Order DEs-

Unit II

Linear Models: Initial-Value Problems-Spring/Mass Systems: Free Undamped Motion-Spring/Mass Systems: Free Damped Motion-Spring/Mass Systems: Driven Motion-Series Circuit Analogue-Linear Models: Boundary-Value Problems

Text: Dennis G Zill, *A first course in differential equations with modeling applications*

References: Shepley L. Ross, *Differential Equations*

I. Sneddon, *Elements of Partial Differential Equations*

GE-1 LATTICE THEORY BS: 502

Credits: 2
Theory: 2 hours /week

Objective: Students will be exposed to elements of theory of lattices.
Outcome : Students apply their knowledge to solve some problems on switching circuits.

Unit I

Lattices: Properties and Examples of Lattices - Distributive Lattices – Boolean Algebras - Boolean Polynomials - Ideals, Filters, and Equations - Minimal Forms of Boolean Polynomials

Unit II

Applications of Lattices – Switching Circuits - Applications of Switching Circuits . - More Applications of Boolean Algebras

Text : Rudolf Lidl and Gunter Pilz, *Applied Abstract Algebra (2e)*

References: Davey and Priestly, *Introduction to Lattices and Order*

DSC-1E LINEAR ALGEBRA BS: 503

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours /week and Practicals: 2 hours /week

Objective: The students are exposed to various concepts like vector spaces , bases , dimension, Eigen values etc .

Outcome: After completion this course students appreciate its interdisciplinary nature.

Unit I

Vector Spaces : Vector Spaces and Subspaces -Null Spaces, Column Spaces, and Linear Transformations -Linearly Independent Sets; Bases -Coordinate Systems -The Dimension of a Vector Space

Unit II

Rank-Change of Basis - Eigenvalues and Eigenvectors - The Characteristic Equation

Unit III

Diagonalization -Eigenvectors and Linear Transformations -Complex Eigenvalues - Applications to Differential Equations -Orthogonality and Least Squares : Inner Product, Length, and Orthogonality -Orthogonal Sets

Text : David C Lay , *Linear Algebra and its Applications 4e*

References: S Lang, *Introduction to Linear Algebra*

Gilbert Strang, Linear *Algebra and its Applications*

Stephen H Friedberg et al*, Linear Algebra*

Kuldeep Singh, *Linear Algebra*

Sheldon Axler, *Linear Algebra Done Right*

Linear Algebra

Practicals Question Bank

UNIT-I

1

Let H be the set of all vectors of the form $\begin{bmatrix} -2t \\ 5t \\ 3t \end{bmatrix}$. Find a vector $\mathbf{v}$ in $\mathbb{R}^3$ such that $H = \text{Span}\{\mathbf{v}\}$. Why does this show that H is a subspace of $\mathbb{R}^3$?

2

Let V be the first quadrant in the xy-plane; that is, let

$$V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x \geq 0, y \geq 0 \right\}$$

a. If $\mathbf{u}$ and $\mathbf{v}$ are in V, is $\mathbf{u} + \mathbf{v}$ in V? Why?

b. Find a specific vector $\mathbf{u}$ in V and a specific scalar c such

3

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 7 \\ -9 \end{bmatrix}$. Determine if $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis for $\mathbb{R}^3$. Is $\{\mathbf{v}_1, \mathbf{v}_2\}$ a basis for $\mathbb{R}^2$?

4

The set $\mathcal{B} = \{1 + t^2, t + t^2, 1 + 2t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = 1 + 4t + 7t^2$ relative to $\mathcal{B}$.

5

The set $\mathcal{B} = \{1 - t^2, t - t^2, 2 - t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = 1 + 3t - 6t^2$ relative to $\mathcal{B}$.

6

The vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ -8 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$ span $\mathbb{R}^2$ but do not form a basis. Find two different ways to express $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

7

Find the dimension of the subspace of all vectors in $\mathbb{R}^3$ whose first and third entries are equal.

8

Find the dimension of the subspace H of $\mathbb{R}^2$ spanned by $\begin{bmatrix} 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ 10 \end{bmatrix}, \begin{bmatrix} -3 \\ 15 \end{bmatrix}$.

9

Let H be an n-dimensional subspace of an n-dimensional vector space V. Show that $H = V$.

10

Let H be an n-dimensional subspace of an n-dimensional vector space V. Show that $H = V$.

UNIT-II

11

If a 4×7 matrix A has rank 3, find $\dim \operatorname{Nul} A$, $\dim \operatorname{Row} A$, and rank A^T.

12

If a 7×5 matrix A has rank 2, find $\dim \operatorname{Nul} A$, $\dim \operatorname{Row} A$, and rank A^T.

13

If the null space of an 8×5 matrix A is 3-dimensional, what is the dimension of the row space of A?

14

If A is a 3×7 matrix, what is the smallest possible dimension of $\operatorname{Nul} A$?

15

Let $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Find $\mathbf{v}$ in $\mathbb{R}^3$ such that $\begin{bmatrix} 1 & -3 & 4 \\ 2 & -6 & 8 \end{bmatrix} = \mathbf{u}\mathbf{v}^T$.

16

If A is a 7×5 matrix, what is the largest possible rank of A? If A is a 5×7 matrix, what is the largest possible rank of A? Explain your answers.

17

Without calculations, list rank A and $\dim \operatorname{Nul} A$

$$A = \begin{bmatrix} 2 & 6 & -6 & 6 & 3 & 6 \\ -2 & -3 & 6 & -3 & 0 & -6 \\ 4 & 9 & -12 & 9 & 3 & 12 \\ -2 & 3 & 6 & 3 & 3 & -6 \end{bmatrix},$$

18

Use a property of determinants to show that A and A^T have the same characteristic polynomial.

19

Find the characteristic equation of

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

20

Find the characteristic polynomial and the real eigenvalues of

$$\begin{bmatrix} 4 & 0 & -1 \\ 0 & 4 & -1 \\ 1 & 0 & 2 \end{bmatrix}, \quad \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

UNIT-III

21

let $A = PDP^{-1}$ and compute A^4

$\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$, $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

22

Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ and $\mathcal{D} = \{\mathbf{d}_1, \mathbf{d}_2\}$ be bases for vector spaces V and W, respectively. Let $T : V \to W$ be a linear transformation with the property that

$$T(\mathbf{b}_1) = 3\mathbf{d}_1 - 5\mathbf{d}_2, \quad T(\mathbf{b}_2) = -\mathbf{d}_1 + 6\mathbf{d}_2, \quad T(\mathbf{b}_3) = 4\mathbf{d}_2$$

Find the matrix for T relative to $\mathcal{B}$ and $\mathcal{D}$.

23

Let $\mathcal{D} = \{\mathbf{d}_1, \mathbf{d}_2\}$ and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ be bases for vector spaces V and W, respectively. Let $T : V \to W$ be a linear transformation with the property that

$$T(\mathbf{d}_1) = 3\mathbf{b}_1 - 3\mathbf{b}_2, \quad T(\mathbf{d}_2) = -2\mathbf{b}_1 + 5\mathbf{b}_2$$

Find the matrix for T relative to $\mathcal{D}$ and $\mathcal{B}$.

24

Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be a basis for a vector space V and let $T : V \to \mathbb{R}^2$ be a linear transformation with the property that

$$T(x_1\mathbf{b}_1 + x_2\mathbf{b}_2 + x_3\mathbf{b}_3) = \begin{bmatrix} 2x_1 - 3x_2 + x_3 \\ -2x_1 + 5x_3 \end{bmatrix}$$

Find the matrix for T relative to $\mathcal{B}$ and the standard basis for $\mathbb{R}^2$.

25

Let $T : \mathbb{P}_2 \to \mathbb{P}_3$ be the transformation that maps a polynomial $\mathbf{p}(t)$ into the polynomial $(t + 3)\mathbf{p}(t)$.

a. Find the image of $\mathbf{p}(t) = 3 - 2t + t^2$.

b. Show that T is a linear transformation.

c. Find the matrix for T relative to the bases $\{1, t, t^2\}$ and $\{1, t, t^2, t^3\}$.

