PRIYADARSHINI GOVT WOMENS DEGREE COLLEGE

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Subject : physics.

Stoke's Theorem :

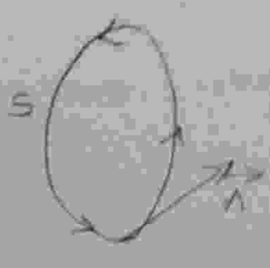
⇒ The line integral of vector A around a closed curve is equal to the surface integral of the curl of vector A taken over the surface "S" surrounded by the closed curve "C"

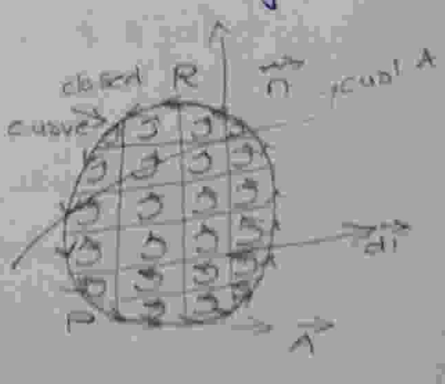
$$\oint_C \bar{A}\cdot\bar{dl} = \iint_S curl\ \bar{A}\cdot\bar{ds} = \iint_S (\nabla \times \bar{A})\cdot ds$$

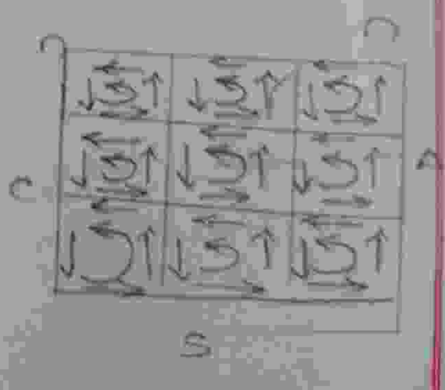
The theorem provides a method for connecting line integral to the surface integral (double integral)

Proof :

Let the surfaces S is bounded by a curve C. Divide the surface S into a large no. of small infinitesimal areas $dS_1, dS_2, dS_3 \ldots$ these areas are bounded by curves $C_1, C_2, C_3 \ldots\ldots$ respectively as shown in figure.







From the definition of curl of a vector A is

$$\nabla \times A = \frac{\oint_c A.dl}{ds} \hat{n}$$

From the above equation curl of vector for the surface ds_1 is

$$(\nabla \times A)_1 = \frac{\oint_{c_1} A.dl}{ds_1} \hat{n}$$

Similarly, for the area ds_2 $(\nabla \times A)_2 . \bar{ds}_2 = \int_{c_2}^{c_1} \bar{A}.\bar{dl}$

Similarly, for the area ds_i $(\nabla \times A)_i . \bar{ds}_i = \int_{c_i} \bar{A}.\bar{dl}$

Adding all LHS terms = All the RHS terms

$$(\nabla \times \bar{A})_1 . \bar{ds}_1 + (\nabla \times \bar{A})_2 . \bar{ds}_2 + (\nabla \times \bar{A})_3 . \bar{ds}_3 + \ldots\ldots + (\nabla \times \bar{A})_i \, ds_i$$

$$= \oint_{c_1} \bar{A}.\bar{dl} + \oint_{c_2} \bar{A}.\bar{dl} + \oint_{c_3} \bar{A}.\bar{dl} + \ldots\ldots + \oint_{c_i} \bar{A}.\bar{dl}$$

$$\sum_{i=1}^{n} (\nabla \times A)_i . ds_i = \oint_c \bar{A}.\bar{dl}$$

$$\therefore \oint_{c_1} \bar{A}.\bar{dl} + \oint_{c_2} \bar{A}.\bar{dl} + \oint_{c_3} \bar{A}.\bar{dl} + \ldots \oint_{c_i} \bar{A}.\bar{dl} = \oint_c \bar{A}.\bar{dl}$$

Replacing the summation by integration and extending the integeration to the total surfaces

$$\oint_c \bar{A}.\bar{dl} = \iint (\nabla \times \bar{A}).\bar{ds}.$$

The line integral values on all the common sides will get cancel with each other.

