GOVERNMENT DEGREE COLLEGE KORUTLA

DEPARTMENT OF PHYSICS

JIGNASA – STUDENT STUDY PROJECT 2021-2022

TITLE OF THE PROJECT

PLANCK'S LAW OF RADIATION

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CONTENT

PLANCK'S STATEMENT:

Planck's radiation law, a mathematical relationship formulated in 1900 by German physicist Max Planck to explain the spectral-energy distribution of radiation emitted by a blackbody

(a hypothetical body that completely absorbs all radiant energy falling upon it, reaches some equilibrium temperature, and then reemits that energy as quickly as it absorbs it).

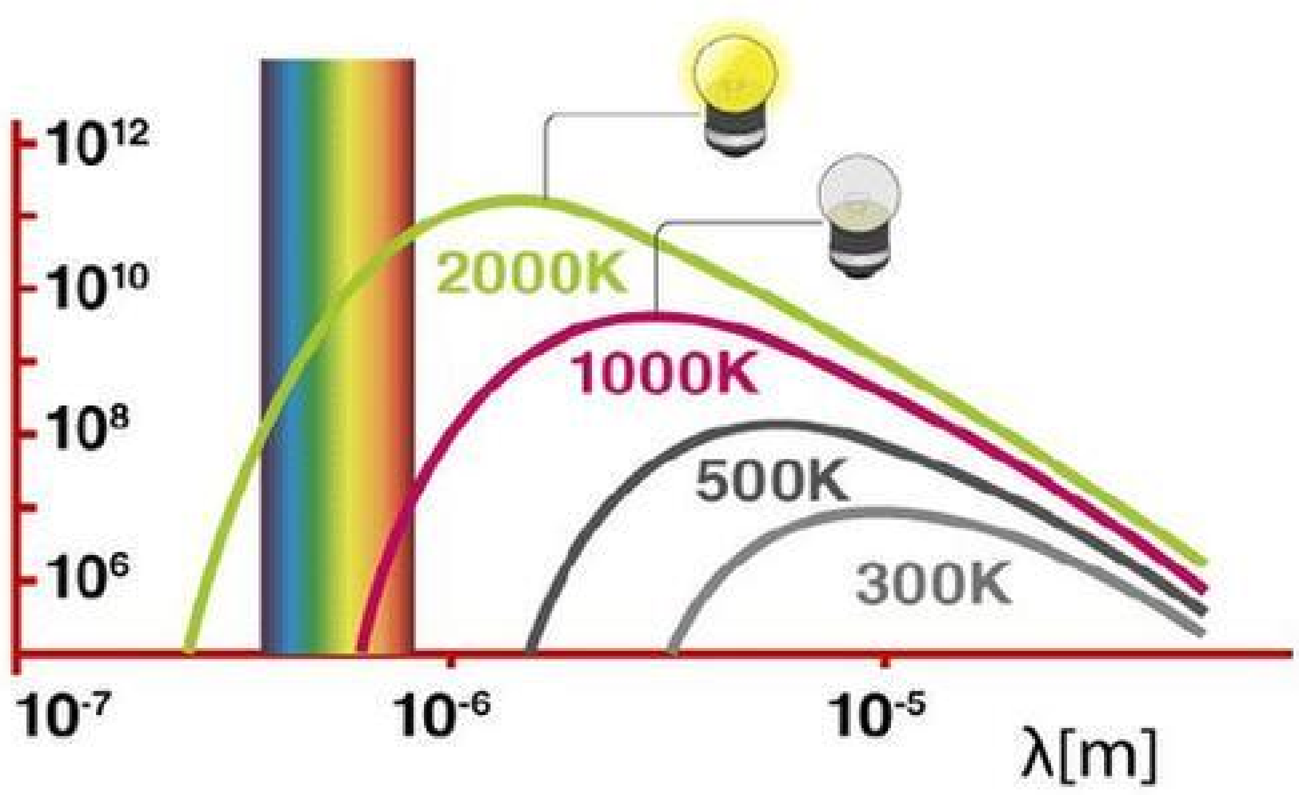
Planck assumed that the sources of radiation are atoms in a state of oscillation and that the vibrational energy of each oscillator may have any of a series of discrete values but never any value between. Planck further assumed that when an oscillator changes from a state of energy E_1 to a state of lower energy E_2, the discrete amount of energy $E_1 - E_2$, or quantum of radiation, is equal to the product of the frequency of the radiation, symbolized by the Greek letter ν and a constant h, now called Planck's constant, that he determined from blackbody radiation data; i.e., $E_1 - E_2 = h\nu$.

Planck's law for the energy E_λ radiated per unit volume by a cavity of a blackbody in the wavelength interval λ to λ + Δλ (Δλ denotes an increment of wavelength) can be written in terms of Planck's constant (h), the speed of light (c), the Boltzmann constant (k), and the absolute temperature (T):

$$E_\lambda = \frac{8\pi hc}{\lambda^5} \times \frac{1}{\exp\left(\frac{hc}{KT\lambda}\right) - 1}$$

PLANCK'S HYPOTHESIS:

- A black body radiator consists of tiny atomic harmonic oscillators of all possible frequencies ,each oscillator having its own characteristic frequency
- According to Planck's quantum theory
- Different atoms and molecules can emit or absorb energy in discrete quantities only. The smallest amount of energy that can be emitted or absorbed in the form of electromagnetic radiation is known as quantum
- The energy of the radiation absorbed or emitted is directly proportional to the frequency of the radiation



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Meanwhile, the energy of radiation is expressed in terms of frequency as,

$E = h\nu$

Where,

E = Energy of the radiation

h = Planck's constant (6.626×10^{-34} J.s)

ν= Frequency of radiation

AIMS AND OBJECTIVES:

To find the spectral density of electromagnetic radiation and to observe the spectral distribution of radiation from a black body with the wavelength

THE DERIVATION OF THE PLANCK FORMULA:

Derivation of Planck's law :

The photoelectric effect demonstrates that light waves have particle properties and that the light quanta, or photons, of a particular frequency ν each have energy $h\nu$. We need to reconcile this picture with the classical picture of electromagnetic waves in a box. In the classical picture, the energy associated with the waves is stored in the oscillating electric and magnetic fields. We found it necessary to impose the constraint that only certain modes are permitted by the boundary conditions – the waves are constrained to fit into the box with whole numbers of half wavelengths in the x, y, z directions.

