BEST PRACTICES

Mathematics

Dr BRR Govt Degree College,

Jadcherla, Mahabubnagr

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Analytical Solid Geometry

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1 Basics

1. The distance between two points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ is
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

2. The Point which divides the line segment joining the points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ in $m : n$ is
$$\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}, \frac{mz_2 + nz_1}{m + n}\right)$$

3. **Directional Cosine (dcs)**: Let α, β, γ be the angles which any line makes with positive direction of the Coordinate axes $X, Y and Z$ then $l = \cos\alpha, m = \cos\beta, n = \cos\gamma$ are direction cosines of the given line.

4. $l^2 + m^2 + n^2 = 1$

5. **Direction Ratios(drs)**: The numbers which are proportional to directional cosines are directional ratios.

6. The directional drs of line which passes through $(x_1, y_1, z_1), (x_2, y_2, z_2)$ is $x_2 - x_1, y_2 - y_1, z_2 - z_1$

1.1 Plane

7. The Plane:$ax + by + cz + d = 0$ represents the plane. Here $a, b, c \in \mathbf{R}$. Here a, b, c are drs to the normal of the plane.

8. **(Perpendicular distance)** The perpendicular distance form (x_1, y_1, z_1) to the plane $ax+by+cz+d = 0$ is
$$\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

9. The equation of the plane passing through (x_1, y_1, z_1) and having a, b, c as dr's of its perpendicular line is
$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

10. The equation of the plane passing through the points $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$ is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$

11. The equation of the plane having a, b, c as$x-$ intercept, $y-$ intercept and $z-$ intercept is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

12. **Angle between two planes**:If θ is an angle between the planes $a_1x + b_1y + c_1z + d_1 = 0, a_2x + b_2y + c_2z + d_2 = 0$ then
$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$
.
Here we get two angles θ and $\pi - \theta$
   (a) The above planes are parallel $\iff a_1 : b_1 : c_1 = a_2 : b_2 : c_2$
   (b) The above planes are perpendicular $\iff a_1a_2 + b_1b_2 + c_1c_2 = 0$
   (c) The above plane represents the same plane $\iff \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{d_1}{d_2}$

13. The equation to the plane parallel to $ax + by + cz + d = 0$ is $ax + by + cz + k = 0$, for k is some real number.

1.2 Line

14. Equation to the line passing through given point (x_1, y_1, z_1) and having directional cosines l, m, n is (Symmetrical form)

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

15. Equation to the line passing through $(x_1, y_1, z_1), (x_2, y_2, z_2)$ is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

16. Line can be describes as the locus of common points to any two planes as unsymmetrical form.
Line equation is $a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2 = 0$

2 The Sphere

17. A **Sphere** is the locus of a point which remains at a constant distance from a fixed point. Here Constant distance is radius and fixed point is centre.

18. The General equation to the sphere $S = 0$ is

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0 \tag{1}$$

(provided $u^2 + v^2 + w^2 - d \geq 0$

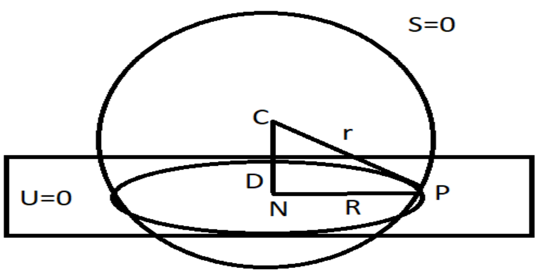
   (a) Centre $C(-u, -v, -w)$
   (b) Radius $r = \sqrt{U^2 + v^2 + w^2 - d}$

19. Equation to a sphere line joining $(x_1, y_1, z_1), (x_2, y_2, z_2)$ as a diameter is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + (z - z_1)(z - z_2) = 0$

20. Equation to a sphere passing through $(0, 0, 0), (a, 0, 0), (0, b, 0), (0, 0, c)$ is

$$x^2 + y^2 + z^2 - ax - by - cz = 0$$

21. Plane section of a sphere is a circle $S = 0; U = 0$.(From Figure r is radius of the sphere,R is radius of the circle,D is perpendicular distance from centre of sphere C to the plane $U = 0$ and N is the centre of Circle.)



22.

23. Equation to the tangent plane to the sphere $S = 0$ at the point (α, β, γ) is

$$\alpha x + \beta y + \gamma z + u(x + \alpha) + v(y + \beta) + w(z + \gamma) + d = 0$$

24. If a plane $U = 0$ is a tangent plane to the sphere $S = 0$ then radius of the sphere is equal to perpendicular distance from centre of the sphere to plane $U = 0$

25. The angle of intersection of two spheres at a common point is the angle between the tangent planes to them at that point.

26. The spheres are said to be **Orthogonal** if the angle of intersection of the spheres is a right angle. i.e $CC'^2 = CP^2 + C'P^2$

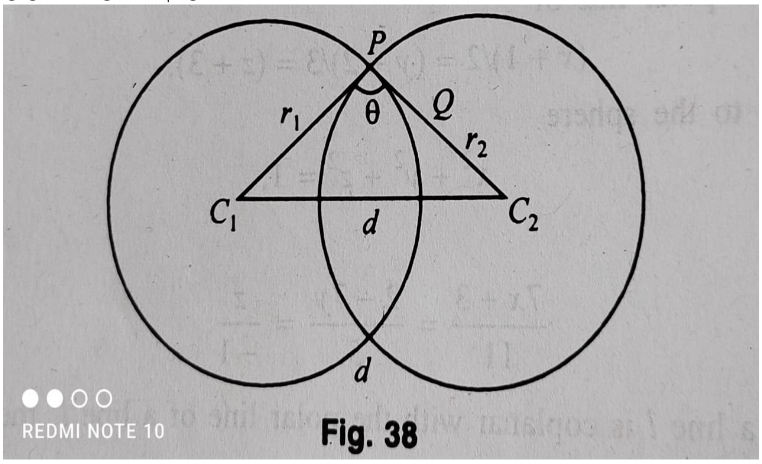


Fig. 38

27. Condition for the orthogonality of the spheres $x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ and $x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$ is

$$2(u_1u_2+v_1v_2+w_1w_2)=d_1+d_2$$

28. **Power of a point:** From a fixed point A, Chords can be drawn in any direction to a sphere in P and Q then $AP.AQ$ is called the power of point.
If $A(\alpha,\beta,\gamma)$ then Power of A to the sphere $S=0$ is

$$S_{11}=\alpha^2+\beta^2+\gamma^2+2u\alpha+2v\beta+2w\gamma+d$$

29. **Radical Plane:** The locus of a point with respect to two spheres are equal is a plane perpendicular to its centre is radical plane.
The Radical equation to the spheres $S_1=0, S_2=0$ is

$$S_1-S_2=0$$

30. **Radical Line**: The three radical planes of three spheres intersect in a line called radical line.
The Radical line of $S_1=0, S_2=0, S_3=0$ is $S_1-S_2=0, S_2-S_3=0$

31. **Radical Centre:** The four radical lines of four spheres taken three by three intersect at a point is called radical centre.
The radical centre we get by solving $S_1-S_2=0, S_2-S_3=0, S_1-S_3=0, S_2-S_4=0$

32. **Co-axal System:** A System of spheres any two members of which have the same radical plane is called a co-axal system of spheres.

33. **Limiting Points:** These are point spheres of co-axal system of spheres.

3 Cone

34. A **Cone** is a surface generated by by a straight line which passes through a fixed point and satisfies one more condition; for instance, it may intersect a given curve or touch a given surface.
The fixed point is called the **vertex** and the given curve the **Guiding Curve** of the cone.
An individual straight line on the surface of a cone is called its **generator**

35. **Enveloping cone:** The Cone formed by the tangent lines to a surface, drawn from a given point is called Enveloping cone of the surface with given point as its vertex.