Hence, the Stoke's theorem is proved

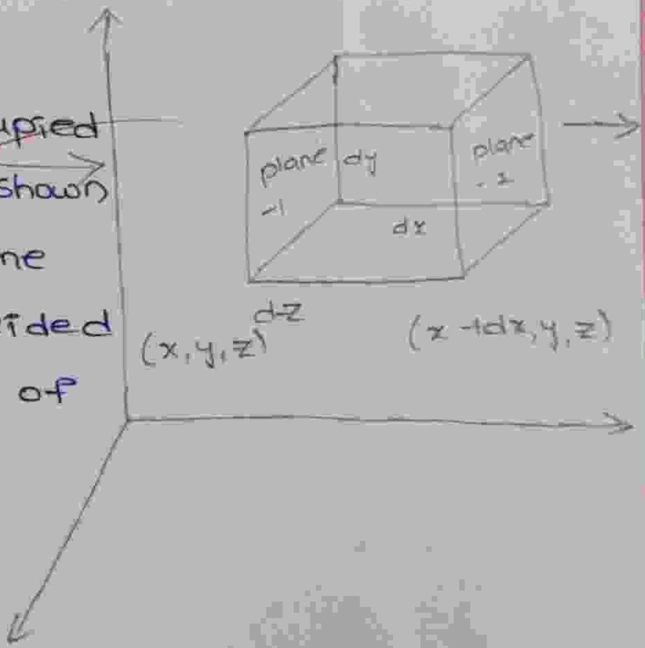
Gauss's Divergence Theorem:

The surface integral of the normal component of vector A taken over a closed surface S is equal to the volume integral of the divergence vector A over the volume enclosed by the surface 'S'

$$\iint_S \bar{A}.\bar{ds} = \iiint_V \text{div}\,\bar{A}\,dv = \iiint_V (\nabla.\bar{A})\,dv$$

The theorem provides a method for connecting volume integral (triple integral) to surface integral (double integral).

Let V be the volume occupied by a closed surface S as shown in fig. Now, the whole volume may be assumed to be divided into a very large number of cubical volume elements adjoining to each other.



Consider a small cubical elements with sides dx, dy, dz.

The value of vector along x-axis on the left face (plane 1) is Ax

The area of left face (plane 1) = dydz

∴ The flux entering into the plane 1 of area dydz = Axdydz

The rate of change of A_x along (x-axis) $= \left(\frac{\partial A_x}{\partial x}\right)$

The increase in the component A_x from face 1 to 2

$$= \left(\frac{\partial A_x}{\partial x}\right) dx$$

The value of vector along the x-axis on the right face (plane 2) of the element is

$$\left(A_x + \frac{\partial A_x}{\partial x} dx\right)$$

The flux coming out of the right face (plane 2) of the area $dydz = \left(A_x + \frac{\partial A_x}{\partial x} dx\right) dydz$

The net flux along x-axis $= \left(A_x + \frac{\partial A_x}{\partial x} dx\right) dydz - A_x dydz$

$$= \frac{\partial A_x}{\partial x} dxdydz$$

Similarly the flux along y-axis $= \frac{\partial A_y}{\partial y} dxdydz$

The flux along the z-axis $= \frac{\partial A_z}{\partial z} dxdydz$

The total flux coming out of volume element

$$(dxdydz) = d\phi = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) dxdydz$$

The total flux coming out of entire volume is

$$\phi = \iiint_V \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) dxdydz \quad \text{——(1)}$$

The flux coming out areas occupying a volume

$$V = \iint_S \bar{A} \cdot \bar{ds} \quad \text{——} (2)$$

The flux coming out of areas occupying a volume V must be equal to flux coming from volume V since the flux is conserved.

$$\therefore eq(1) = eq(2)$$

$$\therefore \iint_S \bar{A} \cdot \bar{ds} = \iiint_V \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) dx\,dy\,dz.$$

But $\left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) = \nabla \cdot \bar{A}$ $dx\,dy\,dz = dv$ = volume element.

$$\therefore \iint_S \bar{A} \cdot \bar{ds} = \iiint_V (\nabla \cdot \bar{A})\, dV = \iiint_V \text{div } A\, dV$$

This is Gauss's theorem of divergence of Gauss's Divergence theorem.

Green's Theorem :

⇒ ϕ, ψ are the two scalar functions consider a closed surface 'S' over which the functions are continuous and differentiable

⇒ The first identify of green's theorem is

1. $$\iiint_V (\phi \nabla^2 \psi + \nabla \phi . \nabla \psi) dv = \iint_S (\phi \nabla \psi). \bar{ds}$$

⇒ The second identity of green's theorem is

2. $$\iiint_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dv = \iint_S (\phi \vec{\nabla} \psi - \psi \nabla \phi) . \vec{ds}$$

