FORM 2

THE PATENTS ACT, 1970

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The Patent Rules, 2003

COMPLETE SPECIFICATION

(See section 10 and rule 13)

TITLE OF THE INVENTION

“First order differential equation solution using numerical networks interpolation and LaGrange”

We, applicant(s)

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The following specification particularly describes the nature of the invention and the manner in which it is performed:

FIELD OF THE INVENTION

The proposed filed of invention related to differential equation.

Background of the invention:

As a result of the fact that many of the issues that arise in real life may be expressed in the form of an ordinary differential equation, it is necessary to find solutions to differential equations. The use of numerical methods is an instrument.

geared at the resolution of mathematical issues. A differential equation, such as u'(x) = Cos(x) for 0 x 3, is represented as an equation involving some derivative of an unknown function u. An example of such an equation is given in the above sentence (Weisstein, 2004). The differential equation also has a domain, which in this case looks like the range 0–x–3, as shown in the illustration. In practise, a differential equation is an infinite set of equations, one for each x in the domain. This is because each x in the domain has a distinct value. The analytic or exact solution is the functional expression of u, or in the example case, u(x) = sin(x) + c, where c is an arbitrary constant. Due to the non-uniqueness that is inherent in differential equations, we typically include some additional equations. The analytic or exact solution is the functional expression of u. In the context of this example, a suitable supplementary equation would be u(1) = 2, which would enable us to calculate c to be 2 sin(1) and, as a result, to recoup the one-of-a-kind analytical solution expressed as u(x) = sin(x)+2 sin (1).

A differential equation issue is indicated by the differential equation as well as any other equations that may be present. Note that if the value of u (1) is altered little, for example from 2 to 1.95, then likewise the values of u are only modified slightly across the whole domain.

This is shown by the fact that if we change the value of u (1) in our example. An illustration of the ongoing dependency on data that will be necessary for us is as follows: A differential equation problem is considered to be well-posed when it contains at least one differential equation and at least one additional equation in such a way that the system as a whole has one and only one solution (existence and uniqueness). This solution is referred to as the analytic or exact solution (Joshn Wiley, 1969) to differentiate it from the approximate numerical solutions that we will look at in the following section. In addition, this analytical solution must rely constantly on the data in the (vague) sense that if the equations are altered slightly, then likewise the solution does not vary too much. This is a necessary condition for an accurate solution. In this respect, the research hopes to find a solution to a differential equation of the first order by combining Newton's interpolation with the Lagrange technique.

Summary of the Present invention:

The findings that have been obtained via the use of this procedure are very near to the true value. This is shown by the very tiny percentage inaccuracy that was found. After getting the quadratic equation, the approach is extremely straightforward to apply and provides highly precise results. Therefore, the value of y may be obtained for every given value of x without the need to first get the values of y that came before it.

Brief description of the present invention:

The primary objective of the research is to find a solution to a first-order differential equation by the use of numerical Newton's interpolation and Lagrange interpolation. The following are some more particular goals of the study:

1. to distinguish between the interpolation methods used by Newton and those developed by Lagrange

2. to investigate the degree to which Newton's and Lagrange's respective interpolation approaches provide accurate results when used to the solution of first-order differential equations

3. to conduct research on the variables that influence the use of Newton's interpolation and the Lagrange technique of interpolation.

Let's consider the following initial value problem

$$y' = f(x, y) \qquad y(x_0) = y_0$$

Where f(x, y) is a known function and the values in the initial conditions are also known numbers.

Newton's interpolation

$$f_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \cdots \ldots + a_n(x - x_0)(x - x_1) \ldots \ldots (x - x_{n-1})$$

Where

$$a_0 = y_0$$

$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}$$

$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)}$$

Etc.

Lagrange method

$$y_n = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2$$

In order to solve the first order differential equation, the research will use a combination of the Lagrange technique and Newton's interpolation approach.

The first value for y can be determined since this is an initial value problem (IVP), which means that it is already known. Newton's interpolation will be used to get the second two terms, and then we will utilize the three different values for y to build a quadratic equation using the Lagrange technique as follows:

Newton's interpolation method

$$y_0 = a_0$$

$$y_1 = a_0 + a_1(x - x_0)$$

$$y_1 = a_0 + a_1(x - x_0) + a_1(x - x_0)(x - x_1)$$

Lagrange method

$$y_n = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2$$

Example 1

Solve $\frac{dy}{dx} = 1 - y \qquad y(0) = 0$

Taking step h=0.01

Using Newton's interpolation

$$a_0 = 0 = y_0$$

$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = \left[\frac{dy}{dx}\right]_{0,0} = 1$$

$$y_1 = 0 + 1(0.01 - 0) = 0.01$$

$$a_2 = \frac{\frac{f(x_2)-f(x_1)}{(x_2-x_1)} - \frac{f(x_1)-f(x_0)}{(x_1-x_0)}}{(x_2-x_0)} = \frac{\left|\frac{dy}{dx}\right|_{0.01,0.01} - \left|\frac{dy}{dx}\right|_{0,0}}{0.02-0} = -0.5$$