36. **Enveloping Cone of a Sphere:** The Equation of the cone whose vertex is at the point (α,β,γ) and whose generators touch the sphere $x^2+y^2+z^2=a^2$ is $SS_1=T^2$

$$(x^2+y^2+z^2-a^2)(\alpha^2+\beta^2+\gamma^2-a^2)=(\alpha x+\beta y+\gamma z-a^2)^2$$

37. The general equation to a cone which passes through the three axes is

$$fyz + gzx + hxy = 0$$

38. **Condition for Tangency:** The condition that the plane $lx + my + nz = 0$ should touch the cone

$$ax^2 + by^2 + cz^2 + afyz + 2gzx + 2hxy = 0 \tag{2}$$

is

$$Al^2 + Bm^2 + Cn^2 + 2Fmn + 2Gnl + 2Hlm = 0 \tag{3}$$

Where A, B, C, E, F, G, H are cofactors of a, b, c, f, g, h respectively in the determinant

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

where

$$A = bc - f^2, B = ca - g^2, C = ab - h^2, F = gh - af, G = hf - bg, H = fg - ch$$

39. **Reciprocal Cones:** The equations (2), (3) such that each is the locus of the normals drawn through the origin to the tangent planes to the other and they are, called Reciprocal cones.

40. **The Right Circular Cone:** A Right circular cone is a surface generated by a line which passes through a fixed point and makes a constant angle with a fixed line through the fixed point.

41. **Equation of a right circular cone:**The Equation of the right circular cone whose vertex is athe point (α, β, γ) and whose axis is the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and semi vertical θ is

$$[l(x-\alpha) + m(y-\beta) + n(z-\gamma)]^2 = (l^2 + m^2 + n^2)[(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2]\cos^2\theta$$

(a) If the vertex $(\alpha, \beta, \gamma) = (0, 0, 0)$ then the right circular cone equation is

$$(lx + my + nz)^2 = (l^2 + m^2 + n^2)(x^2 + y^2 + z^2)\cos^2\theta$$

4 The Cylinder

42. **Cylinder:** A Cylinder is a surface generated by a straight line which is always parallel to a fixed line and subject to one more condition; it may intersect a given curve or touch given surface.

43. **Enveloping Cylinder:** Equation to the cylinder whose generators touch the sphere $x^2 + y^2 + z^2 = a^2$ and are parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ is

$$(lx + my + nz)^2 = (l^2 + m^2 + n^2)(x^2 + y^2 + z^2 - a^2)$$

44. **Right Circular Cylinder:** A Right circular cylinder is a surface generated by a line which intersects a fixed circle called the guiding circle, and is perpendicular to its plane.

45.

Algebra

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1 Basics

1. Natural Numbers $\mathbb{N} = \{1, 2, 3, \dots\}$
2. Integers $(\mathbb{Z}) = \{\dots, -2, -1, 0, 1, 2 \dots\}$
3. Rational Numbers$(\mathbb{Q}) = \{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\}$
   Example : $\frac{1}{2}, \frac{-3}{4}, 1.034376, 0.34343434.....$ (Terminating or Non-terminating but repeating)
4. Irrational Numbers$(\mathbb{R} - \mathbb{Q})$= A Real Number which is not a rational is called irrational.
   $\sqrt{2}, \pi, e, 3.4563452\dots$ (Non-Terminating and Non-repeating)
5. Real Numbers$(\mathbb{R}) = \mathbb{R} \cup (\mathbb{R} - \mathbb{Q})$
6. Complex Numbers $(\mathbb{C})$= $\{a + ib : a, b \in \mathbb{R}\}$

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

2 Unit-I

7. **Binary operation:** Let G be a set, A Binary operation on G is a function that assigns each order pair of elements of G an element of G.
   Addition on Natural number set, Multiplication on Non zero Rational numbers.
8. **Group**: G be a non empty set with binary operation $'.'$ then $(G, .)$ is a group if the following properties are satisfied.
   (a) **Closure Property:** $ab \in G, \forall a, b \in G$
   (b) **Associative Property:** $(ab)c = a(bc), \forall a, b, c \in G$
   (c) **Identity element:** There exist element $e \in G$ such that $ae = a = ea, \forall a \in G$
   Additive identity is 0 and multiplicative identity is 0
   (d) **Inverse property:** For every $a \in G$ there exist $b \in G$ such tat $ab = e = ba$. It is denoted by $b = a^{-1}$
   Additive inverse of a is $-a$ and multiplicative inverse of a is $\frac{1}{a}$
9. **Abelian Group:** $(G, .)$ be a group is said to be abelian group if $ab = ba, \forall a, b \in G$
   (a) $(\mathbb{Z}, +), (\mathbb{Q}, +), (\mathbb{R}, +), (\mathbb{C}, +)$ are additive abelian groups.
   (b) $(\mathbb{Q}^+, \times), (\mathbb{R}^+, \times)$, are Multiplicative abelian groups.
10. **Unit Group** $U(n)$:$\{a \in \mathbb{N} : (a, n) = 1, 1 \leq a \leq n\}$
    $(U(n), \overset{n}{\times})$ forms a multiplicative group module n
11. **Cancellation Laws:** In a group G, the right and left cancellation laws hold; that is,
    (a) $ba = ca \implies b = c$
    (b) $ab = ac \implies b = c \forall a, b, c \in G$.

12. **Order of a group:** The number of elements of a group is called its order. We will denote it by $|G|$ to denote the order of G

    (a) **Finite Group:** If the number of elements of a group are finite.
    $G = \{1, -1, i, -i\}$ with respect to multiplication is finite group of order $|G| = 4$

    (b) **Infinite Group:** If the number of elements of a group are infinite.
    $(\mathbb{Z}, +), (\mathbb{Q}, +), (\mathbb{R}, +), (\mathbb{C}, +)$ are infinite groups.

13. **Order of an element:** The order of an element g in a group G is the smallest positive integer n such that $g^n = e$.(In addition $ng = e$). Denoted by $|g| = n$.If no such integer exist then we say infinite order.

    (a) $U(15) = \{1, 2, 4, 7, 8, 11, 13, 14\}$ order of 7 is 4 as 4 is the least positive integer such that $7^4 = 1$ .Here 1 is identity

    (b) In the group $(\mathbb{Z}_{10}, \overset{+}{_{10}})$ order of 2 is 5 as $5.2 \equiv 0$. Here 0 is identity.

14. **Subgroup:** If a subset H of a group G is itself a group under the operation of G, we say H is a subgroup of G

    (a) $(\mathbb{Z}, +)$ is a sub group of $(\mathbb{Q}, +)$

    (b) $H = \{1, -1\}$ is a subgroup of $G = \{1, -1, i, -i\}$

15. **Set generated by an element:** $< a >= \{a^n : n \in \mathbb{Z}\}$

16. **Centre of a group:** The center, $Z(G)$, of a group G is the subset of elements in G that commute with every element of G.
    $Z(G) = \{a \in G : ax = xa \forall x \in G\}$

17. **Centralizer of a in G:** Let a fixed element of group G. The Centralizer of $a \in G$,is the set of all elements in G that commutes with a.
    Denoted by $C(a) = \{g \in G : ga = ag\}$

18. **Cyclic Group:** G be a cyclic group group if there is an element $a \in G$ such that $G =< a >= \{a^n : n \in G\}$.Here a is called generator of the group G

    (a) In $(\mathbb{Z}, +)$ is a cyclic group generated by 1.