Proff : From Gauss divergence theorem

$$\iiint (\nabla . A) dv = \oiint \vec{A} \, \vec{ds} \quad \ldots (1)$$

Put $\vec{A} = \phi \nabla \psi$

$$iA_x + jA_y + kA_z = \phi \left(\vec{i} \frac{\partial \psi}{\partial x} + \vec{j} \frac{\partial \psi}{\partial y} + k \frac{\partial \psi}{\partial z} \right)$$

$$A_x = \phi \frac{\partial \psi}{\partial x}, \quad A_y = \phi \frac{\partial \psi}{\partial y}, \quad A_z = \phi \frac{\partial \psi}{\partial z}$$

$$\nabla . \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla . \bar{A} = \frac{\partial}{\partial x} \left(\phi \frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial y} \left(\phi \frac{\partial \psi}{\partial y} \right) + \frac{\partial}{\partial z} \left(\phi \frac{\partial \psi}{\partial z} \right)$$

$$\nabla . \bar{A} = \phi \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) + \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y} \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial z} \frac{\partial \phi}{\partial z} \quad (2)$$

Substituting this in the equation (1) we have

$$\iiint (\phi \nabla^2 \psi + \nabla \phi) dv = \iint_S (\phi \nabla \psi) \bar{ds}$$

This is the first identity of greens theorem. By interchanging ϕ and ψ in the above.

$$\iiint_V (\psi \nabla^2 \phi + \vec{\nabla}\phi \cdot \nabla\psi)\, dV = \oiint_S (\psi \vec{\nabla} \phi) \cdot ds \ldots$$

subtracting eq (iv) from eq (iii)

$$\iiint_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi)\, dV = \iint (\phi \nabla \psi - \psi \nabla \phi)\, \vec{ds}$$

This is the green's second Identity.

Galilean Transformations

Galilean transformations are used to transform the co-ordinates of a particle from one inertial frame to another. They relate the observations of position and time made by two of observers, located in two different inertial frames.

⇒ Let us consider two inertial frames S and S'. S is at rest and S' is moving with a constant velocity v relative to S. Let an event is happening at point P at a particular time.

⇒ Let the co-ordinates of P with respect to S is (x, y, z, t) and with respect to S' is (x', y', z', t').

⇒ If $t = t' = 0$

the origins of the both reference frames coincides with each other S' frames travelled a distance of 'vt' after 't' seconds of time with respect to S-frame.

⇒ Let us choose our axes so that x and x' are parallel to v. Then the relation between these two frames can be written as.

$x' = x - vt,$

$y' = y$ (there is no relative motion along y and z axes)

$z' = z$

$t' = t$ (time is independent of space co-ordinate system)

⇒ The above equations are known as Galilean Transformations.

⇒ The inverse Galilean Transformations can be expressed as under

$x = x' + vt$

$y = y'$

$z = z'$

$t = t'$

Galilean transformation for velocity can be written as

$$u'_x = \frac{dx'}{dt} = \frac{d(x - vt)}{dt} = \frac{dx}{dt} - v$$

$u'_x = u_x - v$ in vector form $u' = u - v$

$u'_y = u_y$, $u'_z = u_z$.

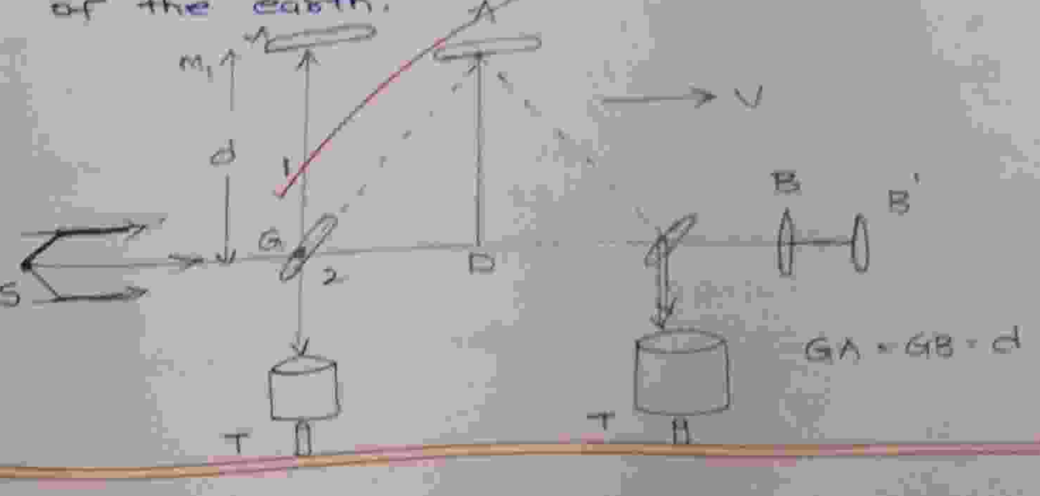
Michelson - Morley Experiment :

⇒ Michelson - Morley experiment is aimed to determine the velocity of the earth with respect to ether frame that means, to detect the real existence of ether, which was proposed by Huygens.

⇒ Michelson and morley conducted this experiment by using michelson interferometer. Experiments laid the experimental foundations of theory of relativity.