Now we have a further constraint. The quantisation of electromagnetic radiation means that the energy of a particular mode of frequency ν cannot have any arbitrary value but only those energies which are multiples of $h\nu$, in other words the energy of the mode is $E(\nu) = nh\nu$, where we associate n photons with this mode.

We now consider all the modes (and photons) to be in thermal equilibrium at temperature T. In order to establish equilibrium, there must be ways of exchanging energy between the modes (and photons) and this can occur through interactions with any particles or oscillators within the volume or with the walls of the enclosure.

We now use the Boltzmann distribution to determine the expected occupancy of the modes in thermal equilibrium. The probability that a single mode has energy En = nhν is given by the usual Boltzmann factor

$$p(n) = \frac{exp(-En/KT)}{\sum_{n=0}^{\infty} exp(-En/KT)} \qquad (1)$$

Where the denominator ensures that the total probability is unity, the usual normalisation procedure. In the language of photons, this is the probability that the state contains n photons of frequency ν.

The mean energy of the mode of frequency ν is therefore

$$\mathrm{E\nu} = \sum_{n=0}^{\infty} \mathrm{En\ p(n)} = \frac{\sum_{n=0}^{\infty} \mathrm{En\ exp\ (-En/kT)}}{\sum_{n=0}^{\infty} \mathrm{exp\ (-En/kT)}}$$

$$= \frac{\sum_{n=0}^{\infty} \mathrm{nh\ \nu exp\ (-nh\nu/kT)}}{\sum_{n=0}^{\infty} \mathrm{exp\ (-nh\nu/kT)}} \qquad (2)$$

To simplify the calculation, let us substitute x = exp(−hν/kT).

Then (2) becomes

$$\overline{\mathrm{E\nu}} = \mathrm{h\nu}\ \frac{\sum_{n=0}^{\infty} \mathrm{nxn}}{\sum_{n=0}^{\infty} \mathrm{xn}} = \mathrm{h\nu}\ \frac{(x+2x^2+3x^3+\cdots)}{(1+x+x^2+\cdots)}$$

$$= \mathrm{h\nu}\, x\, \frac{(1+2x+3x^2+\cdots)}{(1+x+x^2+\cdots)} \qquad (3)$$

Now, we remember the following series expansions:

[Note (5) can be found from (4) by differentiation with respect to x.]

$$\frac{1}{(1-x)} = (1 + x + x^2 + \cdots) \qquad (4)$$

$$\frac{1}{(1-x)^2} = (1 + 2x + 3x^2 + \cdots) \qquad (5)$$

Hence, the mean energy of the mode is

$$\bar{E} = \frac{h\nu x}{(1-x)} = \frac{h\nu}{(x^{-1}-1)} = \frac{h\nu}{\left(e^{\frac{h\nu}{KT}}-1\right)}. \qquad (6)$$

Average energy of a mode of frequency ν according to quantum theory

$$\bar{E} = \frac{h\nu}{\left(e^{\frac{h\nu}{KT}}-1\right)}.$$

This is the result we have been seeking. To find the classical limit, we allow the energy quanta hν to tend to zero.

Expanding $e^{\frac{h\nu}{KT}} - 1$ for small values of $\frac{h\nu}{KT}$,

$$e^{\frac{h\nu}{KT}} - 1 = 1 + \frac{h\nu}{KT} + 1 = \frac{h\nu}{KT}$$

and so $E = \frac{h\nu}{\left(e^{\frac{h\nu}{KT}} - 1\right)} = \frac{h\nu}{\left(\frac{h\nu}{KT}\right)} = kT$

Thus, if we take the classical limit, we recover exactly the expression for the average energy of a harmonic oscillator in thermal equilibrium, E = kT. We can now complete the determination of Planck's radiation formula. We have already shown that the number of modes in the frequency interval ν to ν +dν is

$(8\pi\nu^2/c^3)\, d\upsilon$ per unit volume. The energy density of radiation in this frequency range is $u(\nu)\, d\nu = \frac{8\pi\nu^2}{c^3} \overline{E\nu} d\nu$

$$= \frac{8\pi h\nu^3}{c^3} \frac{1}{exp(h\nu/KT) - 1} \quad (7)$$

This is the Planck distribution function.

RESULT OF PLANCK'S RADIATION LAW:

The wavelength of the emitted radiation is inversely proportional to its frequency or $\lambda=c/v$. At higher temperatures , the total radiated energy increases and the intensity peak of the emitted spectrum shifts to shorter wavelengths so that a significant portion is radiated as visible light.

CONCLUSION OF PLANCK'S RADIATION LAW:

Planck postulated that the energy of light is proportional to the frequency and the constant that relates them is known as Planck's constant (h).His work led to Albert Einstein determining that light exists in discrete quanta of energy or photons

Thank you