# APPLICATIONS OF FIRST ORDER DIFFERENTIAL EQUATIONS IN VARIOUS FIELDS 

## STUDENTS' STUDY PROJECT SUBMITTED TO THE COMMISSIONER OF COLLEGIATE EDUCATION, HYD.

 UnderJIGNASA 2022-23


Submitted by
B.Deepika , N.Varshasri , A.Anjanna , A.Anil Kumar

## Under the Supervision of

SWAPNA NOLLA
Lecturer in Mathematics

## DEPARTMENT OF MATHEMATICS

GOVTERNMENT DEGREE COLLEGE
CHENNOOR, TELANGANA

## DECLARATION

We do hereby declare that the work presented in this study project entitled "APPLICATIONS OF FIRST ORDER DIFFERENTIAL EQUATIONS IN VARIOUS FIELDS" has been originally carried out by us under the supervision of Swapna Noolla, Lecturer in Mathematics, Govt. Degree College, Chennoor and has not been submitted either in part or in full for any study project work to any other Govt. Degree colleges in Telangana state.

Date: 12-2022

Place: Chennoor

1. B.Deepika
2.N.Varshasri
3.A.Anjanna
4.A.Anil Kumar

## CERTIFICATE

This is to certify that the JIGNASA- Students' study project entitled "APPLICATIONS OF FIRST ORDER DIFFERENTIAL EQUATIONS IN VARIOUS FIELDS" has been carried out by the Students of B.Sc. (MPCs) under my supervision. I further certify that the study project work done by them is original and has not been submitted for any study project work either in part or in full to any other degree college.

Date: -12-2022
Place: Chennoor
(SWAPNA NOOLLA)
Study Project Supervisor

# APPLICATIONS OF FIRST ORDER DIFFERENTIAL EQUATIONS IN VARIOUS FIELDS 


#### Abstract

This study project is mainly focus on applications of the first order differential equations in various fields, which considers some linear and non linear models, such as equations with separable variables, homogeneous and Bernoulli's equations with first order linear differential equations are discussed and also modeling phenomena for real world problems which are described by firstorder differential equations is discussed in detail. The models includes Newton's cooling law, growth and decay, radio carbon dating, flow of liquid from a small orifice and epidemic problems.


## OBJECTIVES:

Main objectives of the study project carried out are as follows
$>$ To study and understand various differential equations.
$>$ To study and understand various solutions of differential
equations.
$>$ To study and understand first order differential equations and its
solutions
$>$ To study and find problems in various fields regarding to first order differential equations.
$>$ To find the solutions of problems in various fields by using first order differential equations

## INTRODUCTION

An equation with a function in one independent variable as unknown, containing not only the unknown function itself, but also its derivatives of various orders is known as Differential Equation.

The term "differential equations" was proposed in 1676 by G. Leibniz. The first studies of these equations were carried out in the late 17th century in the context of certain problems in mechanics and geometry.

Ordinary differential equations have important applications and are a powerful tool in the study of many problems in the natural sciences and in technology.

They are extensively employed in mechanics, astronomy, physics, and in many problems of chemistry and biology..

Many problems in engineering and science can be formulated in terms of differential equations. The formulation of mathematical models is basically to address real-world problems which have been one of the most important aspects of applied mathematics.

There are many applications of 1st order differential equations, especially separable ones.

1) Newton's law of cooling: $d T / d t=k\left(T-T_{i}\right)$, where $T$ is temperature of body and $T_{i}$ is temperature of surroundings.
2) Mechanics problems involving variable acceleration can be solved using separable equations or linear 1st order equation.

3 ) First order equations are also used to find the growth of bacteria at time ' $t$ ', and the rate of decay of a radioactive substance.
4) We can find the spreading and control of a contagious disease.

## PROBLEMS RELATED TO VARIOUS FIELDS BY USING FIRST ORDER DIFFERENTIAL EQUATIONS

## TEMPERATURE RATE OF CHANGE (NEWTON'S LAW OF COOLING):

When a hot object is placed in a cool room, the object dissipates heat to the surroundings, and its temperature decreases. Newton's Law of Cooling states that the rate at which the object's temperature decreases is proportional to the difference between the temperature of the object and the ambient temperature. At the beginning of the cooling process, the difference between these temperatures is greatest, so this is when the rate of temperature decrease is greatest. However, as the object cools, the temperature difference gets smaller, and the cooling rate decreases; thus, the object cools more and more slowly as time passes. To formulate this process mathematically, let $T(t)$ denote the temperature of the object at time $t$ and let $T_{0}$ denote the (essentially constant) temperature of the surroundings. Newton's Law of Cooling then says

$$
\frac{d T}{d t} \propto\left(\mathrm{~T}-\mathrm{T}_{0}\right) \Rightarrow \frac{d T}{d t}=\mathrm{k}\left(\mathrm{~T}-\mathrm{T}_{0}\right)
$$