⇒ The principle of the experiment lies finding the shift in frings in the michelson interferometer due to difference in time taken by light to travel along and opposite to the direction of motion of the earth.

⇒ The time taken by a beam of light to travel along the direction of motion of the earth is greater than that to travel distance opposite to the direction of motion of the earth.



Experimental Arrangement :

The experimental arrangement of michelson-morley is shown in figure. S is a monochromatic light source. The monochromatic light from S is incident on a semi silvered glass plate G. Which is kept at an inclined angle of 45° with the light. The light falls on the semi silvered glass plate and split into two rays, they are reflected and transmitted rays.

The reflected ray (ray 1) towards the plane mirror M_1 which is at a distance of d from the glass plate G. The transmitted ray (ray 2) travels towards plane mirror M_2 which is also at a distance of d from glass plate G.

⇒ The two rays are incident normally on the mirrors M_1 and M_2 respectively and they are reflected back along their original paths and meet at the surface of the semi silvered glass plate G.

⇒ If the experimental set up at rest in ether, then the two reflected rays take equal time to reach Glass plate G. But the whole setup is moving along with earth

⇒ Consider the direction of motion of earth is in the direction of the intial beam.

⇒ Due to the motion of the earth, the optical paths travelled by the two rays are not the same

⇒ Then the reflections at the mirrors M_1 and M_2 do not take place at A and B, but take place at A' and B' respectively.

⇒ Let the velocities of the light and the apparatus (earth) are c and v respectively. It is obvious from fig. that the reflected ray 1 from glass plate G. will move along GA' and strikes the mirror M_1 at A' instead of A due to the motion of the earth.

after reflection the total path taken by the ray is $GA'G'$

from the $\Delta GA'D$ we have

$$(GA')^2 = (GD)^2 + (A'D)^2 \quad \text{But} \quad GD = AA'$$

$$\therefore (GA')^2 = (AA')^2 + (A'D)^2 \text{ ——— } (1)$$

If t is the time taken by the ray to move from G to A', then eq (1) becomes

$$(ct)^2 = (vt)^2 + (d)^2$$

$$= t^2(c^2 - v^2) = d^2$$

$$\therefore t = \frac{d}{\sqrt{c^2 - v^2}}$$

If t_1 is the taken by the ray 1 to travel whole path $GA'G'$ then

$$t_1 = 2t$$

$$= \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{2d}{c}\left[1 - \frac{v^2}{c^2}\right]^{1/2}$$

$$t_1 = \frac{2d}{c}\left[1 + \frac{v^2}{2c^2}\right] \text{ — } (2)$$

Let the experimental setup is moving with earth velocity v in the direction of incident light. Then the transmitted light ray 2 is traveling with a velocity $(c-v)$ from glass plate G to plane mirror M_2 with respect to interferometer.

Similarly, it is reflected from M_2 with a velocity of $(c+v)$ and falls on G. The distance between G and M_2 is d

Then $t_2 = \frac{d}{c-v} + \frac{d}{c+v} = \frac{d(c+v+c-v)}{(c-v)(c+v)}$

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$$\frac{2dc}{c^2-v^2} = \frac{2dc}{c^2\left[1-\frac{v^2}{c^2}\right]}$$

$$= \frac{2d}{c}\left(1-\frac{v^2}{c^2}\right)$$

$$t_2 = \frac{2d}{c}\left(1+\frac{v^2}{c^2}\right) \quad \text{— (3)}$$

The difference in the Times of Travels of ray 1 and ray 2 is.

$$\Delta t = t_1 - t_2$$

$$\Delta t = t_1 - t_2$$

$$= \frac{2d}{c}\left[1+\frac{v^2}{c^2}\right] - \frac{2d}{c}\left[1+\frac{v^2}{2c^2}\right]$$

$$= \frac{2d}{c}\left[1+\frac{v^2}{c^2} - 1 - \frac{v^2}{2c^2}\right]$$

$$\Delta t = \frac{2d}{c}\left[\frac{v^2}{2c^2}\right] = \frac{dv^2}{c^3} \quad \text{— (4)}$$

The optical path difference between two rays is given by

Optical difference = velocity × Δt

$$= \frac{dv^2}{c^3} \times c = \frac{dv^2}{c^2}$$

If the wave length of the light is λ, then the path difference in terms of wavelength $= \frac{dv^2}{\lambda c^2}$

The path difference gives the fringe shift. When the apparatus is rotated to 90°. Hence the Mirrors M_1 and M_2 exchange their positions. It means initially if a ray travels lesser distance by rotating. It travels longer distance. Hence, the path difference becomes negative by rotating the apparatus the resultant path difference results the fringe shift to equal to $= \frac{2dv^2}{\lambda c^2}$