26

Assume the mapping $T : \mathbb{P}_2 \to \mathbb{P}_2$ defined by

$$T(a_0 + a_1 t + a_2 t^2) = 3a_0 + (5a_0 - 2a_1)t + (4a_1 + a_2)t^2$$

is linear. Find the matrix representation of T relative to the basis $\mathcal{B} = \{1, t, t^2\}$.

27

Define $T : \mathbb{P}_3 \to \mathbb{R}^4$ by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(-2) \\ \mathbf{p}(3) \\ \mathbf{p}(1) \\ \mathbf{p}(0) \end{bmatrix}$.

a. Show that T is a linear transformation.

b. Find the matrix for T relative to the basis $\{1, t, t^2, t^3\}$ for $\mathbb{P}_3$ and the standard basis for $\mathbb{R}^4$.

28

Let A be a 2×2 matrix with eigenvalues -3 and -1 and corresponding eigenvectors $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Let $\mathbf{x}(t)$ be the position of a particle at time t. Solve the initial value problem $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

29

construct the general solution of $\mathbf{x}' = A\mathbf{x}$

$A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$, $\begin{bmatrix} -7 & 10 \\ -4 & 5 \end{bmatrix}$

30

Compute the orthogonal projection of $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ onto the line through $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ and the origin.

DSE-1E/A ANALYTICAL SOLID GEOMETRY BS: 506

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours /week and Practicals: 2 hours /week

Objective: Students learn to describe some of the surfaces by using analytical geometry.

Outcome: Students understand the beautiful interplay between algebra and geometry.

Unit I

Sphere: Definition-The Sphere Through Four Given Points-Equations of a Circle-Intersection of a Sphere and a Line-Equation of a Tangent Plane-Angle of Intersection of Two Spheres-Radical Plane

Unit II

Cones and Cylinders: Definition-Condition that the General Equation of second degree Represents a Cone-Cone and a Plane through its Vertex –Intersection of a Line with a Cone- The Right Circular Cone-The Cylinder- The Right Circular Cylinder

Unit III

The Conicoid: The General Equation of the Second Degree-Intersection of Line with a Conicoid-Plane of contact-Enveloping Cone and Cylinder

Text : Shanti Narayan and P K Mittal , *Analytical Solid Geometry (17e)*

References: Khaleel Ahmed , *Analytical Solid Geometry*

S L Loney, *Solid Geometry*

Smith and Minton, *Calculus*

Solid Geometry
Practicals Question Bank

UNIT-I

1

Find the equation of the sphere through the four points
$$(4, -1, 2), (0, -2, 3), (1, -5, -1), (2, 0, 1).$$

2

Find the equation of the sphere through the four points
$$(0, 0, 0), (-a, b, c), (a, -b, c), (a, b, -c)$$

3

Find the centre and the radius of the circle
$$x+2y+2z=15,\ x^2+y^2+z^2-2y-4z=11.$$

4

Show that the following points are concyclic :—

(*i*) (5, 0, 2), (2, −6, 0), (7, −3, 8), (4, −9, 6).

(*ii*) (−8, 5, 2), (−5, 2, 2), (−7, 6, 6), (−4, 3, 6).

5

Find the centres of the two spheres which touch the plane
$$4x+3y=47$$
at the point (8, 5, 4) and which touch the sphere
$$x^2+y^2+z^2=1.$$

6

Show that the spheres
$$x^2+y^2+z^2=25$$
$$x^2+y^2+z^2-24x-40y-18z+225=0$$
touch externally and find the point of the contact.

7

Find the equation of the sphere that passes through the two points
$$(0, 3, 0), (-2, -1, -4)$$
and cuts orthogonally the two spheres
$$x^2+y^2+z^2+x-3z-2=0,\ 2(x^2+y^2+z^2)+x+3y+4=0.$$

8

Find the limiting points of the co-axal system of spheres
$$x^2+y^2+z^2-20x+30y-40z+29+\lambda(2x-3y+4z)=0.$$

9

Find the equations to the two spheres of the co-axal system
$$x^2+y^2+z^2-5+\lambda(2x+y+3z-3)=0,$$
which touch the plane
$$3x+4y=15.$$

10

Show that the radical planes of the sphere of a co-axal system and of any given sphere pass through a line.

UNIT-II

11

Find the equation of the cone whose vertex is the point (1, 1, 0) and whose guiding curve is
$$y=0,\ x^2+z^2=4.$$

12

The section of a cone whose vertex is P and guiding curve the ellipse $x^2/a^2+y^2/b^2=1,\ z=0$ by the plane $x=0$ is a rectangular hyperbola. Show that the locus of P is
$$\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1.$$

13

Find the enveloping cone of the sphere
$$x^2+y^2+z^2-2x+4z=1$$
with its vertex at (1, 1, 1).

14

Find the equation of the quadric cone whose vertex is at the origin and which passes through the curve given by the equations

$$ax^2+by^2+cz^2=1,\ lx+my+nz=p.$$

15

Find the equation of the cone with vertex at the origin and direction cosines of its generators satisfying the relation

$$3l^2-4m^2+5n^2=0.$$

16

Find the equation of the cylinder whose generators are parallel to

$$x=-\tfrac{1}{2}y=\tfrac{1}{3}z$$

and whose guiding curve is the ellipse

$$x^2+2y^2=1,\ z=3.$$

17

Find the equation of the right circular cylinder of radius 2 whose axis is the line

$$(x-1)/2=(y-2)=(z-3)/2.$$

18

The axis of a right circular cylinder of radius 2 is

$$\frac{x-1}{2}=\frac{y}{3}=\frac{z-3}{1};$$

show that its equation is

$$10x^2+5y^2+13z^2-12xy-6yz-4zx-8x+30y-74z+59=0.$$

19

Find the equation of the circular cylinder whose guiding circle is

$$x^2+y^2+z^2-9=0,\ x-y+z=3.$$

20

Obtain the equation of the right circular cylinder described on the circle through the three points (1, 0, 0), (0, 1, 0), (0, 0, 1) as guiding circle.

UNIT-III

21

Find the points of intersection of the line

$$-\tfrac{1}{3}(x+5)=(y-4)=\tfrac{1}{7}(z-11)$$

with the conicoid

$$12x^2-17y^2+7z^2=7.$$

22

Find the equations to the tangent planes to

$$7x^2-3y^2-z^2+21=0,$$

which pass through the line,

$$7x-6y+9=3,\ z=3.$$

23

Obtain the tangent planes to the ellipsoid

$$x^2/a^2+y^2/b^2+z^2/c^2=1,$$

which are parallel to the plane

$$lx+my+nz=0.$$

24

Show that the plane $3x+12y-6z-17=0$ touches the conicoid $3x^2-6y^2+9z^2+17=0$, and find the point of contact.

25

Find the equations to the tangent planes to the surface

$$4x^2-5y^2+7z^2+13=0,$$

26

Find the equations to the tangent planes to the surface

$$4x^2-5y^2+7z^2+13=0,$$

parallel to the plane

$$4x+20y-21z=0.$$

Find their points of contact also.

27

Find the locus of the perpendiculars from the origin to the tangent planes to the surface

$$x^2/a^2+y^2/b^2+z^2/c^2=1$$

which cut off from its axes intercepts the sum of whose reciprocals is equal to a constant $1/k$.

28

If the section of the enveloping cone of the ellipsoid

$$x^2/a^2+y^2/b^2+z^2/c^2=1,$$

whose vertex is P by the plane $z=0$ is a rectangular hyperbola, show that the locus of P is

$$\frac{x^2+y^2}{a^2+b^2}+\frac{z^2}{c^2}=1.$$

29

Find the locus of points from which three mutually perpendicular tangent lines can be drawn to the conicoid $ax^2+by^2+cz^2=1$.

30

$P(1, 3, 2)$ is a point on the conicoid,

$$x^2-2y^2+3z^2+5=0.$$

Find the locus of the mid-points of chords drawn parallel to OP.

DSE-1E/B INTEGRAL CALCULUS BS: 506

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours /week and Practicals: 2 hours /week

Objective: Techniques of multiple integrals will be taught.

Outcome: Students will come to know about its applications in finding areas and volumes of some solids.