## Estimating the time of murder:

Problem 1: The body of a murder victim was discovered at 9:00AM. The forensic expert took the temperature of the body was $95.8^{\circ} \mathrm{F}$ at 9:45AM. He again took temperature after 45 minutes, it showed $94.8^{\circ} \mathrm{F}$ and noticed that room temperature is $65^{\circ} \mathrm{F}$. Estimate the time of death. (Normal temperature of human body $=98.4^{\circ} \mathrm{F}$ )

## Solution:-

We get the solution of the above problem by using Newton's law of cooling.
We have the differential equation, $\frac{d T}{d t}=k\left(T-T_{0}\right) \Rightarrow \frac{d T}{T-T_{0}}=k d t$

Integrating on both sides,

$$
\int \frac{d T}{\left(T-T_{0}\right)}=\int k d t \Rightarrow \mathrm{~T} \quad=\quad \mathrm{T}_{0}+c \mathrm{e}^{\mathrm{kt}}
$$

To find the ' $c$ ' value,
By substituting the values, when $\mathrm{t}=0, \mathrm{~T}=95.8$ and $\mathrm{T}_{0}=65$
in the equation $T_{0}+c e^{k t}=T$ we get $c=30.8$
Next we find " $k$ " value by the following information.
When $t=45$ minutes $\mathrm{T}=94.8^{\circ} \mathrm{F}$ and this gives

$$
\begin{aligned}
& c e^{k t}=T-T_{0} \\
& 30.8 e^{k(45)}=94.8-65 \\
& k=-0.000733
\end{aligned}
$$

Now using the values $c, k$ and $T=98.4, T_{0}=65$
We can find the estimated time ' t ' of death
by using the solution of the given differential equation $T=T_{0}+c e^{k t}$

$$
\begin{aligned}
98.4=65+(30.8) e^{-0.000733 t} & \Rightarrow \mathrm{t}=-110.5 \mathrm{~min} \\
& \Rightarrow \mathrm{t}=-1 \text { hour } 50 \mathrm{~min}
\end{aligned}
$$

Therefore, estimated time of the death is $=9: 45 \mathrm{AM}-1$ hour 50 minutes $=$ 7:55 A.M.

## GROWTH AND DECAY PROBLEMS:

The initial value problem $\quad \frac{d x}{d t}=\mathrm{kx}, \quad \mathrm{x}\left(\mathrm{t}_{0}\right)=\mathrm{x}_{0}$
Where $k$ is a constant, occurs in many physical theories involving either growth or decay. For example, in biological it is often observed that the rate at which certain bacteria grow is proportional to the number of bacteria present at any time.

The following graph is showing of Growth and Decay.


## Finding the time of increase of bacteria:

Problem 1: Bacteria in certain culture increase at rate proportional to the number present .if the number doubles in 2 hours, how long does it take for the number to triple ?

## Solution:

The above problem will be solved by using differential equation of growth and decay

$$
\frac{d y}{d t}=\mathrm{ky} \Rightarrow \mathrm{y}=\mathrm{ce}^{\mathrm{kt}}
$$

from the condition $t=0, y=y_{0}$, we get, $c=y_{0}$.

Now we have when $t=2, y=2 y_{0}$

Substituting these values in $y=c e^{k t}$ we get the value of $k$

$$
2 y_{0}=y_{0} \mathrm{e}^{\mathrm{k}(2)} \Rightarrow \mathrm{k}=\frac{\log 2}{2},
$$

Hence, $\mathrm{y}=\mathrm{y}_{0} e^{t \frac{\log 2}{2}}$

Now we can find how much time for number to triple by substituting $\mathrm{y}=3 \mathrm{y}_{0}$ in the above equation, we get $\mathrm{t}=3.1694 \mathrm{hrs}$.

Therefore after 3hr 10 min the bacteria will be triple.