    (b) $U(10) = \{1, 3, 5, 7\}$ is multiplication group mod 10 generated by $U(10) =< 7 >$ as $\{7^1 = 7, 7^2 \equiv 9, 7^3 \equiv 3, 7^4 \equiv 1\}$

19. **Fundamental Theorem of cyclic groups:** Every subgroup of a cyclic group is cyclic. Moreover, if $| < a > | = n$, then the order of any subgroup of $< a >$ is a divisor of n; for each positive divisor k of n, the group $< a >$ has exactly one subgroup of order k-namely,$< a^{\frac{n}{k}} >$

20. If $|a| = n$ then $|a^k| = \frac{n}{(n.K)}$. Here (n, k) is GCD of n and k.

21. The Number of generators of Cyclic group of order n is $\phi(n)$

3 UNIT-II

22. **Permutation**: A Permutation of a set A is a function from A to A that is both one-one and onto.

23. **Permutation group**: A Permutation group of a set A is a set of permutations of A that forms a group under function composition.
    S_3 forms a group on symbols $A = \{1, 2, 3\}$

24. **Even Permutation**: A Permutation that can be expressed as a product of an even number of 2-cycles is called even permutation.

25. **Odd Permutation**: A Permutation that can be expressed as a product of an odd number of 2-cycles is called even permutation.

26. **Isomorphism**: An Isomorphism $\phi : G \to \bar{G}$ is a function from group G to a group $\bar{G}$ which is

    (a) ϕ is Homomorphism $(\phi(xy) = \phi(x)\phi(y))$ for all $x, y \in G$.

(b) ϕ is one-one.

(c) ϕ is onto.

27. **Caley's Theorem**: Every group is isomorphic to a group of permutations.

28. **Automorphism**:An Isomorphism from a group G onto itself is called an automorphism of G.

29. **Coset**: G be a group and H be a subgroup of G and for every $a \in G$then

    (a) $aH = \{ah/h \in H\}$ is left coset of H in G.

    (b) $Ha = \{ha/h \in H\}$ is right coset of H in G.

30. **Lagrange's Theorem**: If G is a finite group and H is a subgroup of G, then $|H|$ divides $|G|$, Moreover, the number of distinct left(right) cosets of H in G is $\frac{|G|}{|H|}$

4 Unit-III

31. **Normal Subgroup**: A Subgroup of H of a group G is called a normal subgroup of G if $aH = Ha$ for all a in G.
    (or) A subgroup H of G is normal subgroup in G iff $xHx^{-1} \subseteq H$.

32. **Factor Group**: Let G be a group and H a normal subgroup of G. The set of $\frac{G}{H} = \{aH/a \in G\}$ is a group under the operation $(aH)(bH) = abH$

33. **Kernal of Hkomomorphism**: The Kernal of a homomorphism ϕ from a group G to a group $\bar{G}$ is the set $Ker\phi = \{x \in G/\phi(x) = \bar{e}\}$ where $\bar{e}$ is identity in $\bar{G}$

34. **First Isomorphism Theorem**: Let ϕ be a group homomorphism from G to $\bar{G}$. Then the mapping from $\frac{G}{Ker\phi} \to \phi(G)$ given by $\phi(\text{gKer } \phi) = \phi(g)$ is an isomorphism, In Symbols $\frac{G}{Ker\phi} \approx \phi(G)$

35. **Rings**: A Ring $(R, +, .)$ is said to be ring if

    (a) $(R, +)$ is an abelian group.

    (b) $(R, .)$ is a semi group.(Closure and associative)

    (c) i. Left distributive law :$a(b + c) = ab + ac$.
        ii. Right distributive law :$(a + b)c = ab + ac$ for all $a, b, c \in R$

36. **Commutative Ring**: $(R, +, .)$ is said to be commutative ring if $ab = ba$ for all $a, b \in R$ (commutative on second operation)

37. **Subring**: A subset S of a ring R is a subring of R if S itself a ring with the operations of R

38. **Zero Divisors**: An nonzero element a in a commutative ring R is called a zero divisor if there is a nonzero element b in R such that $ab = 0$

39. **Integral Domain**: A commutative ring with unity is said to be an integral domain if it has no-zero divisors.

40. **Unit**: A nonzero element in Ring R is unit if it has inverse on second operation.

41. **Field**: A commutative ring with unity is called a field if for every non zero element is a unit.

42. **Characteristic of a Ring**: The Characteristic of a ring R is the least positive integer n such that $nx = 0$ for all $x \in R$. If no such integer exists, we say that R has characteristic 0.It is denoted by $CharR$

5 Unit-IV

43. **Ideal**: A Subring A of a ring R is called (two-sided) ideal of R if it for every $r \in R$ and for every $a \in A$ both $ra, ar \in A$

44. **Factor Ring**: Let R be a ring and A be a subring of R. The set of cosets $\{r + A/r \in R\}$ is a ring under the operations $(s + A) + (t + A) = s + t + A$ and $(s + A)(t + A) = st + A$ if and only if A is an ideal of R.

45. **Prime Ideal**: A Proper ideal A of a commutative ring R is said be a prime ideal of R if $a, b \in R$ and $ab \in A$ implies $a \in A$ or $b \in A$.

46. **Maximal Ideal**:A Proper ideal A of a commutative ring R is said be a Maximal ideal of R if, whenever B is an ideal of R and $A \subseteq B \subseteq B$ then $A = B$ or $B = R$

Differential Equations

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Exact &Non Exact D.E

1. $M(x, y)dx + N(x, y)dy = 0$ is said to be Exact if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
   General Solution for Exact D.E is $\int^x Mdx + \int N(\text{terms not containg x})dy = 0$

2. $M(x, y)dx + N(x, y)dy = 0$ is said to be non Exact if $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$. If it is non exact D.E we have to multiply Integrating Factor(I.F) to make it as an Exact D.E.

3. If a non Exact D.E is in the form of $M(x, y)dx + N(x, y)dy = 0$ and homogeneous (sum of powers in each term are equal) then I.F $= \dfrac{1}{Mx + Ny}$.

4. If a non exact D.E is in the form of $yf(xy)dx + xg(xy)dy = 0$ then I.F is $\dfrac{1}{Mx - Ny}$.

5. If a non Exact D.E is in the form of $M(x, y)dx + N(x, y)dy = 0$ and $\dfrac{1}{N}\left(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right) = f(x)$ or k(constant) then I.F $=e^{\int f(x)dx}$ or $e^{\int kdx}$

6. If a non Exact D.E is in the form of $M(x, y)dx + N(x, y)dy = 0$ and $\dfrac{1}{M}\left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right) = g(y)$ or k(constant) then I.F $=e^{\int g(y)dy}$ or $e^{\int kdy}$

7. Linear D.E in y : $\frac{\mathrm{d}y}{\mathrm{d}x} + P(x)y = Q(x)$ then I.F $= e^{\int P(x)dx}$ and General Solution is $y.I.F = \int Q(x).I.Fdx$. Where $P(x)$ and $Q(x)$ are differentiable functions of x.

8. Linear D.E in x : $\frac{\mathrm{d}x}{\mathrm{d}y} + P_1(y)x = Q_1(y)$ then I.F $= e^{\int P_1(y)dy}$ and General Solution is $x.I.F = \int Q_1(y).I.Fdy$. Where $P_1(y)$ and $Q_1(y)$ are differentiable functions of y.

9. Bernouli's equation of the form $\dfrac{\mathrm{d}y}{\mathrm{d}x} + P(x)y = Q(x)y^n$, n is any real number. We have to solve it by dividing y^n and convert it into linear differential equation.

Finding General Solution to $f(D)y = 0$, where $D = \frac{\mathrm{d}}{\mathrm{d}x}$

1. If $m_1, m_2 (m_1 \neq m_2)$ are two real roots of Auxiliary equation $f(m) = 0$ then $y = c_1e^{m_1x} + c_2e^{m_2x}$ is solution for $f(D)y = 0$.