Unit I

Areas and Volumes: Double Integrals-Double Integrals over a Rectangle-Double Integrals over General Regions in the Plane-Changing the order of Integration

Unit II

Triple Integrals: The Integrals over a Box- Elementary Regions in Space-Triple Integrals in General

Unit III

Change of Variables: Coordinate Transformations-Change of Variables in Triple Integrals

Text: Susan Jane Colley, *Vector Calculus(4e)*

References: Smith and Minton , *Calculus*

Shanti Narayan and Mittal*, Integral calculus*

Ulrich L. Rohde , G. C. Jain , Ajay K. Poddar and A. K. Ghosh, *Introduction to Integral Calculus*

Integral Calculus
Practicals Question Bank

Unit-I

1. Let $R = [-3, 3] \times [-2, 2]$. Without explicitly evaluating any iterated integrals, determine the value of $\iint_R (x^5 + 2y)\, dA$.

2. Integrate the function $f(x, y) = 3xy$ over the region bounded by $y = 32x^3$ and $y = \sqrt{x}$.

3. Integrate the function $f(x, y) = x + y$ over the region bounded by $x + y = 2$ and $y^2 - 2y - x = 0$.

4. Evaluate $\iint_D xy\, dA$, where D is the region bounded by $x = y^3$ and $y = x^2$.

5. Evaluate $\iint_D e^{x^2}\, dA$, where D is the triangular region with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$.

6. Evaluate $\iint_D 3y\, dA$, where D is the region bounded by $xy^2 = 1$, $y = x$, $x = 0$, and $y = 3$.

7. Evaluate $\iint_D (x - 2y)\, dA$, where D is the region bounded by $y = x^2 + 2$ and $y = 2x^2 - 2$.

8. Evaluate $\iint_D (x^2 + y^2)\, dA$, where D is the region in the first quadrant bounded by $y = x$, $y = 3x$, and $xy = 3$.

9. Consider the integral

$$\int_0^2 \int_{x^2}^{2x} (2x + 1)\, dy\, dx.$$

a) Evaluate this integral.

b) Sketch the region of integration.

c) Write an equivalent iterated integral with the order of integration reversed. Evaluate this new integral and check that your answer agrees with part (a).

10. Find the volume of the region under the graph of

$$f(x, y) = 2 - |x| - |y|$$

and above the xy-plane.

Unit-II

Integrate the following over the indicated region W.

11. $f(x, y, z) = 2x - y + z$; W is the region bounded by the cylinder $z = y^2$, the xy-plane, and the planes $x = 0, x = 1, y = -2, y = 2$.

12. $f(x, y, z) = y$; W is the region bounded by the plane $x + y + z = 2$, the cylinder $x^2 + z^2 = 1$, and $y = 0$.

13. $f(x, y, z) = 8xyz$; W is the region bounded by the cylinder $y = x^2$, the plane $y + z = 9$, and the xy-plane.

14. $f(x, y, z) = z$; W is the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the planes $y = x, x = 0$, and $z = 0$.

15. $f(x, y, z) = 1 - z^2$; W is the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$.

16. $f(x, y, z) = 3x$; W is the region in the first octant bounded by $z = x^2 + y^2$, $x = 0$, $y = 0$, and $z = 4$.

17. $f(x, y, z) = x + y$; W is the region bounded by the cylinder $x^2 + 3z^2 = 9$ and the planes $y = 0$, $x + y = 3$.

18. $f(x, y, z) = z$; W is the region bounded by $z = 0$, $x^2 + 4y^2 = 4$, and $z = x + 2$.

Unit-III

19. $f(x, y, z) = 4x + y$; W is the region bounded by $x = y^2$, $y = z$, $x = y$, and $z = 0$.

20. $f(x, y, z) = x$; W is the region in the first octant bounded by $z = x^2 + 2y^2$, $z = 6 - x^2 - y^2$, $x = 0$, and $y = 0$.

Let $\mathbf{T}(u, v) = (3u, -v)$.

21. (a) Write $\mathbf{T}(u, v)$ as $A\begin{bmatrix} u \\ v \end{bmatrix}$ for a suitable matrix A.

22. (b) Describe the image $D = \mathbf{T}(D^*)$, where D^* is the unit square $[0, 1] \times [0, 1]$.

23. Determine the value of

$$\iint_D \sqrt{\frac{x + y}{x - 2y}}\, dA,$$

where D is the region in $\mathbf{R}^2$ enclosed by the lines

24. Evaluate

$$\iint_D \frac{(2x+y-3)^2}{(2y-x+6)^2}\, dx\, dy,$$

where D is the square with vertices $(0,0)$, $(2,1)$, $(3,-1)$, and $(1,-2)$. (Hint: First sketch D and find the equations of its sides.)

25. Evaluate

$$\iint_D \cos(x^2+y^2)\, dA,$$

where D is the shaded region in Figure 5.106.

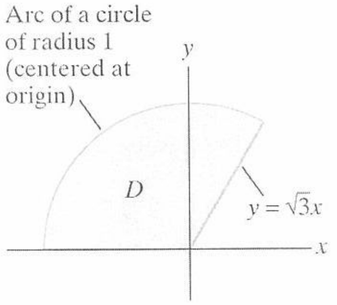

Figure 5.106 The region D of Exercise 25.

26. Evaluate

$$\iint_D \frac{1}{\sqrt{4-x^2-y^2}}\, dA,$$

where D is the disk of radius 1 with center at $(0,1)$. (Be careful when you describe D.)

27. Determine the value of $\iiint_W \frac{z}{\sqrt{x^2+y^2}}\, dV$, where W is the solid region bounded by the plane $z = 12$ and the paraboloid $z = 2x^2 + 2y^2 - 6$.

SEC-4G BOOLEAN ALGEBRA BS: 601

Credits: 2
Theory: 2 hours /week

Objective: Students will be exposed to Elements of theory of lattices.

Outcome : Students apply their Knowledge in solving some problems on switching circuits.

Unit I

Lattices: Properties and Examples of Lattices - Distributive Lattices – Boolean Algebras - Boolean Polynomials - Ideals, Filters, and Equations - Minimal Forms of Boolean Polynomials

Unit II

Applications of Lattices – Switching Circuits - Applications of Switching Circuits . - More Applications of Boolean Algebras

Text : Rudolf Lidl and Gunter Pilz, *Applied Abstract Algebra (2e)*

References: Davey and Priestly, *Introduction to Lattices and Order*

SEC-4H **GRAPH THEORY** BS: 601

Credits: 2
Theory: 2 hours /week

Objective: The students will be exposed To some basic ideas of group theory.

Outcome: Students will be able to appreciate the subject learnt.

Unit I

Graphs: A Gentle Introduction - Definitions and Basic Properties - Isomorphism

Unit II

Paths and Circuits: Eulerian Circuits - Hamiltonian Cycles -The Adjacency Matrix Shortest Path Algorithms

Text : Edgar Goodaire and Michael M. Parmenter, *Discrete Mathematics with Graph Theory (2e)*

References: Rudolf Lidl and Gunter Pilz, *Applied Abstract Algebra*

S Pirzada*, Introduction to Graph Theory*

GE-2 ELEMENTS OF NUMBER THEORY BS:602

Credits: 2
Theory: 2 hours /week

Objective: Students will be exposed to some elements of number theory.

Outcome : Students apply their knowledge problems on check digits, modular designs.

Unit I

The Division Algorithm- Number Patterns- Prime and Composite Numbers- Fibonacci and Lucas' numbers- Fermat Numbers- GCD-The Euclidean Algorithm- The Fundamental Theorem of Arithmetic- LCM- Linear Diophantine Equations Congruences- Linear Congruences

Unit II

The Pollard Rho Factoring Method- Divisibility Tests- Modular Designs- Check Digits- The Chinese Remainder Theorem- General Linear Systems- 2X2 Systems Wilson's Theorem- Fermat's Little Theorem- Pseudo primes- Euler's Theorem

Text : Thomas Koshy, *Elementary Number Theory with Applications*

References: David M Burton, *Elementary Number Theory*

DSC-1F NUMERICAL ANALYSIS BS: 603

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours /week and Practicals: 2 hours /week

Objective: Students will be made to understand some methods of numerical analysis.

Outcome: Students realize the importance of the subject in solving some problems of algebra and calculus.