## RADIO CARBON DATING OR CARBON DATING OR CARBON-14 DATING

The atmosphere of the earth is continuously bombarded by cosmic rays. These cosmic rays produce neutrons in the earth's atmosphere, and these neutrons combine with nitrogen to produce ${ }^{14} C$. This radiocarbon ( ${ }^{14} C$ ) is incorporated in carbon dioxide and thus moves through the atmosphere to be absorbed by plants. In turn, radiocarbon is built in animal tissues by eating the plants. In living tissues, the rate of ingestion of ${ }^{14} C$ exactly balances the rate of disintegration of ${ }^{14} C$.When an organism dies, though, it ceases to ingest ${ }^{14} C$, its ${ }^{14} C$ concentration being to decease through disintegration of the ${ }^{14} C$ present. Now, it is a physically accepted fact that the rate of bombardment of the earth's atmosphere by cosmic rays has always been constant.

Let $\mathrm{N}(\mathrm{t})$ denotes the amount of carbon-14 present in a sample at timet, and $N_{0}$ denotes the amount present at time $\mathrm{t}=0$ when the sample was formed. If $k$ denotes the decay constant of ${ }^{14} C$ (half-life 5568 years), then $d N(t), N(0)=N_{0}$ and, consequently, $\mathrm{N}(t)=\mathrm{N}_{0} \mathrm{e}^{-\mathrm{kt}}$.

Problem 1: It is found that 1 percent of radium disappears in 24 years. What percentage will disappear in 10,000 years ?

Solution: Let $A$ be the quantity of radium in grams, present after $t$ years then $\frac{d A}{d t}$ represents the rate of disintegration of radium. According to law of radioactive decay, we have

$$
\frac{d A}{d t} \propto A(\text { or }) \frac{d A}{d t}=c \mathrm{cA}
$$

Since $A$ is positive and is decreasing, then $\frac{d A}{d t}<0$ and we see that constant of proportionality $c$ must be negative, writing $c=-k$, we get

$$
\frac{d A}{d t}=-\mathrm{kA} \Rightarrow \mathrm{~A}=\mathrm{ce}-\mathrm{kt}
$$

Let $A_{0}$ be the amount in grams of radium present initially,
then $0.01 \mathrm{~A}_{0} \mathrm{~g}$ disappears in 24 years, so that $0.99 \mathrm{~A}_{0} \mathrm{~g}$ remains,

We thus have at $t=0, A=A_{0}$ and at $t=24$ years, $A=0.99 A_{0} g$.

We get the solution of the above problem with the equation $A=c e^{-k t}$

Since at $t=0, A=A_{0}$ then we get $c=A_{0}$, and hence $A=A_{0} e^{-k t}$

Also at $t=24$ years and $A=0.99 A_{0}$, then
$0.99 \mathrm{~A}_{0}=\mathrm{A}_{0} \mathrm{e}^{-24 \mathrm{k}} \Rightarrow \mathrm{e}^{-24 \mathrm{k}}=0.99$
We get, $k=0.000418$
Substituting ' $k$ ' value in the equation $A=A_{0} e^{-k t}$
Hence we get $A=A_{0} e^{-0.000418 t}$
When $\mathrm{t}=10,000, \mathrm{e}^{-0.000418 \mathrm{t}}=0.015$
from (1) we get, $A=A_{0} \times 10,000 \times 0.015$

$$
\Rightarrow A=0.015 A_{0}
$$

So that $0.015 \mathrm{~A}_{0}$ grams remains in 10,000 years

## A PROBLEM IN EPIDEMIOLOGY:

An important problem in biology and medicine deals with the occurrence, spreading and control of a contagious disease i.e., one which can be transmitted form one individual to another. The science that deals with this study is called epidemiology, and if a large number of populations get the disease, we say that there is an epidemic.

Let $N_{i}$ denotes the number of infected students at any time $t$ and $N_{u}$ the uninfected students. Then, if $N$ is the total number of students (assumed to be constant), we have

$$
\begin{equation*}
N=N_{i}+N_{u^{-}}^{-} \tag{1}
\end{equation*}
$$

Here, $d N_{i} / d t$ is the rate of change in number of infected students and should depend in some way on $N_{i}$, and thus $N_{u}$. Assuming that $d N_{i} / d t$ is a quadratic function of $N_{i}$ as an approximation. We get

$$
\begin{equation*}
\frac{d N_{i}}{d t}=\mathrm{a}_{0}+\mathrm{a}_{1} N_{i}+\mathrm{a}_{2} N_{i}^{2} \tag{2}
\end{equation*}
$$