2. If m, m are two real roots of Auxiliary equation $f(m) = 0$ then $y = (c_1 + c_2x)e^{mx}$ is solution for $f(D)y = 0$.

3. If $m_1 \pm m_2i$ are two complex roots of Auxiliary equation $f(m) = 0$ then $y = e^{m_1x}(c_1 \cos m_2x + c_2 \sin m_2x)$ is solution for $f(D)y = 0$.

4. If $1, 2, 2, -3\pm4i$ are roots of auxiliary equation $f(m) = 0$ then $y_c = c_1e^x + (c_2 + c_3x)e^{2x} + e^{-3x}(c_4 \cos 4x + c_5 \sin 4x)$ is solution for $f(D)y = 0$

Finding Particular Integral (y_p)

1. $y_p = \frac{1}{f(D)}e^{ax} = \frac{e^{ax}}{f(a)}$ when $f(a) \neq 0$.
2. $y_p = \frac{1}{(D-a)^k}e^{ax} = \frac{x^k}{k!}e^{ax}$.
3. $y_p = \frac{1}{f(D^2)}\sin bx = \frac{1}{f(-b^2)}\sin bx$ when$f(-b^2) \neq 0$.
4. $y_p = \frac{1}{f(D^2)}\cos bx = \frac{1}{f(-b^2)}\cos bx$ when$f(-b^2) \neq 0$
5. $y_p = \frac{1}{D^2+b^2}\sin bx = \frac{-x}{2b}\cos bx$
6. $y_p = \frac{1}{D^2+b^2}\cos bx = \frac{x}{2b}\sin bx$
7. $(1-D)^{-1} = 1 + D + D^2 + D^3 + D^4 + \cdots$
8. $(1+D)^{-1} = 1 - D + D^2 - D^3 + D^4 + \cdots$
9. $(1-D)^{-2} = 1 + 2D + 3D^2 + 4D^3 + 5D^4 + \cdots$
10. $(1+D)^{-2} = 1 - 2D + 3D^2 - 4D^3 + 5D^4 + \cdots$
11. $y_p = \frac{1}{1+f(D)}x^m = (1+f(D))^{-1}x^m$
12. $y_p = \frac{1}{f(D)}e^{ax}V = e^{ax}\frac{1}{f(D+a)}V$
13. $y_p = \frac{1}{f(D)}xV = x\frac{1}{f(D)}V - \frac{f^1(D)}{(f(D)^2}V$
14. $y_p = \frac{1}{f(D)}x^m \sin x$ =Imaginary part of $\frac{1}{f(D)}e^{ix}x^m$
15. $y_p = \frac{1}{f(D)}x^m \cos x$ =Real part of $\frac{1}{f(D)}e^{ix}x^m$
16. In variation of parameters $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + P\frac{\mathrm{d}y}{\mathrm{d}x} + Qy = R$ if $y_c = c_1u + c_2v$ then $y_p = Au + Bv$ is general solution where $A = -\int \frac{vR\mathrm{dx}}{u\frac{\mathrm{d}v}{\mathrm{d}x} - v\frac{\mathrm{d}u}{\mathrm{d}x}}$ and $B = \int \frac{uR\mathrm{dx}}{u\frac{\mathrm{d}v}{\mathrm{d}x} - v\frac{\mathrm{d}u}{\mathrm{d}x}}$ and $y = y_c + y_p$
17. Cauchy Euler Equation: $x^n\frac{\mathrm{d}^ny}{\mathrm{d}x^n} + P_1x^{n-1}\frac{\mathrm{d}^{n-1}y}{\mathrm{d}x^{n-1}} + ... + P_ny = Q$ where $P_1, P_2...P_n$ real constants and Q is function of x. It can be solved by $x = e^z$.i.e $xD = \theta, x^2D^2 = \theta(\theta-1), x^3D^3 = \theta(\theta-1)(\theta-2)$
18. Orthogonal Trajectories: let $f(x,y,c) = 0$ be the equation of given family of curves. Then differentiate w.r.t x and eliminate parameter c, replace $\frac{\mathrm{d}y}{\mathrm{d}x}$ by $-\frac{\mathrm{d}x}{\mathrm{d}y}$ then solve.
19. let $f(r,\theta,c) = 0$ be the equation of given family of curves. Then differentiate w.r.t θ and eliminate parameter c, replace $\frac{\mathrm{d}r}{\mathrm{d}\theta}$ by $-r^2\frac{\mathrm{d}\theta}{\mathrm{d}r}$ then solve.
20. Clairaut's equation: It is in the form of $y = xp + f(p)$. General solution to this equation is $y = xc + f(c)$.

Linear Algebra Formula Sheet

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1 Unit-I

1. A **Vector Space** is a non empty set V called vectors on which we define vector addition and multiplication by scalars subject to 10 axioms . For all $\bar{u}, \bar{v}, \bar{w} \in V$ and for all $c, d \in \mathbb{R}$ (Scalars)

   1-5. $(V.+)$ is an Abelian group.
   6. $c\bar{u} \in V, \forall c \in \mathbb{R}$
   7. $c(\bar{u} + \bar{v}) = c\bar{u} + c\bar{v}$
   8. $(c + d)\bar{u} = c\bar{u} + d\bar{u}$
   9. $c(d\bar{u}) = (cd)\bar{u}$
   10. $1\bar{u} = \bar{u}$

2. A **Sub Space** of a vector space V of a subset H of V that has three properties
   (a) Zero vector in H. that is $\bar{0} \in H$.
   (b) $\bar{u} + \bar{v} \in H$ for every $\bar{u}, \bar{v} \in H$
   (c) $c\bar{u} \in H$ for every $c \in \mathbb{R}, \bar{u} \in H$

3. **Linear Combination** of $\bar{v}_1, \bar{v}_2 ... \bar{v}_n$ of a vector space is $c_1\bar{v}_1 + c_2\bar{v}_2 + ... c_n\bar{v}_n$ for $c_1, c_2 ... c_n \in \mathbb{R}$

4. **Span** of vectors $\bar{v}_1, \bar{v}_2 ... \bar{v}_n$ of a vector space is set of all linear combinations of vectors.
   Span$\{\bar{v}_1, \bar{v}_2 ... \bar{v}_n\} = \{c_1\bar{v}_1 + c_2\bar{v}_2 + ... c_n\bar{v}_n : \forall c_1, c_2 ... c_n \in \mathbb{R}\}$

5. The **Null space** of a an $m \times n$ matrix A, written as **Null**A, is the set of all solution to the homogeneous equation $A\bar{x} = \bar{0}$

$$NullA = \{\bar{x} : \bar{x} \in \mathbb{R}^n \quad \text{and} \quad A\bar{x} = \bar{0}\}$$

6. The **Column Space** of an $m \times n$ matrix A, written as **Col**A, is the set of all linear combinations of the columns of A. If $A = [a_1, ... a_n]$ (Columns of A) then

$$ColA = \{\bar{b} : \bar{b} = A\bar{x} \quad \text{for some } \bar{x} \text{ in} \mathbb{R}^n\}$$

7. A **Linear Transformation** T from a vector space V into a vector space W such that
   (a) $T(\bar{u} + \bar{v}) = T(\bar{u}) + T(\bar{v})$ for all $\bar{u}, \bar{v} \in V$
   (b) $T(c\bar{u}) = cT(\bar{u})$ for all $c \in \mathbb{R}, \bar{u} \in \mathbb{R}$
   (c) An Isomorphism T from a vector space V into a vector space W if
      i. T is a linear transform.
      ii. T is one-one(Injective)
      iii. T is onto (Surjective)

8. An indexed set of vectors $\{\bar{v}_1, \bar{v}_2 ... \bar{v}_p\}$ in V is said to be **Linearly Independent** if the equation

$$c_1\bar{v}_1 + c_2\bar{v}_2 + ... c_p\bar{v}_p = \bar{0}$$

has only the trivial solution $c_1 = c_2 = ... = c_p = 0$

9. An indexed set of vectors $\{\bar{v}_1, \bar{v}_2 ... \bar{v}_p\}$ in V is said to be **Linearly Dependent** if the equation

$$c_1\bar{v}_1 + c_2\bar{v}_2 + ...c_p\bar{v}_p = \bar{0}$$

then at least one of $c_1, c_2...c_p$ is non zero.