Unit – I

Solutions of Equations in One Variable : The Bisection Method - Fixed-Point Iteration - Newton's Method and Its Extensions - Error Analysis for Iterative Methods - Accelerating Convergence - Zeros of Polynomials and Müller's Method - Survey of Methods and Software

Unit – II

Interpolation and Polynomial Approximation: Interpolation and the Lagrange Polynomial - Data Approximation and Neville's Method - Divided Differences - Hermite Interpolation - Cubic Spline Interpolation

Unit – III

Numerical Differentiation and Integration: Numerical Differentiation - Richardson's Extrapolation - Elements of Numerical Integration- Composite Numerical Integration - Romberg Integration - Adaptive Quadrature Methods - Gaussian Quadrature

Text : Richard L. Burden and J. Douglas Faires, *Numerical Analysis (9e)*

References: M K Jain, S R K Iyengar and R k Jain, *Numerical Methods for Scientific and Engineering computation*

B.Bradie*, A Friendly introduction to Numerical Analysis*

Numerical Analysis

Practicals Question Bank

UNIT-I

1

Use the Bisection method to find p_3 for $f(x) = \sqrt{x} - \cos x$ on $[0, 1]$.

2

Let $f(x) = 3(x+1)(x-\frac{1}{2})(x-1)$. Use the Bisection method on the following intervals to find p_3.

a. $[-2, 1.5]$ **b.** $[-1.25, 2.5]$

3

Use the Bisection method to find solutions accurate to within 10^{-5} for the following problems.

a. $x - 2^{-x} = 0$ for $0 \le x \le 1$

b. $e^x - x^2 + 3x - 2 = 0$ for $0 \le x \le 1$

c. $2x\cos(2x) - (x+1)^2 = 0$ for $-3 \le x \le -2$ and $-1 \le x \le 0$

4

1. Use algebraic manipulation to show that each of the following functions has a fixed point at p precisely when $f(p) = 0$, where $f(x) = x^4 + 2x^2 - x - 3$.

a. $g_1(x) = \left(3 + x - 2x^2\right)^{1/4}$ **b.** $g_2(x) = \left(\dfrac{x + 3 - x^4}{2}\right)^{1/2}$

5

Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on $[1, 2]$. Use $p_0 = 1$.

6

Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^3 - x - 1 = 0$ on $[1, 2]$. Use $p_0 = 1$.

7

Use a fixed-point iteration method to find an approximation to $\sqrt{3}$ that is accurate to within 10^{-4}.

8

The equation $x^2 - 10\cos x = 0$ has two solutions, ± 1.3793646. Use Newton's method to approximate the solutions to within 10^{-5} with the following values of p_0.

a. $p_0 = -100$ **b.** $p_0 = -50$ **c.** $p_0 = -25$

d. $p_0 = 25$ **e.** $p_0 = 50$ **f.** $p_0 = 100$

9

The equation $4x^2 - e^x - e^{-x} = 0$ has two positive solutions x_1 and x_2. Use Newton's method to approximate the solution to within 10^{-5} with the following values of p_0.

10

Use each of the following methods to find a solution in $[0.1, 1]$ accurate to within 10^{-4} for

$$600x^4 - 550x^3 + 200x^2 - 20x - 1 = 0.$$

a. Bisection method **c.** Secant method **e.** Müller's method

b. Newton's method **d.** method of False Position

UNIT-II

11

For the given functions $f(x)$, let $x_0 = 0$, $x_1 = 0.6$, and $x_2 = 0.9$. Construct interpolation polynomials of degree at most one and at most two to approximate $f(0.45)$, and find the absolute error.

a. $f(x) = \cos x$ **c.** $f(x) = \ln(x + 1)$

12

For the given functions $f(x)$, let $x_0 = 1$, $x_1 = 1.25$, and $x_2 = 1.6$. Construct interpolation polynomials of degree at most one and at most two to approximate $f(1.4)$, and find the absolute error.

a. $f(x) = \sin \pi x$ **c.** $f(x) = \log_{10}(3x - 1)$

13

Let $P_3(x)$ be the interpolating polynomial for the data $(0, 0)$, $(0.5, y)$, $(1, 3)$, and $(2, 2)$. The coefficient of x^3 in $P_3(x)$ is 6. Find y.

14

Neville's method is used to approximate $f(0.4)$, giving the following table.

$x_0 = 0$	$P_0 = 1$			
$x_1 = 0.25$	$P_1 = 2$	$P_{01} = 2.6$		
$x_2 = 0.5$	P_2	$P_{1,2}$	$P_{0,1,2}$	
$x_3 = 0.75$	$P_3 = 8$	$P_{2,3} = 2.4$	$P_{1,2,3} = 2.96$	$P_{0,1,2,3} = 3.016$

Determine $P_2 = f(0.5)$.

15

Neville's method is used to approximate $f(0.5)$, giving the following table.

$x_0 = 0$	$P_0 = 0$		
$x_1 = 0.4$	$P_1 = 2.8$	$P_{0,1} = 3.5$	
$x_2 = 0.7$	P_2	$P_{1,2}$	$P_{0,1,2} = \frac{27}{7}$

Determine $P_2 = f(0.7)$.

16

Neville's Algorithm is used to approximate $f(0)$ using $f(-2)$, $f(-1)$, $f(1)$, and $f(2)$. Suppose $f(-1)$ was overstated by 2 and $f(1)$ was understated by 3. Determine the error in the original calculation of the value of the interpolating polynomial to approximate $f(0)$.

17

Use the Newton forward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

a. $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$

b. $f(0.18)$ if $f(0.1) = -0.29004986$, $f(0.2) = -0.56079734$, $f(0.3) = -0.81401972$, $f(0.4) = -1.0526302$

18

Use the Newton backward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

a. $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$

b. $f(0.25)$ if $f(-1) = 0.86199480$, $f(-0.5) = 0.95802009$, $f(0) = 1.0986123$, $f(0.5) = 1.2943767$

19

Determine the natural cubic spline S that interpolates the data $f(0) = 0$, $f(1) = 1$, and $f(2) = 2$.

20

Determine the clamped cubic spline s that interpolates the data $f(0) = 0$, $f(1) = 1$, $f(2) = 2$ and satisfies $s'(0) = s'(2) = 1$.

UNIT-III

21

Use the forward-difference formulas and backward-difference formulas to determine each missing entry in the following tables.

a.

x	$f(x)$	$f'(x)$
0.5	0.4794	
0.6	0.5646	
0.7	0.6442	

b.

x	$f(x)$	$f'(x)$
0.0	0.00000	
0.2	0.74140	
0.4	1.3718	

22

Derive a method for approximating $f'''(x_0)$ whose error term is of order h^2 by expanding the function f in a fourth Taylor polynomial about x_0 and evaluating at $x_0 \pm h$ and $x_0 \pm 2h$.

23

The forward-difference formula can be expressed as

$$f'(x_0) = \frac{1}{h}[f(x_0 + h) - f(x_0)] - \frac{h}{2}f''(x_0) - \frac{h^2}{6}f'''(x_0) + O(h^3).$$

Use extrapolation to derive an $O(h^3)$ formula for $f'(x_0)$.

24

Show that

$$\lim_{h\to 0}\left(\frac{2+h}{2-h}\right)^{1/h} = e.$$

25

Approximate the following integrals using the Trapezoidal rule.

a. $\int_{0.5}^{1} x^4\, dx$ **b.** $\int_{0}^{0.5} \frac{2}{x-4}\, dx$

c. $\int_{1}^{1.5} x^2 \ln x\, dx$ **d.** $\int_{0}^{1} x^2 e^{-x}\, dx$

26

The Trapezoidal rule applied to $\int_0^2 f(x)\, dx$ gives the value 5, and the Midpoint rule gives the value 4. What value does Simpson's rule give?

27

The quadrature formula $\int_0^2 f(x)\, dx = c_0 f(0) + c_1 f(1) + c_2 f(2)$ is exact for all polynomials of degree less than or equal to 2. Determine c_0, c_1, and c_2.

28

Romberg integration is used to approximate

$$\int_2^3 f(x)\, dx.$$

If $f(2) = 0.51342$, $f(3) = 0.36788$, $R_{31} = 0.43687$, and $R_{33} = 0.43662$, find $f(2.5)$.