Where $a_{0}, a_{1}, a_{2}$ are constants. Now we expect $d N_{i} / d t=0$, where $N_{i}=0$, i.e. there are no infected students and where $N_{i}=N$, i.e., all students are infected. Then form equation (2), we have

$$
a_{0}=0 \quad \text { and } \quad a_{1} N+a_{2} N^{2}=0 \quad \text { or } \quad a_{2}=-a_{1} / N
$$

So that equation becomes

$$
\begin{equation*}
\frac{d N_{i}}{d t}=\mathrm{a}_{1} N_{i-} \frac{a_{1} N_{i}^{2}}{N}=\frac{a_{1}}{N} N_{i}\left(N-N_{i}\right)=\mathrm{k} N_{i}\left(N-N_{i}\right) \tag{3}
\end{equation*}
$$

Where $\mathrm{k}=\mathrm{a}_{1} / \mathrm{N}$ and the initial conditions are

$$
\begin{equation*}
N_{i}=N_{0} \quad \text { at } \quad t=0- \tag{4}
\end{equation*}
$$

From the above equations we get

$$
\Rightarrow N_{i}=\frac{N}{1+\left(\frac{N}{N_{0}}-1\right) e^{-N k t}}
$$



## Spread of Flu Virus:

Problem 1: A student carrying flu virus returns to an isolated college hostel of 750 students. If it is assumed that the rate at which the virus spreads is proportional not only to the number of infected students $N_{i}$ but also to the students not infected. Find the number of infected students after 8 days when it is further observed that after 5 days $N_{i}(5)=32$.

## Solution:-

The above problem will be solved by using the following equation

$$
\begin{aligned}
& N_{i}=\frac{N}{1+\left(\frac{N}{N_{0}}-1\right) e^{-N k t}} \\
& \Rightarrow \mathrm{~N}_{\mathrm{i}}=\mathrm{N}(\mathrm{t})=\frac{750}{1+\left(\frac{750}{1}-1\right) e^{-N k t}}=\frac{750}{1+(749) e^{-750 k t}}
\end{aligned}
$$

Now we can determine ' $k$ ' value by using $N_{i}=N(5)=32$

$$
\mathrm{N}(5)=32==\frac{750}{1+(749) e^{-750 k t}}==\frac{750}{1+(749) e^{-750 k \times 5}}
$$

$$
k=0.001006
$$

Now we find the number of infected students after 8 days

$$
\begin{aligned}
& \mathrm{t}=8, \mathrm{k}=0.001006, \mathrm{~N}=750, \mathrm{~N}_{\mathrm{i}}=? \\
& \mathrm{~N}_{\mathrm{i}}=\mathrm{N}(8)=\frac{750}{1+(749) e^{-750 \times 0.001006 \times 8}}=\frac{750}{1+(749) e^{-6.036}} \\
& \quad=268 \text { Approximately }
\end{aligned}
$$

The number of infected students after 8 days is 268.

## Flow of liquid from a small orifice:

The liquid in the vessel flows through out a small sharp edged orifice. If there were no loss of energy, the speed of the escaping water would be the same as
the speed of a freely falling body, namely $\sqrt{2 \mathrm{gh}}$, where ' $h$ ' denotes the height (ft) of thesurface above the orifice at time $t$. Because of the friction and the surface tension,the actual speed has been found (approx.)to be $0.6 \sqrt{2 \mathrm{gh}}$ or $4.8 \sqrt{\mathrm{~h}} \mathrm{ft} / \mathrm{s}$, when $\mathrm{g}=32 \mathrm{ft} / \mathrm{s}$ is the acceleration due to gravity. This is known as Torricelli's law. Thus, if the orifice has an area A, the fluid leaves at $4.8 \mathrm{Ah}^{1 / 2}$ $\mathrm{ft}^{3} / \mathrm{s}$. Hence, if V denotes the volume of liquid in the vessel at time t , then

$$
\frac{d V}{d t}=-4.8 \mathrm{Ah}^{1 / 2}
$$

The minus indicates that V decreases with time t .

Problem 1: A tank 9 ft deep has a rectangular cross - section $5 \times 7$. The tank is initially filled with water, which runs out through an orifice radius 0.5 in located in the bottom of the tank.
a) Find the required time for the tank to empty.
b) Find the required time when water is half to the tank.
c) Find the height of the water to the tank after 15 minutes.