10. A Subset $H = \{(a, b, c), (d, e, f), (g, h, i)\}$ of $\mathbb{R}^3$ and $D = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ then

    (a) H is Linearly independent iff $D \neq 0$

    (b) H is Linearly Dependent iff $D = 0$

11. A non empty set $\beta = \{\bar{b}_1, \bar{b}_2, ...\bar{b}_n\}$ is a **Basis** of a vector space V if

    (a) β is a linearly independent.

    (b) β spans V.$\{\text{span}\beta = V\}$.

12. Suppose $\beta = \{\bar{b}_1, \bar{b}_2, ...\bar{b}_n\}$ is a basis for V and x is in V. The **Coordinates of** x relative to the basis β or β coordinates of x are $[x]_\beta = c_1, c_2...c_n \in \mathbb{R}$ such that

$$x = c_1\bar{b}_1 + c_2\bar{b}_2...c_n\bar{b}_n$$

13. If V is spanned by a finite set, then V is said to be **Finite Dimensional**, and the **Dimensional** of V, written as **dim**V is the number of vectors in a basis for V.If V is not spanned by a finite set then V is said to be **infinite dimensional**

14. The dimension of Null$A$ is the number of free variables in the equation $A\bar{x} = \bar{0}$ and dimension of Col$A$ is the number of pivot columns in A (Row reduced form)

2 Unit-II

15. The set of all linear combinations of an $m \times n$ matrix A is the set of all linear combination of the row vectors is called the **Row Space** of A

16. If two matrices A and B are **row equivalent** then their row spaces are the same.

17. The **rank** of A is the dimension of the column space of A.

18. The **Rank Theorem**: The dimensions of the column space and the row space of an $m \times n$ matrix A are equal. This common dimension, the rank of A, also equals the number of pivot positions in A

$$rankA + dimNulA = n$$

19. An **Eigen vector** of an $n \times n$ matrix A is a nonzero $\bar{x}$ such that $A\bar{x} = \lambda\bar{x}$ for some scalar λ. A scalar λ is called an **Eigen value** of A if there is a nontrivial solutions of $\bar{x}$ of $A\bar{x} = \lambda\bar{x}$ such an $\bar{x}$ is called an **Eigen vector corresponding** to λ.

20. A scalar λ is an eigenvalue of an $n \times n$ matrix A if and only if λ satisfies the **Characteristic equation**

$$det(A - \lambda I) = 0$$

21. If A and B are $n \times n$ matrices , then A **is similar to** B if there is an invertible matrix P such that

$$P^{-1}AP = B$$

3 Unit-III

22. Square matrix A is said to be **diagonalizable** if A is similar to a diagonal matrix, that is, if $P^{-1}DP = B$ for some invertible matrix P and some diagonal matrix D.

4 Unit-IV

23. Let $\bar{u}, \bar{v}$ are two $n \times 1$ matrices in $\mathbb{R}^n$ then **Inner product** is defined as

$$\bar{u}.\bar{v} = \bar{u}^T\bar{v} = u_1v_1 + u_2v_2 + ...u_nv_n$$

where $\bar{u} = \begin{bmatrix} u_1 \\ u_2 \\ . \\ . \\ u_n \end{bmatrix}$ and $\bar{v} = \begin{bmatrix} v_1 \\ v_2 \\ . \\ . \\ v_n \end{bmatrix}$

24. $\bar{u}, \bar{v}, \bar{w}$ are vectors in $\mathbb{R}^n$ and c is a scalar then

(a) $\bar{u}.\bar{v} = \bar{v}.\bar{u}$

(b) $(\bar{u} + \bar{v}).\bar{w} = \bar{u}.\bar{w} + \bar{v}.\bar{w}$

(c) $(c\bar{u}).\bar{v} = c(\bar{u}.\bar{v}) = \bar{u}.(c\bar{v})$

(d) $\bar{u}.\bar{u} \geq 0$ and $\bar{u}.\bar{u} = 0$ if and only if $\bar{u} = 0$

25. The **Length (Norm)** of $\bar{v}$ is the non negative scalar $||\bar{v}||$ defined by

$$||\bar{v}|| = \sqrt{\bar{v}.\bar{v}} = \sqrt{\bar{v_1}^2 + \bar{v_2}^2 ... + \bar{v_n}^2} \quad \text{and} \quad ||\bar{v}||^2 = \bar{v}.\bar{v}$$

26. For $\bar{u}$ and $\bar{v}$ in $\mathbb{R}^n$ the Distance between $\bar{u}$ and $\bar{v}$, written as dist$(\bar{u}, \bar{v})$ is the length of the vector $\bar{u} - \bar{v}$. that is

$$dist(\bar{u}, \bar{v}) = ||\bar{u} - \bar{v}||$$

27. Two vectors $\bar{u}$ and $\bar{v}$ in $\mathbb{R}^n$ are **Orthogonal** (to each other) if $\bar{u}.\bar{v} = 0$. If every distinct vectors are orthogonal then the set is orthogonal set.

28. **The Pythagorean Theorem** Two vectors $\bar{u}$ and $\bar{v}$ are orthogonal if and only if $||\bar{u}+\bar{v}||^2 = ||\bar{u}||^2 + ||\bar{v}||^2$

29. If $\bar{u}$ and $\bar{v}$ are non zero vectors in $\mathbb{R}^2, \mathbb{R}^3$ and the **Angle between** them is θ then

$$\bar{u}.\bar{v} = ||u||||v|| \cos\theta$$

30. A Set $\{\bar{u_1}, \bar{u_2} ... \bar{u_p}\}$ is an **Orthonormal Set** if it is orthogonal set of unit vectors that is for every distinct vectors $\bar{u_i}, \bar{u_j}$

$$\bar{u_i}.\bar{u_j} = \bar{o}, i \neq j \quad \text{and} \quad ||\bar{u_i}|| = 1 \quad \forall i$$

31. The **Gram Schmidt process** is used to construct orthogonal and orthonormal set of $\mathbb{R}^n$.
Given a basis $\{\bar{x_1}, \bar{x_2} ... \bar{x_p}\}$ for a subspace W of $\mathbb{R}^n$. Define

$$\begin{aligned}
\bar{v_1} &= \bar{x_1} \\
\bar{v_2} &= \bar{x_2} - \frac{\bar{x_2}.\bar{v_1}}{\bar{v_1}.\bar{v_1}}\bar{v_1} \\
\bar{v_3} &= \bar{x_3} - \frac{\bar{x_3}.\bar{v_1}}{\bar{v_1}.\bar{v_1}}\bar{v_1} - \frac{\bar{x_3}.\bar{v_2}}{\bar{v_2}.\bar{v_2}}\bar{v_2} \\
\bar{v_p} &= \bar{x_p} - \frac{\bar{x_p}.\bar{v_1}}{\bar{v_1}.\bar{v_1}}\bar{v_1} - \frac{\bar{x_p}.\bar{v_2}}{\bar{v_2}.\bar{v_2}}\bar{v_2} ... - \frac{\bar{x_p}.\bar{v}_{p-1}}{\bar{v}_{p-1}.\bar{v}_{p-1}}\bar{v}_{p-1}
\end{aligned}$$