29

Use Romberg integration to compute $R_{3,3}$ for the following integrals.

a. $\int_{1}^{1.5} x^2 \ln x\, dx$ **b.** $\int_{0}^{1} x^2 e^{-x}\, dx$

30

Use Romberg integration to compute $R_{3,3}$ for the following integrals.

a. $\int_{-1}^{1} (\cos x)^2\, dx$ **b.** $\int_{-0.75}^{0.75} x \ln(x+1)\, dx$

DSE-1F/A COMPLEX ANALYSIS BS: 606

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours /week and Practicals: 2 hours /week

Objective: Analytic Functions, contour integration and calculus of residues will be introduced to the students.
Outcome: Students realize calculus of residues is one of the power tools in solvng some problems, like improper and definite integrals, effortlessly.

Unit – I

Regions in the Complex Plane - Analytic Functions - Functions of a Complex Variable - Mappings - Mappings by the Exponential Function - Limits - Theorems on Limits - Limits Involving the Point at Infinity - Continuity - Derivatives - Differentiation Formulas - Cauchy–Riemann Equations - Sufficient Conditions for Differentiability - Polar Coordinates-Harmonic Functions

Elementary Functions: The Exponential Function - The Logarithmic Function - Branches and Derivatives of Logarithms - Some Identities Involving Logarithms Complex Exponents - Trigonometric Functions - Hyperbolic Functions

Unit – II

Integrals: Derivatives of Functions w(t) - Definite Integrals of Functions w(t) - Contours - Contour Integrals - Some Examples - Examples with Branch Cuts - Upper Bounds for Moduli of Contour Integrals -Antiderivatives

Unit – III

Cauchy–Goursat Theorem - Proof of the Theorem - Simply Connected Domains - Multiply Connected Domains - Cauchy Integral Formula - An Extension of the Cauchy Integral Formula - Some Consequences of the Extension - Liouville's Theorem and the Fundamental Theorem of Algebra- Maximum Modulus Principle

Text: James Ward Brown and Ruel V. Churchill, *Complex Variables and Applications (8e)*

References: Joseph Bak and Donald J Newman, Complex *analysis*

Lars V Ahlfors, *Complex Analysis*

S.Lang, Complex *Analysis*

B Choudary, *The Elements Complex Analysis*

Complex Analysis
Practicals Question Bank

UNIT-I

1

Sketch the following sets and determine which are domains:

(a) $|z - 2 + i| \le 1$; *(b)* $|2z + 3| > 4$;
(c) $\operatorname{Im} z > 1$; *(d)* $\operatorname{Im} z = 1$;

2

Sketch the region onto which the sector $r \le 1, 0 \le \theta \le \pi/4$ is mapped by the transformation *(a)* $w = z^2$; *(b)* $w = z^3$; *(c)* $w = z^4$.

3

find all roots of the equation

(a) $\sinh z = i$; *(b)* $\cosh z = \frac{1}{2}$.

4

Find all values of z such that

(a) $e^z = -2$; *(b)* $e^z = 1 + \sqrt{3}i$; *(c)* $\exp(2z - 1) = 1$.

,

5

Show that

$$\lim_{z \to z_0} f(z)g(z) = 0 \quad \text{if} \quad \lim_{z \to z_0} f(z) = 0$$

and if there exists a positive number M such that $|g(z)| \le M$ for all z in some neighborhood of z_0.

6

show that $f'(z)$ does not exist at any point if *(a)* $f(z) = \overline{z}$; *(b)* $f(z) = z - \overline{z}$; *(c)* $f(z) = 2x + ixy^2$; *(d)* $f(z) = e^x e^{-iy}$.

7

verify that each of these functions is entire:

(a) $f(z) = 3x + y + i(3y - x)$; *(b)* $f(z) = \sin x \cosh y + i \cos x \sinh y$;

(c) $f(z) = e^{-y} \sin x - ie^{-y} \cos x$; *(d)* $f(z) = (z^2 - 2)e^{-x}e^{-iy}$.

8

State why a composition of two entire functions is entire. Also, state why any *linear combination* $c_1 f_1(z) + c_2 f_2(z)$ of two entire functions, where c_1 and c_2 are complex constants, is entire.

9

Show that $u(x, y)$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$ when

(a) $u(x, y) = 2x(1 - y)$; *(b)* $u(x, y) = 2x - x^3 + 3xy^2$;
(c) $u(x, y) = \sinh x \sin y$; *(d)* $u(x, y) = y/(x^2 + y^2)$.

10

Show that if v and V are harmonic conjugates of $u(x, y)$ in a domain D, then $v(x, y)$ and $V(x, y)$ can differ at most by an additive constant.

UNIT-II

11

evaluate

$$\int_C f(z)\,dz.$$

$f(z) = (z+2)/z$ and C is

(a) the semicircle $z = 2\,e^{i\theta}$ $(0 \le \theta \le \pi)$;

(b) the semicircle $z = 2\,e^{i\theta}$ $(\pi \le \theta \le 2\pi)$;

(c) the circle $z = 2\,e^{i\theta}$ $(0 \le \theta \le 2\pi)$.

12

$f(z)$ is defined by means of the equations

$$f(z) = \begin{cases} 1 & \text{when } y < 0, \\ 4y & \text{when } y > 0, \end{cases}$$

and C is the arc from $z = -1 - i$ to $z = 1 + i$ along the curve $y = x^3$.

13

Let C denote the line segment from $z = i$ to $z = 1$. By observing that of all the points on that line segment, the midpoint is the closest to the origin, show that

$$\left| \int_C \frac{dz}{z^4} \right| \le 4\sqrt{2}$$

without evaluating the integral.

14

Let C_R denote the upper half of the circle $|z| = R$ $(R > 2)$, taken in the counterclockwise direction. Show that

$$\left| \int_{C_R} \frac{2z^2 - 1}{z^4 + 5z^2 + 4}\,dz \right| \le \frac{\pi R(2R^2 + 1)}{(R^2 - 1)(R^2 - 4)}.$$

Then, by dividing the numerator and denominator on the right here by R^4, show that the value of the integral tends to zero as R tends to infinity.

15

By finding an antiderivative, evaluate each of these integrals, where the path is any contour between the indicated limits of integration:

(a) $\displaystyle\int_i^{i/2} e^{\pi z}\,dz$; *(b)* $\displaystyle\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz$; *(c)* $\displaystyle\int_1^3 (z-2)^3\,dz$.

16

Use an antiderivative to show that for every contour C extending from a point z_1 to a point z_2,

$$\int_C z^n\,dz = \frac{1}{n+1}(z_2^{n+1} - z_1^{n+1}) \qquad (n = 0, 1, 2, \ldots).$$

17

Let C_0 and C denote the circles

$$z = z_0 + Re^{i\theta} \ (-\pi \le \theta \le \pi) \quad \text{and} \quad z = Re^{i\theta} \ (-\pi \le \theta \le \pi),$$

respectively.

(a) Use these parametric representations to show that

$$\int_{C_0} f(z - z_0)\, dz = \int_C f(z)\, dz$$

18

evaluate the integral

$$\int_C z^m \, \overline{z}^{\,n} dz,$$

where m and n are integers and C is the unit circle $|z| = 1$, taken counterclockwise.

19

$f(z) = 1$ and C is an arbitrary contour from any fixed point z_1 to any fixed point z_2 in the z plane.

evaluate

$$\int_C f(z) \ dz.$$

20

$f(z) = \pi \exp(\pi \overline{z})$ and C is the boundary of the square with vertices at the points $0, 1, 1+i$, and i, the orientation of C being in the counterclockwise direction.

evaluate

$$\int_C f(z) \ dz.$$

UNIT-III

21

Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate each of these integrals:

(a) $\displaystyle\int_C \frac{e^{-z}\, dz}{z - (\pi i/2)}$; *(b)* $\displaystyle\int_C \frac{\cos z}{z(z^2+8)}\, dz$; *(c)* $\displaystyle\int_C \frac{z\, dz}{2z+1}$;

22

Find the value of the integral of $g(z)$ around the circle $|z - i| = 2$ in the positive sense when

(a) $g(z) = \dfrac{1}{z^2+4}$; *(b)* $g(z) = \dfrac{1}{(z^2+4)^2}$.