Solution: We have an equation

$$
\begin{aligned}
& \frac{d V}{d t}=-4.8 A \sqrt{h} \\
& \text { Here } A=\pi r^{2}
\end{aligned}
$$

According to the problem

$$
\text { Height of the tank }(\mathrm{h})=9 \mathrm{ft}
$$

Since $r=0.5$ inches we will get $A=\pi(0.5 / 12)^{2}=\frac{\pi}{(24)^{2}}$

$$
\begin{aligned}
& \frac{d V}{d t}=-4.8 \frac{\pi}{(24)^{2}} \sqrt{h} \\
& \frac{d V}{d t}=-\frac{\pi}{120} \sqrt{h}
\end{aligned}
$$

We have $l=5, b=7, h=h$

Since V=5x7xh=35h

Differentiate both sides with respect ' t '

$$
\frac{d V}{d t}=35 \frac{d h}{d t}
$$

But we have

$$
\begin{aligned}
& \frac{d V}{d t}=\frac{-\pi}{120} \sqrt{h} \\
& 35 \frac{d h}{d t}=\frac{-\pi}{120} \sqrt{h}
\end{aligned}
$$

Integrates both sides

$$
\begin{aligned}
& 2 \sqrt{\mathrm{~h}}=-\frac{\pi}{4200} \mathrm{t}+\mathrm{c} \\
& \text { When } \mathrm{t}=0, \mathrm{~h}=9 \text {, then } \mathrm{c}=\text { ? }
\end{aligned}
$$

Substitute the above values in 2 nd equation

Then we get, $\Rightarrow \mathrm{c}=6$

And again the above value Substitute in 2nd equation

$$
\begin{equation*}
2 \sqrt{\mathrm{~h}}=-\frac{\pi}{4200} \mathrm{t}+6 \tag{3}
\end{equation*}
$$

(a) To find the required time for the tank to empty means $\mathrm{h}=0, \mathrm{t}=$ ?

Substitute $\mathrm{h}=0$ in 3rd equation, then we get

$$
\begin{aligned}
& 2 \sqrt{0}=-\frac{\pi}{4200} \mathrm{t}+6 \Rightarrow \mathrm{t}=8018.18 \text { Seconds=133.63minutes } \\
& \Rightarrow \mathrm{t}=2.2272 \mathrm{hrs}
\end{aligned}
$$

Required time to empty the tank is 2 hrs 14 min .
(b) To find the required time when water is half to the tank

$$
\begin{aligned}
& \text { Thus } \mathrm{h}=9 / 2=4.5, \mathrm{t}=\text { ? } \\
& \text { From the equation(3), we get } \\
& \Rightarrow 2 \sqrt{4.5}=-\frac{\pi}{4200} \mathrm{t}+6 \Rightarrow \mathrm{t}=2352 \mathrm{sec} \Rightarrow \mathrm{t}=39.2 \mathrm{~min}
\end{aligned}
$$

The required time for water is half to the tank is 39.2 min .
(c) To find the height of the water in the tank after 15 minutes

$$
\begin{aligned}
& \text { when } t=15 \mathrm{~min}, \mathrm{~h}=\text { ? } \\
& \mathrm{t}=15 \mathrm{~min} \times 60=900 \mathrm{~s}
\end{aligned}
$$

substituting the above values in the equation(3), we get

$$
2 \sqrt{\mathrm{~h}}=-\frac{\pi}{4200}(900)+6 \Rightarrow \mathrm{~h}=6.7 \mathrm{ft}
$$

The height of the water in the tank after 15 minutes is 6.7 ft .

## CONCLUSION:

In this study project we found solutions to many problems relating to various areas with the help of ordinary differential equations of first order. Many other problems relating to many fields will be solved by higher order differential equations and partial differential equations. Further we will study such type of differential equations and find solutions of many other relating problems.

## REFERENCES

1) Advanced Engineering Mathematics, Erwin Kreyszig, Blackwell Publishing.
2) Differential Equations and Their Applications, Zafar Ahsan, Prentice-Hall of India, 2005.
3) Differential Equations with Applications and Historical Notes, George F. Simmons, McGraw Hill Education.
4) Differential Equations with boundary value problems, Dennis G. Zill \& Michael R. Cullen, Thomson books.
5) Differential Equations text book for graduate students, Dr. Srinivas Vangala \& Madhu Rajesh, Spectrum University press.
6) Ordinary Differential Equations and Applications, W S Weiglhofer, K A Lindsay, Woodhead publishing India