Then $\{\bar{v_1}, \bar{v_2} ... \bar{v_p}\}$ is an orthogonal basis for W. In addition

$$span\{\bar{v_1}, \bar{v_2} ... \bar{v_p}\} = span\{\bar{x_1}, \bar{x_2} ... \bar{x_p}\}$$

Reference: Linear Algebra and its applications,Third edition by David C.Lay
You can download this file by blog
https://0953176gdc.blogspot.com/2023/09/linear-algebra-formulae-sheet.html

Numerical Analysis-Formulas

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Bisection Method

For a equation $f(x) = 0$ on $[a, b]$ as $f(a), f(b)$ have opposite signs then $x_1 = \frac{a+b}{2}$
If $f(a), f(x_1)$ have opposite signs then next approximation $x_2 = \frac{a+x_1}{2}$
If $f(x_1), f(b)$ have opposite signs then next approximation $x_2 = \frac{x_1+b}{2}$
and so on

We have to find number of iterations in this method are $n > \frac{ln\frac{b-a}{\epsilon}}{ln2}$

Error

Maximum error after n iterations in bisection method is $|\frac{b-a}{2^n}|$

Fixed point method

A number α is called fixed point if $g(\alpha) = \alpha$
If $g \in [a, b]$ and $g(x) \in [a, b]$ then g has a fixed point in $[a, b]$
If $|g'(x)| \leq K, 0 < k < 1$ then g has a unique fixed point in $[a, b]$

Iteration formula is
$p_n = g(p_{n-1}), n \geq 1$ and p_0 is initial approximation.

Newton Formula

If $f(x) = 0$ is given then its root is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, n \geq 0$$

Secant method

If the given function $y = f(x) = 0$ and $f(a), f(b)$ have opposite signs and $x_0 = a, x_1 = b$ then

$$x_{n+1} = \frac{x_{n-1}y_n - x_n y_{n-1}}{y_n - y_{n-1}}$$

or

$$x_{n+1} = \frac{x_{n-1}f_n - x_n f_{n-1}}{f_n - f_{n-1}}$$

Modified Newton Method

If $f(x) = 0$ is given function then its root is

$$x_{n+1} = \frac{f(x_n)f'(x_n)}{(f'(x_n))^2 - f(x_n)f''(x_n)}, n \geq 0$$

Lagrange interpolating polynomial of at most 1 degree

If $y = f(x)$ is given function then polynomial
$p(x) = l_0(x)y_0 + l_1(x)y_1$ where $y_0 = f(x_0), y_1 = f(x_1)$
$l_0(x) = \frac{(x-x_1)}{(x_0-x_1)}, l_1(x) = \frac{(x-x_0)}{(x_1-x_0)}$

Lagrange interpolating polynomial of at most 2 degree

If $y = f(x)$ is given function then polynomial
$p(x) = l_0(x)y_0 + l_1(x)y_1 + l_2(x)$ where
$y_0 = f(x_0), y_1 = f(x_1), y_2 = f(x_2)$

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}, l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

Divided difference

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$f[x_{n-1}, x_n] = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

Frame Title

Differential and Integral calculus(Semester-I)

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1 Basics

1.

$\sin 2x = 2\sin x\cos x$	$\cos 2x = \cos^2 x - \sin^2 x$	$\cos 2x = 2\cos^2 x - 1$	$\cos 2x = 1 - 2\sin^2 x$
$\sin 3x = 3\sin x - 4\sin^3 x$	$\cos 3x = 4\cos^3 x - 3\cos x$	$\sin n\pi = 0, \forall n \in \mathbb{Z}$	$\cos n\pi = (-1)^n, \forall n \in \mathbb{Z}$

2.

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$	$2\sin A\sin B = \cos(A-B) - \cos(A+B)$
$2\cos A\sin B = \sin(A+B) - \sin(A-B)$	$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

3.

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$	$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$
$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$	$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

4.

FUNCTION	DERIVATIVE
Constant	0
x	1
x^n	nx^{n-1}
e^x	e^x
a^x	$a^x \log a$
$\log x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-cosec^2 x$
$\sec x$	$\sec x \tan x$
$cosecx$	$-cosecx \cot x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
fg	$fg' + f'g$
$\frac{f}{g}$	$\frac{f'g - fg'}{g^2}$
$f(g(x))$	$f'(g(x))g'(x)$

Table 1: Derivative Table

FUNCTION	INTEGRATION
dx	$x + c$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1} + c$
e^x	$e^x + c$
$\frac{1}{x}$	$\log\|x\| + c$
a^x	$\frac{a^x}{\log a} + c$
$\sin x$	$-\cos x + c$
$\cos x$	$\sin x + c$
$\tan x$	$\log\|\sec x\| + c$
$\cot x$	$\log\|\sin x\| + c$
$\sec x$	$\log\|\sec x + \tan x\| + c$
$cosecx$	$\log\|cosecx - \cot x\| + c$
$\sec^2 x$	$\tan x + c$
$cosec^2 x$	$-\cot x + c$
$\sec x \tan x$	$\sec x + c$
$cosecx \cot x$	$-cosecx + c$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + c$
$-\frac{1}{\sqrt{1-x^2}}$	$\cos^{-1} x + c$
$\frac{1}{1+x^2}$	$\tan^{-1} x$ +c
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\log\frac{x-a}{x+a} + c, \|x\| > a > 0$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\log\frac{a+x}{a-x} + c, \|x\| < a$
fg	$f\int g - f'\int g$

Table 2: Integration Table

5. $\int fg = fg_1 - f'g_{11} + f''g_{111} - ...$
   Here f' means derivative of f and g_1 means integration of g

6. **Indeterminate forms**: $\frac{0}{0}, \frac{\infty}{\infty}, 0.\infty, \infty - \infty, 0^0, 1^\infty, \infty^0$ are called Indeterminate forms.

7. **L'Hospital Rule:**If $\lim_{x\to 0}\frac{f(x)}{g(x)}$ is in Indeterminate form then $\lim_{x\to 0}\frac{f(x)}{g(x)} = lim_{x\to 0}\frac{f'(x)}{g'(x)}$

2 UNIT-I

1. If $f(x,y)$ is function of two variables then
   (a) $\frac{\partial f}{\partial x} = f_x = \lim_{h\to o} \frac{f(x+h,y)-f(x,y)}{h}$
   (b) $\frac{\partial f}{\partial y} = f_y = \lim_{k\to o} \frac{f(x,y+k)-f(x,y)}{h}$
2. **Homogeneous Functions**: $f(x,y) = a_0x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + ...a_ny^n$ is homogeneous since degree of each of the term is same (n)
   $f(x,y) = x^3 + x^2y - 4y^3$ of degree 3
3. $f(x,y)$ is homogeneous function of degree n if $f(x,y) = x^n f(\frac{y}{x})$
   $f(x,y) = \frac{\sqrt{y}+\sqrt{x}}{y+x} = x^{\frac{-1}{2}} \left(\frac{1+\sqrt{\frac{y}{x}}}{1+\frac{y}{x}}\right)$ so the degree is $\frac{-1}{2}$
4. **Euler's Theorem on Homogeneous Function**: If $z = f(x,y)$ be a Homogeneous function of x,y of degree n then
   $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$ for all x,y in the domain of the function.
5. If $z = f(x,y)$ is homogeneous function of x,y of degree n, then $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$