23

Let C be the circle $|z| = 3$, described in the positive sense. Show that if

$$g(z) = \int_C \frac{2s^2 - s - 2}{s - z}\, ds \qquad (|z| \ne 3),$$

then $g(2) = 8\pi i$. What is the value of $g(z)$ when $|z| > 3$?

24

Let C be any simple closed contour, described in the positive sense in the z plane, and write

$$g(z) = \int_C \frac{s^3 + 2s}{(s - z)^3}\, ds.$$

Show that $g(z) = 6\pi i z$ when z is inside C and that $g(z) = 0$ when z is outside.

25

Show that if f is analytic within and on a simple closed contour C and z_0 is not on C, then

$$\int_C \frac{f'(z)\,dz}{z-z_0} = \int_C \frac{f(z)\,dz}{(z-z_0)^2}.$$

26

Let C be the unit circle $z = e^{i\theta}\,(-\pi \le \theta \le \pi)$. First show that for any real constant a,

$$\int_C \frac{e^{az}}{z}\,dz = 2\pi i.$$

Then write this integral in terms of θ to derive the integration formula

$$\int_0^{\pi} e^{a\cos\theta}\cos(a\sin\theta)\,d\theta = \pi.$$

27

Suppose that $f(z)$ is entire and that the harmonic function $u(x, y) = \text{Re}[f(z)]$ has an upper bound u_0; that is, $u(x, y) \le u_0$ for all points (x, y) in the xy plane. Show that $u(x, y)$ must be constant throughout the plane.

28

Let a function f be continuous on a closed bounded region R, and let it be analytic and not constant throughout the interior of R. Assuming that $f(z) \neq 0$ anywhere in R, prove that $|f(z)|$ has a *minimum value* m in R which occurs on the boundary of R and never in the interior. Do this by applying the corresponding result for maximum

29

Let the function $f(z) = u(x, y) + iv(x, y)$ be continuous on a closed bounded region R, and suppose that it is analytic and not constant in the interior of R. Show that the component function $v(x, y)$ has maximum and minimum values in R which are reached on the boundary of R and never in the interior, where it is harmonic.

30

Let f be the function $f(z) = e^z$ and R the rectangular region $0 \le x \le 1, 0 \le y \le \pi$. Illustrate results in Sec. 54 and Exercise 6 by finding points in R where the component function $u(x, y) = \text{Re}[f(z)]$ reaches its maximum and minimum values.

DSE-1F/B VECTOR CALCULUS BS: 606

Theory: 3 credits and Practicals: 1 credits

Theory: 3 hours /week and Practicals: 2 hours /week

Objective: Concepts like gradient, divergence, curl and their physical relevance will be taught.

Outcome: Students realize the way vector calculus is used to addresses some of the problems of physics.

Unit I

Line Integrals: Introductory Example : Work done against a Force-Evaluation of Line Integrals-Conservative Vector Fields-Surface Integrals: Introductory Example : Flow Through a Pipe-Evaluation of Surface Integrals

Unit II

Volume Integrals: Evaluation of Volume integrals

Gradient, Divergence and Curl: Partial differentiation and Taylor series-Partial differentiation-Taylor series in more than one variable-Gradient of a scalar field-Gradients, conservative fields and potentials-Physical applications of the gradient

Unit III

Divergence of a vector field -Physical interpretation of divergence-Laplacian of a scalar field-Curl of a vector field-Physical interpretation of curl-Relation between curl and rotation-Curl and conservative vector fields.

Text: P.C. Matthews, Vector *Calculus.*

References: G.B. Thomas and R.L. Finney, *Calculus*
H. Anton, I. Bivens and S. Davis, *Calculus*

Vector Calculus
Practicals Question Bank

UNIT-I

1

Evaluate the line integral

$$\int_C \boldsymbol{F} \times \boldsymbol{dr},$$

where $\boldsymbol{F}$ is the vector field $(y, x, 0)$ and C is the curve $y = \sin x$, $z = 0$, between $x = 0$ and $x = \pi$.

2

Evaluate the line integral

$$\int_C x + y^2 \, \boldsymbol{dr},$$

where C is the parabola $y = x^2$ in the plane $z = 0$ connecting the points $(0, 0, 0)$ and $(1, 1, 0)$.

3

Evaluate the line integral

$$\int_C \boldsymbol{F} \cdot \boldsymbol{dr} \qquad \text{where} \qquad \boldsymbol{F} = (5z^2, 2x, x + 2y)$$

and the curve C is given by $x = t$, $y = t^2$, $z = t^2$, $0 \leq t \leq 1$.

4

Find the line integral of the vector field $\boldsymbol{u} = (y^2, x, z)$ along the curve given by $z = y = e^x$ from $x = 0$ to $x = 1$.

5

Evaluate the surface integral of $\boldsymbol{u} = (y, x^2, z^2)$, over the surface S, where S is the triangular surface on $x = 0$ with $y \geq 0$, $z \geq 0$, $y + z \leq 1$, with the normal $\boldsymbol{n}$ directed in the positive x direction.

6

Find the surface integral of $\boldsymbol{u} = \boldsymbol{r}$ over the part of the paraboloid $z = 1 - x^2 - y^2$ with $z > 0$, with the normal pointing upwards.

7

If S is the entire x, y plane, evaluate the integral

$$I = \iint_S e^{-x^2 - y^2} \, dS,$$

by transforming the integral into polar coordinates.

8

Find the line integral $\oint_C \boldsymbol{r} \times \boldsymbol{dr}$ where the curve C is the ellipse $x^2/a^2 + y^2/b^2 = 1$ taken in an anticlockwise direction. What do you notice about the magnitude of the answer?

9

By considering the line integral of $\boldsymbol{F} = (y, x^2 - x, 0)$ around the square in the x, y plane connecting the four points $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$, show that $\boldsymbol{F}$ cannot be a conservative vector field.

10

Evaluate the line integral of the vector field $\boldsymbol{u} = (xy, z^2, x)$ along the curve given by $x = 1 + t$, $y = 0$, $z = t^2$, $0 \leq t \leq 3$.

UNIT-II

11

A cube $0 \leq x, y, z, \leq 1$ has a variable density given by $\rho = 1 + x + y + z$. What is the total mass of the cube?

12

Find the volume of the tetrahedron with vertices at $(0,0,0)$, $(a,0,0)$, $(0,b,0)$ and $(0,0,c)$.

13

Evaluate the surface integral of $\boldsymbol{u} = (xy, x, x + y)$ over the surface S defined by $z = 0$ with $0 \leq x \leq 1$, $0 \leq y \leq 2$, with the normal $\boldsymbol{n}$ directed in the positive z direction.

14

Find the surface integral of $\boldsymbol{u} = \boldsymbol{r}$ over the surface of the unit cube $0 \leq x, y, z \leq 1$, with $\boldsymbol{n}$ pointing outward.

15

The surface S is defined to be that part of the plane $z = 0$ lying between the curves $y = x^2$ and $x = y^2$. Find the surface integral of $\boldsymbol{u} \cdot \boldsymbol{n}$ over S where $\boldsymbol{u} = (z, xy, x^2)$ and $\boldsymbol{n} = (0,0,1)$.

16

Find the surface integral of $\boldsymbol{u} \cdot \boldsymbol{n}$ over S where S is the part of the surface $z = x + y^2$ with $z < 0$ and $x > -1$, $\boldsymbol{u}$ is the vector field $\boldsymbol{u} = (2y + x, -1, 0)$ and $\boldsymbol{n}$ has a negative z component.

17

Find the volume integral of the scalar field $\phi = x^2 + y^2 + z^2$ over the region V specified by $0 \leq x \leq 1$, $1 \leq y \leq 2$, $0 \leq z \leq 3$.

18

Find the volume of the section of the cylinder $x^2 + y^2 = 1$ that lies between the planes $z = x + 1$ and $z = -x - 1$.

19 Find the unit normal $\boldsymbol{n}$ to the surface $x^2 + y^2 - z = 0$ at the point $(1,1,2)$.

20 Find the gradient of the scalar field $f = xyz$, and evaluate it at the point (1,2,3). Hence find the directional derivative of f at this point in the direction of the vector $(1,1,0)$.