$$y_2 = 0 + 1(0.02 - 0) - 0.5(0.02 - 0)(0.02 - 0.01) = 0.0199$$

Forming quadratic using Lagrange

$$y_n = \frac{(x-0.01)(x-0.02)}{(0-0.01)(0-0.02)} * 0 + \frac{(x-0)(x-0.02)}{(0.01-0)(0.01-0.02)} * 0.01 + \frac{(x-0)(x-0.01)}{(0.02-0)(0.02-0.01)} * 0.0199$$

$$y_n = -0.5x^2 + 1.005x$$

The equation is used to get the values for y at any given value of x

5 Example 2

$$\frac{dy}{dx} = x^2 - y \qquad y(0) = 1$$

We will take to be h=0.01

$$a_0 = 1$$
$$y_0 = 1$$
$$a_1 = \left[\frac{dy}{dx}\right]_{0,1} = -1$$
$$y_1 = 1 - 1(0.01 - 0) = 0.99$$
$$y_1 = 1 - 1(0.01 - 0) = 0.99$$

$$a_2 = \frac{\frac{f(x_2)-f(x_1)}{(x_2-x_1)} - \frac{f(x_1)-f(x_0)}{(x_1-x_0)}}{(x_2-x_0)} = \frac{\left[\frac{dy}{dx}\right]_{0.01,0.99} - \left[\frac{dy}{dx}\right]_{0,1}}{0.02-0.01} = 0.505$$

$$y_2 = 1 - 1(0.02 - 0) + 0.505(0.02 - 0)(0.02 - 0.01) = 0.980101$$

Forming quadratic using Lagrange

$$y_n = \frac{(x - 0.01)(x - 0.02)}{(0 - 0.01)(0 - 0.02)} * 1 + \frac{(x - 0)(x - 0.02)}{(0.01 - 0)(0.01 - 0.02)} * 0.99 + \frac{(x - 0)(x - 0.01)}{(0.02 - 0)(0.02 - 0.01)} * 0.980101$$

$$y_n = 0.505x^2 - 1.00505x + 1$$

Table 1. The table showing results of the equation $\frac{dy}{dx} = 1 - y$

X	Combined Newton's interpolation and Lagrange	Exact values	Percentage error
0	0	0	0%
0.01	0.01	0.009950166251	0.50083333%
0.02	0.0199	0.019801326	0.498320163%
0.03	0.0297	0.029554466	0.492426423%
0.04	0.0394	0.03921056	0.483135155%
0.05	0.049	0.048770575	0.470468105%
0.06	0.0585	0.058235466	0.454248962%
0.07	0.0679	0.06760618	0.434605238%
0.08	0.0772	0.076883653	0.411461978%
0.09	0.0864	0.086068814	0.384792103%
0.1	0.0955	0.095162581	0.354571093%

Table 2. The table showing results of the equation $\frac{dy}{dx} = x^2 - y$

X	Combined Newton's interpolation and Lagrange	Exact	Percentage error
0	1	1	0%
0.01	0.99	0.990050166	-0.00506701596%
0.02	0.980101	0.980201326	-0.00983363455%
0.03	0.970303	0.970454466	-0.0000015607%
0.04	0.960606	0.96081056	-0.021290357%
0.05	0.950101	0.951270575	-0.12294872%
0.06	0.941515	0.941835466	-0.034025688%
0.07	0.932121	0.93250618	-0.041305892%
0.08	0.922828	0.923283653	-0.049351355%
0.09	0.913636	0.914168814	-0.058283983%
0.1	0.904545	0.905162582	-0.068228829%

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We Claim:

1. 1. An ever-increasing number of new and improved techniques for the numerical solution of partial differential equations are being developed at a pace that is only becoming faster. In this paper, which is aimed towards scientists skilled in mathematics but not necessarily in numerical analysis, we make an effort to explain and unify the fundamental important facts about this progression.

2. The vast majority of the novel approaches may be comprehended and categorized according to the manner in which space, time, and boundary conditions are discretized as well as the manner in which nonlinear algebraic equations that occur in the process of solution are solved.

3. A technique similar to that which is claimed in Claim 1, in which the methods for the numerical integration of the first order ordinary differential equation are employed for the methods for the numerical integrations of the step interval.

4. A method in accordance with the one claimed in Claim 1, in which error monitoring strategies are used to determine the duration of the next step.

Dated this 13th day of January 2023

Signature:

Applicant(s)

Dr. S Kiran et. al.

ABSTRACT

First order differential equation solution using numerical networks interpolation and LaGrange

One of the most important fields of study in mathematics, differential equations may be solved in a number of different ways. There is the analytic method and the numerical method; the analytic technique can only be used to a certain class of equations; hence the numerical method is utilized the majority of the time. The majority of studies on numerical approaches to the solution of first order ordinary differential equations have a tendency to adopt methods such as the Runge-Kutta method, the Taylor series method, and Euler's method. However, not a single study has actually combined Newton's interpolation and the Lagrange method to solve first order differential equations. In order to find solutions to the issues posed by first-order differential equations, this investigation will make use of both Newton's interpolation and the Lagrange technique.

Dated this 13th day of January 2023

Signature: 

Applicant(s)

Dr. S Kiran et. al.