3 UNIT-II

6. If $z = f(x,y)$ and $x = \phi(t), y = \psi(t)$ then $\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}.\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}.\frac{\mathrm{d}y}{\mathrm{d}t}$
7. If $z = f(x,y)$ and $x = \phi(u,v), y = \psi(u,v)$ then $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}.\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}.\frac{\partial y}{\partial u}$
   and $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}.\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}.\frac{\partial y}{\partial v}$
8. **Implicit Functions**: If we obtain y as a function of x from $f(x,y) = 0$ then y is implicit function of x and $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-f_x}{f_y}$ if $f_y \neq 0$
   $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\left(\frac{f_{x^2}(f_y)^2 - 2f_{yx}(f_x)(f_y) + f_{y^2}(f_x)^2}{(f_y)^3}\right)$ if $f_y \neq 0$, here $f_{x^2} = f_{xx}, f_{y^2} = f_{yy}$
9. **Lagranges Mean Value Theorem**:If a function f is continuous on $[a, a+h]$ and derivable on $(a, a+h)$ then there exist at least one number $\theta \in (0,1)$ such that $f'(a+\theta h) = \frac{f(a+h)-f(a)}{h}$
10. Generally $f_{xy}(a,b) \neq f_{yx}(a,b)$
11. **Equality of f_{xy} and f_{yx}** : If $f(x,y)$ possesses continuous second order partial derivatives then $f_{xy} = f_{yx}$
12. **Taylors Theorem for a function of two variables**: If f possesses continuous partial derivatives of the third order in a neighbourhood of a point (a,b) and if $(a+h, b+k)$ be a point of this neighbourhood,then there exist $\theta, (0 < \theta < 1)$ such that

$$f(a+h,b+k) = f(a,b) + [hf_x(a,b) + kf_y(a,b)] + \frac{1}{2!}[h^2f_{x^2}(a,b) + 2hkf_{xy}(a,b) + k^2f_{y^2}(a,b)]$$
$$+\frac{1}{3!}[h^3f_{x^3}(u,v) + 3h^2kf_{x^2y}(u,v) + 3hk^2f_{xy^2}(u,v) + k^3f_{y^3}(u,v)]$$

   where $u = a+\theta h, v = b+\theta k$
13. **Another Form**:

$$f(x,y) = f(a,b) + \left[(x-a)\frac{\partial}{\partial x} + (y-b)\frac{\partial}{\partial y}\right] f(a,b) + \frac{1}{2!}\left[(x-a)\frac{\partial}{\partial x} + (y-b)\frac{\partial}{\partial y}\right]^2 f(a,b)$$
$$+\frac{1}{3!}\left[(x-a)\frac{\partial}{\partial x} + (y-b)\frac{\partial}{\partial y}\right]^3 f(a,b) + ... + R_n$$

   where $R_n = \frac{1}{n!}\left[(x-a)\frac{\partial}{\partial x} + (y-b)\frac{\partial}{\partial y}\right]^n f(a+(x-a)\theta, b+(y-b)\theta), 0 < \theta < 1$ called the the Taylor's expansion of $f(x,y)$ about the point (a,b) in powers $(x-a)$ and $(y-b)$

14. A point (a, b) is called a **Stationary point** if $f_x(a, b) = 0, f_y(a, b) = 0$.

15. A Stationary point, if it is a maximum or a minimum is known as an **Extreme point** and the values of the function at an extreme point are called **Extreme Values.**

16. **Lagrange's Condition for maxima and minima**: If u is the given function of two variablesx, y and $r = \frac{\partial^2 u}{\partial x^2}, t = \frac{\partial^2 u}{\partial v^2}, s = \frac{\partial^2 u}{\partial x \partial y}$ then Find the (a, b) where $f_x(a, b) = 0, f_y(a, b) = 0$.

17. **Lagrange's Condition for minimum is**

$$rt - s^2 > 0 \quad \text{and} \quad r > 0 \quad \text{at} \quad (a, b)$$

18. **Lagrange's Condition for Maximum is**

$$rt - s^2 > 0 \quad \text{and} \quad r < 0 \quad \text{at} \quad (a, b)$$

If $rt - s^2 < 0$ then there is neither maximum nor minimum.

4 Unit-III

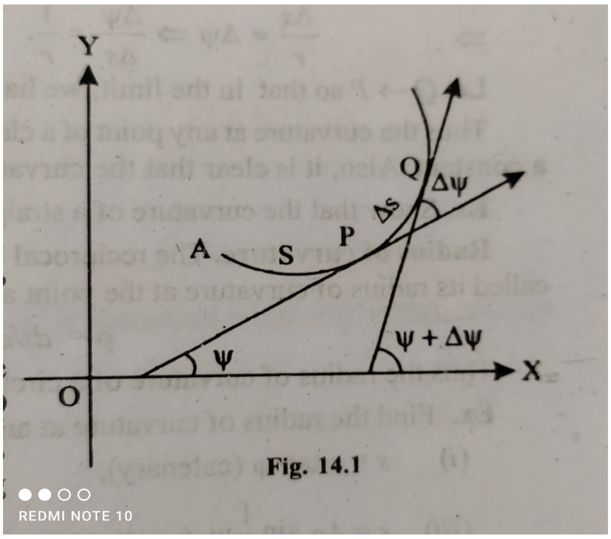


Fig. 14.1

19. The symbol $\Delta\Psi$ denotes the angles through which turns as a point moves along the curve from P to Q

   (a) **Total bending or total curvature**: $\Delta\Psi$
   (b) **Average curvature of the arc PQ**$=\frac{\Delta\Psi}{\Delta s}$
   (c) **The curvature at P**$=\lim_{Q \to P} \frac{\Delta\Psi}{\Delta s} = \frac{d\Psi}{ds}$
   (d) **The radius of curvature**(ρ): $\rho = \frac{ds}{d\Psi}$

20. **Derivative of arc**

   (a) **Cartesian Form**:If $y = f(x)$ then $\frac{ds}{dx} = \sqrt{\left(1 + (\frac{dy}{dx})^2\right)}$
   (b) **Parametric Form**: If $x = x(t), y = y(t)$ then $(\frac{ds}{dt})^2 = (\frac{dx}{dt})^2 + (\frac{dy}{dt})^2$
   (c) **Polar Form**:If $r = f(\theta)$ then $\frac{ds}{d\theta} = \sqrt{(1 + (\frac{dr}{d\theta})^2)}$

21. **Radius of Curvature**

   (a) **Cartesian Form**:If $y = f(x)$ then $\rho = \frac{(1+(y_1)^2)^{\frac{3}{2}}}{y_2}$ where $y_1 = \frac{dy}{dx}, y_2 = \frac{d^2y}{dx^2}$
   (b) **Parametric Form**: If $x = x(t), y = y(t)$ then $\rho = \frac{(x'^2+y'^2)^{\frac{3}{2}}}{x'y''-y'x''}$