UNIT-III

21

Find the divergence of the vector field $\boldsymbol{u} = \boldsymbol{r}$.

22

The vector field $\boldsymbol{u}$ is defined by $\boldsymbol{u} = (xy, z + x, y)$. Calculate $\boldsymbol{\nabla} \times \boldsymbol{u}$ and find the points where $\boldsymbol{\nabla} \times \boldsymbol{u} = \boldsymbol{0}$.

23

Find the gradient $\boldsymbol{\nabla}\phi$ and the Laplacian $\nabla^2\phi$ for the scalar field $\phi = x^2 + xy + yz^2$.

24

Find the gradient and Laplacian of

$$\phi = \sin(kx)\sin(ly)\exp(\sqrt{k^2 + l^2}z).$$

25

Find the unit normal to the surface $xy^2 + 2yz = 4$ at the point $(-2,2,3)$.

26

For $\phi(x,y,z) = x^2+y^2+z^2+xy-3x$, find $\nabla\phi$ and find the minimum value of ϕ.

27

Find the equation of the plane which is tangent to the surface $x^2 + y^2 - 2z^3 = 0$ at the point $(1,1,1)$.

28

Find both the divergence and the curl of the vector fields

(a) $\boldsymbol{u} = (y,z,x)$;

(b) $\boldsymbol{v} = (xyz, z^2, x-y)$.

29

For what values, if any, of the constants a and b is the vector field $\boldsymbol{u} = (y\cos x + axz, b\sin x + z, x^2 + y)$ irrotational?

30

(a) Show that $\boldsymbol{u} = (y^2 z, -z^2\sin y + 2xyz, 2z\cos y + y^2 x)$ is irrotational.

(b) Find the corresponding potential function.

(c) Hence find the value of the line integral of $\boldsymbol{u}$ along the curve $x = \sin \pi t/2$, $y = t^2 - t$, $z = t^4$, $0 \le t \le 1$.

MOOCs Resources

A set of MOOCs resources for ICT based learning and teaching

NPTEL: nptel.ac.in

COURSERA: www.coursera.org

MITOCW: ocw.mit.edu

ACADEMIC EARTH: www.academicearth.org

EdX : www.edx.org

KHAN ACADEMY : www.khanacademy.org

ALISON: www.alison.com

STANFORD ONLINE: www.online.stanford.edu

VIDEO LECTURES: videolectures.net

INTERACTIVE REAL ANALYSIS: mathcs.org

VISUAL CALCULUS: archives.math.utk.edu/visual.calculus

MOOCS CALCULUS: mooculus.osu.edu

Few Math Softwares

Useful for Classroom teaching: Geogebra (Freeware)

Type setting software: LaTeX

High end commercial softwares: Mathematica , Maple , Matlab

Answering search engine: www.wolframalpha.com

Group theory software: group explorer 2.2 (Freeware)

Visualization software: Mathematics Visualization Toolkit (freeware)

Appendices

ज्ञान-विज्ञान विमुक्तये
प्रो. (डॉ.) जसपाल एस. सन्धू
सचिव
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MBBS, MS (Ortho), DSM, FAIS, FASM, FAFSM, FFIMS, FAMS
Secretary

सत्यमेव जयते

विश्वविद्यालय अनुदान आयोग
University Grants Commission
(मानव संसाधन विकास मंत्रालय, भारत सरकार)
(Ministry of Human Resource Development, Govt. of India)
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BY SPEED POST

D.O.No. F. 1-1/2014(Secy) 12th November, 2014

Dear Sir/Madam,

The UGC has embarked on numerous measures to enhance efficiency and excellence in the higher education system in the country. The reforms undertaken in this regard have led to noticeable improvement in the standards of education. However, because of the diversity in the evaluation system followed by different universities in India, students have suffered acceptance of their credentials, at times, across the university system, as well as the employment agencies.

In order to mitigate this procedure, it has been thought that the Choice-Based Credit System (CBCS) proposed by the UGC should be adopted by all the Universities. This would ensure seamless mobility of students across the higher education institutions in the country as well as abroad. The credits earned by the student can be transferred and would be of great value to the students in the event of their seeking migration from one institution to the other.

Even in the universities which have already adopted the CBCS it has come to our notice that there is tremendous diversity in the adoption of the system that inter-university migration of students amongst such universities has also posed problems. Under the situation mentioned, the UGC has formulated Guidelines for adoption of uniform Choice-Based Credit System across all the universities. The Guidelines have been uploaded on the website of the UGC (www.ugc.ac.in).

You are requested that the Guidelines may kindly be accessed from the UGC website and the system introduced in your esteemed university from the academic year 2015-16. All the actions taken in this regard may kindly be communicated to the Secretary, UGC (email: ugc.action@gmail.com).

With kind regards,

Yours sincerely,

(Jaspal S. Sandhu)

The Vice-Chancellors of all Universities.

1953-2013

Appendix-II

UGC GUIDELINES ON ADOPTION OF CHOICE BASED CREDIT SYSTEM

UNIVERSITY GRANTS COMMISSION
BAHADURSHAH ZAFAR MARG NEW
DELHI — 110 002

UGC Guidelines on Adoption of Choice Based Credit System

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1. Preamble

The University Grants Commission (UGC) has initiated several measures to bring equity, efficiency and excellence in the Higher Education System of country. The important measures taken to enhance academic standards and quality in higher education include innovation and improvements in curriculum, teaching-learning process, examination and evaluation systems, besides governance and other matters.

The UGC has formulated various regulations and guidelines from time to time to improve the higher education system and maintain minimum standards and quality across the Higher Educational Institutions (HEIs) in India. The academic reforms recommended by the UGC in the recent past have led to overall improvement in the higher education system. However, due to lot of diversity in the system of higher education, there are multiple approaches followed by universities towards examination, evaluation and grading system. While the HEIs must have the flexibility and freedom in designing the examination and evaluation methods that best fits the the curriculum, syllabi and teaching–learning methods, there is a need to devise a sensible system for awarding the grades based on the performance of students. Presently the performance of the students is reported using the conventional system of marks secured in the examinations or grades or both. The conversion from marks to letter grades and the letter grades used vary widely across the HEIs in the country. This creates difficulty for the acadamia and the employers to understand and infer the performance of the students graduating from different universities and colleges based on grades.

The grading system is considered to be better than the conventional marks system and hence it has been followed in the top instutitions in India and abroad. So it is desirable to introduce uniform grading system. This will facilitate student mobility across institutions within and across countries and also enable potential employers to assess the performance of students. To bring in the desired uniformity, in grading system and method for computing the cumulative grade point average (CGPA) based on the performance of students in the examinations, the UGC has formulated these guidelines.

2. Applicability of the Grading System

These guidel

ines shall apply to all undergraduate and postgraduate level degree, diploma and certificate programmes under the credit system awarded by the Central, State and Deemed to be universities in India.

3. Definitions of Key Words:

1. **Academic Year**: Two consecutive (one odd + one even) semesters constitute one academic year.
2. **Choice Based Credit System (CBCS):** The CBCS provides choice for students to select from the prescribed courses (core, elective or minor or soft skill courses).
3. **Course**: Usually referred to, as 'papers' is a component of a programme. All courses need not carry the same weight. The courses should define learning objectives and

learning outcomes. A course may be designed to comprise lectures/ tutorials/laboratory work/ field work/ outreach activities/ project work/ vocational training/viva/ seminars/ term papers/assignments/ presentations/ self-study etc. or a combination of some of these.