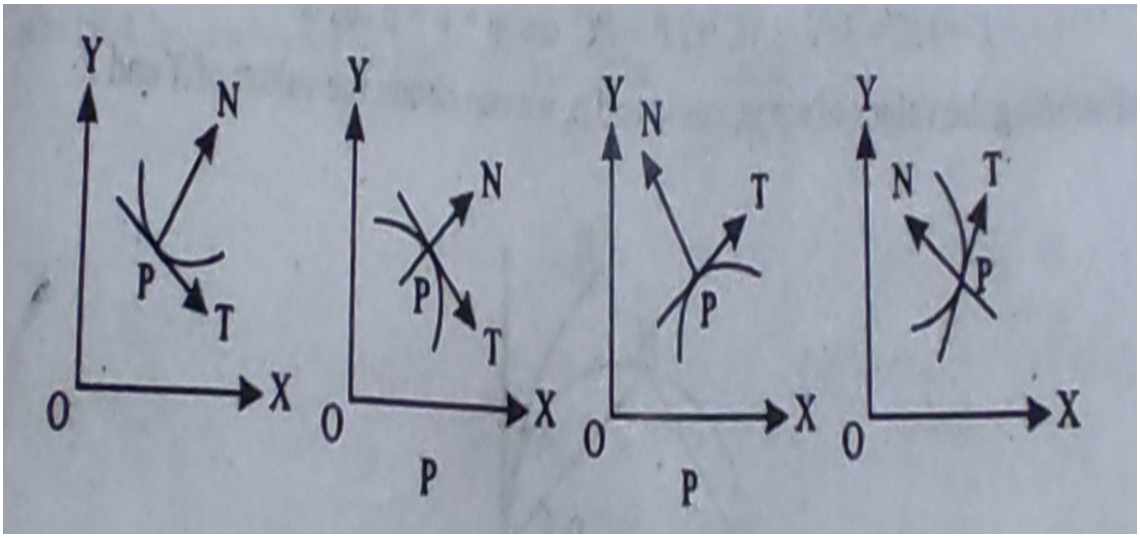
(c) **Polar Form**:If $r = f(\theta)$ then $\rho = \frac{(r^2+r_1^2)^{\frac{3}{2}}}{r^2+2r_1^2-rr_2}$ where $r_1 = f'(\theta), r_2 = f''(\theta)$

22. **Newtonian Method**:

    (a) **Cartesian Form**:If a curve passes through the origin and the axis of x is tangent at the origin, then radius of curvature at origin is $\rho = \lim_{x\to 0} \frac{x^2}{2y}$

    (b) **Polar form**: $\rho = \lim_{\theta\to 0} \frac{r}{2\theta}$

23. **Centre of Curvature:**The centre of curvature at any point P of a curve is the point which lies on the positive direction of the normal at P and a distance, ρ from it.

    (a) 

    (b) The centre of curvature for $y = f(x)$ is (X, Y) where $X = x - \frac{y_1(1+y_1^2)}{y_2}, Y = y + \frac{1+y_1^2}{y_2}$
    Here $y_1 = \frac{dy}{dx}, y_2 = \frac{d^2y}{dx^2}$

24. **Evolutes and Involutes**

    (a) **Evolute:** If the centre of curvature for each point on a curve be taken, we get a new curve called *evolute* of the original one.

    (b) **Involute:** The Original curve, when considered with respect to its evolute, is called an *involute.*

25. **Envelope**: The *envelope* of a one parameter of family curves is the locus of the limiting positions of the points of intersection of any two curves of the family when one of them tends to coinicide with other which is kept fixed.

26. **Determination of Envelope:** If$f(x, y, \alpha) = 0$ is one parameter family of curves with α being a parameter then we get the envelope by eliminating α from
    $f(x, y, \alpha) = 0$ and $f_\alpha(x, y, \alpha) = 0$ where $f_\alpha(x, y, \alpha) = \frac{\partial f}{\partial \alpha}$

5 Unit-IV

27. **Length of a curve**

    (a) **Cartesian Form**:If $y = f(x)$ then length of the curve from $x = a$ to $x = b$ is $\int_a^b \sqrt{\left(1 + (\frac{dy}{dx})^2\right)}dx$

    (b) **Cartesian Form**:If $x = f(y)$ then length of the curve from $y = a$ to $y = b$ is $\int_a^b \sqrt{\left(1 + (\frac{dx}{dy})^2\right)}dy$

    (c) **Parametric Form**: Length of a cuve is $\int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt, a \le t \le b$

    (d) **Polar Form**:If $r = f(\theta), \alpha \le \theta \le \beta$ then Length of the curve is $\sqrt{\left(r^2 + (\frac{dr}{d\theta})^2\right)}d\theta$

28. **Volume of a solid revolution**

(a) Volume obtained by the arc of the curve $y = f(x)$ about X axis between a and b is

$$\int_a^b \pi y^2 dx = \int_a^b \pi (f(x))^2 dx$$

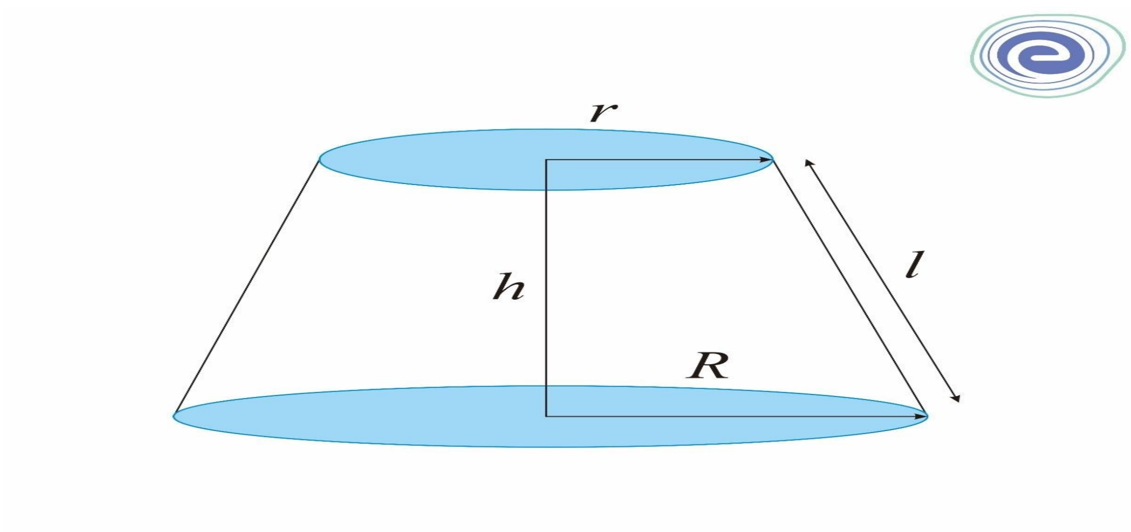
It is assumed that arc doesn't intersect X axis.

(b) Volume obtained by the curve $x = f(y)$ about y axis between a and b is

$$\int_a^b \pi x^2 dy = \int_a^b \pi (f(y))^2 dy$$

. It is assumed that arc doesnt intersect Y axis.

29. The surface Area of the frustum.



(a)

(b) The Surface Area of the frustum is
$\pi l(r + R)$=
=π .Slant height.(sum of radii of two bases.
Here l is slant height and r, R are radii of tho base circles

30. **Surface area of revolution**

(a) The surface are of the arc of the curve $y = f(x)$ obtained by revolving about X axis between a, b is

$$= 2\pi \int_a^b y \frac{ds}{dx} dx = 2\pi \int_a^b f(x)\sqrt{1 + (f'(x)^2} dx$$

(b) **Polar Form**: If $r = f(\theta)$ then surface area is $2\pi \int y \frac{ds}{d\theta} d\theta$

(c) **Parametric Form** :If $x = f(t), y = g(t)$ then surface area is $2\pi \int y \frac{ds}{dt} dt$

31. **Theorems of Pappus**

(a) **Volume of revolution**: If a closed plane curve revolves about a straight line in its plane,(the straight line not intersecting the curve), then the volume of the solid of revolution thus formed is obtained on multiplying the area of the region enclosed by the curve with the length of the path described by the centroid of the region.

(b) **Surface of revolution** :If a closed plane curve revolves about a straight line in its plane,(the straight line not intersecting the curve), then the surface of the solid of revolution thus formed is obtained on multiplying the length of the curve with that of the path described by the centroid of the curve..

6 References

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- Shanti Narayan Integral Calculus, S.CHAND, NEW DELHI