4. **Credit Based Semester System (CBSS)**: Under the CBSS, the requirement for awarding a degree or diploma or certificate is prescribed in terms of number of credits to be completed by the students.
5. **Credit Point**: It is the product of grade point and number of credits for a course.
6. **Credit**: A unit by which the course work is measured. It determines the number of hours of instructions required per week. One credit is equivalent to one hour of teaching (lecture or tutorial) or two hours of practical work/field work per week.
7. **Cumulative Grade Point Average (CGPA)**: It is a measure of overall cumulative performance of a student over all semesters. The CGPA is the ratio of total credit points secured by a student in various courses in all semesters and the sum of the total credits of all courses in all the semesters. It is expressed up to two decimal places.
8. **Grade Point**: It is a numerical weight allotted to each letter grade on a 10-point scale.
9. **Letter Grade:** It is an index of the performance of students in a said course. Grades are denoted by letters O, A+, A, B+, B, C, P and F.
10. **Programme**: An educational programme leading to award of a Degree, diploma or certificate.
11. **Semester Grade Point Average (SGPA)**: It is a measure of performance of work done in a semester. It is ratio of total credit points secured by a student in various courses registered in a semester and the total course credits taken during that semester. It shall be expressed up to two decimal places.
12. **Semester**: Each semester will consist of 15-18 weeks of academic work equivalent to 90 actual teaching days. The odd semester may be scheduled from July to December and even semester from January to June.
13. **Transcript or Grade Card or Certificate:** Based on the grades earned, a grade certificate shall be issued to all the registered students after every semester. The grade certificate will display the course details (code, title, number of credits, grade secured) along with SGPA of that semester and CGPA earned till that semester.

4. Semester System and Choice Based Credit System

The Indian Higher Education Institutions have been moving from the conventional annual system to semester system. Currently many of the institutions have already introduced the choice based credit system. The semester system accelerates the teaching-learning process and enables vertical and horizontal mobility in learning. The credit based semester system provides flexibility in designing curriculum and assigning credits based on the course content and hours of teaching. The choice based credit system provides a 'cafeteria' type approach in which the students can take courses of their choice, learn at their own pace, undergo additional courses and acquire more than the required credits, and adopt an interdisciplinary approach to learning, It is desirable that the HEIs move to CBCS and implement the grading system.

5. Types of Courses:

Courses in a programme may be of three kinds: Core, Elective and Foundation.

1. Core Course:-

There may be a Core Course in every semester. This is the course which is to be compulsorily studied by a student as a core requirement to complete the requirement of a programme in a said discipline of study.

2. Elective Course:-

Elective course is a course which can be chosen from a pool of papers. It may be:

- Supportive to the discipline of study
- Providing an expanded scope
- Enabling an exposure to some other discipline/domain
- Nurturing student's proficiency/skill.

An elective may be "Generic Elective" focusing on those courses which add generic proficiency to the students. An elective may be "Discipline centric"or may be chosen from an unrelated discipline. It may be called an "Open Elective."

3. Foundation Course:-

The Foundation Courses may be of two kinds: Compulsory Foundation and Elective foundation. "Compulsory Foundation" courses are the courses based upon the content that leads to Knowledge enhancement. They are mandatory for all disciplines. Elective Foundation courses are value-based and are aimed at man-making education.

6. Examination and Assessment

The HEIs are currently following various methods for examination and assessment suitable for the courses and programmes as approved by their respective statutory bodies. In assessing the performance of the students in examinations, the usual approach is to award marks based on the examinations conducted at various stages (sessional, mid-term, end-semester etc.,) in a semester. Some of the HEIs convert these marks to letter grades based on absolute or relative grading system and award the grades. There is a marked variation across the colleges and universities in the number of grades, grade points, letter grades used, which creates difficulties in comparing students across the institutions. The UGC recommends the following system to be implemented in awarding the grades and CGPA under the credit based semester system.

6.1. Letter Grades and Grade Points:

i. Two methods -relative grading or absolute grading– have been in vogue for awarding grades in a course. The relative grading is based on the distribution (usually normal distribution) of marks obtained by all the students of the course and the grades are awarded based on a cut-off marks or percentile. Under the absolute grading, the marks are converted to grades based on pre-determined class intervals. To implement the following grading system, the colleges and universities can use any one of the above methods.

ii. The UGC recommends a 10-point grading system with the following letter grades as given below:

Table 1: Grades and Grade Points

Letter Grade	Grade Point

O (Outstanding)	10
A+(Excellent)	9
A(Very Good)	8
B+(Good)	7
B(Above Average)	6
C(Average)	5
P (Pass)	4
F(Fail)	0
Ab (Absent)	0

iii. A student obtaining Grade F shall be considered failed and will be required to reappear in the examination.

iv. For non credit courses 'Satisfactory' or "Unsatisfactory' shall be indicated instead of the letter grade and this will not be counted for the computation of SGPA/CGPA.

v. The Universities can decide on the grade or percentage of marks required to pass in a course and also the CGPA required to qualify for a degree taking into consideration the recommendations of the statutory professional councils such as AICTE, MCI, BCI, NCTE etc.,

vi. The statutory requirement for eligibility to enter as assistant professor in colleges and universities in the disciplines of arts, science, commerce etc., is a minimum average mark of 50% and 55% in relevant postgraduate degree respectively for reserved and general category. Hence, it is recommended that the cut-off marks for grade B shall not be less than 50% and for grade B+, it should not be less than 55% under the absolute grading system. Similarly cut-off marks shall be fixed for grade B and B+ based on the recommendation of the statutory bodies (AICTE, NCTE etc.,) of the relevant disciplines.

6.2. Fairness in Assessment:

Assessment is an integral part of system of education as it is instrumental in identifying and certifying the academic standards accomplished by a student and projecting them far and wide as an objective and impartial indicator of a student's performance. Thus, it becomes bounden duty of a University to ensure that it is carried out in fair manner. In this regard, UGC recommends the following system of checks and balances which would enable Universities effectively and fairly carry out the process of assessment and examination.

i. In case of at least 50% of core courses offered in different programmes across the disciplines, the assessment of the theoretical component towards the end of the semester should be undertaken by external examiners from outside the university conducting examination, who may be appointed by the competent authority. In such courses, the question papers will be set as well as assessed by external examiners.

ii. In case of the assessment of practical component of such core courses, the team of examiners should be constituted on 50 – 50 % basis. i.e. half of the examiners in the team should be invited from outside the university conducting examination.

iii. In case of the assessment of project reports / thesis / dissertation etc. the work should be undertaken by internal as well as external examiners.

7. Computation of SGPA and CGPA

The UGC recommends the following procedure to compute the Semester Grade Point Average (SGPA) and Cumulative Grade Point Average (CGPA):

i. The SGPA is the ratio of sum of the product of the number of credits with the grade points scored by a student in all the courses taken by a student and the sum of the number of credits of all the courses undergone by a student, i.e

SGPA (Si) = $\sum(C_i \times G_i) / \sum C_i$

where C_i is the number of credits of the ith course and G_i is the grade point scored by the student in the ith course.

ii. The CGPA is also calculated in the same manner taking into account all the courses undergone by a student over all the semesters of a programme, i.e.

CGPA = $\sum(Ci \times Si) / \sum Ci$

where Si is the SGPA of the ith semester and Ci is the total number of credits in that semester.

iii. The SGPA and CGPA shall be rounded off to 2 decimal points and reported in the transcripts.

8. Illustration of Computation of SGPA and CGPA and Format for Transcripts

i. Computation of SGPA and CGPA

Illustration **for SGPA**

Course	**Credit**	**Grade letter**	**Grade point**	**Credit Point** (Credit x Grade
Course 1	3	A	8	3 X 8 = 24
Course 2	4	B+	7	4 X 7 = 28
Course 3	3	B	6	3 X 6 = 18
Course 4	3	O	10	3 X 10 = 30
Course 5	3	C	5	3 X 5 = 15
Course 6	4	B	6	4 X 6 = 24
	20			139

Thus, **SGPA =139/20 =6.95**

Illustration **for CGPA**

Semester 1	**Semester 2**	**Semester 3**	**Semester 4**
Credit : 20 SGPA:6.9	Credit : 22 SGPA:7.8	Credit : 25 SGPA: 5.6	Credit : 26 SGPA:6.0

Semester 5	**Semester 6**		
Credit : 26 SGPA:6.3	Credit : 25 SGPA: 8.0		

Thus, **CGPA** = $\frac{20 \times 6.9 + 22 \times 7.8 + 25 \times 5.6 + 26 \times 6.0 + 26 \times 6.3 + 25 \times 8.0}{144}$ = **6.73**

ii. Transcript (Format): Based on the above recommendations on Letter grades, grade points and SGPA and CCPA, the HEIs may issue the transcript for each semester and a consolidated transcript indicating the performance in all semesters.