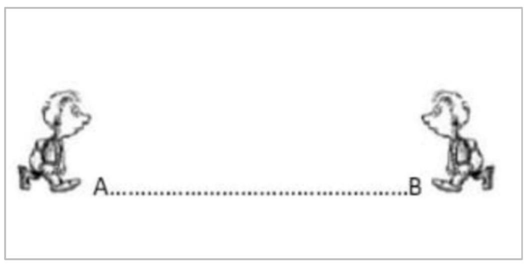
GOVERNMENT DEGREE COLLEGE
SHADNAGAR

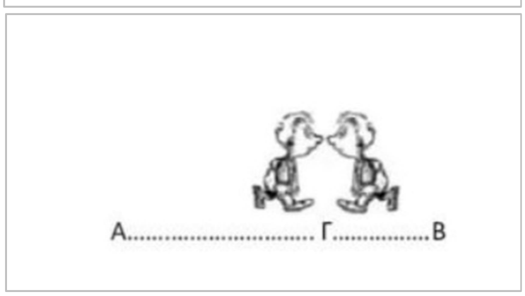
Ranga Reddy – Dist

Department of Mathematics

STUDY PROJECT – 2021 – 22

On





"Story on Location of Roots Theorem"

Government Degree College - Shadnagar

Ranga Reddy (Dist)

Student Study Project

on

"Story on Location of Roots Theorem"

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Certificate

This is to certify that BSc (MPC & MPCs) SEM III students has successfully completed a Study Project on **"Story on Location of Roots Theorem"** for the academic year 2021 - 22 under the Supervision of **T. Sri Krishna, Department of Mathematics**.

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Hence it is certified

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project

Story on location of roots theorem :-

If f is continuous on $[a,b]$ and $f(a), f(b)$ have opposite signs then there exists $c \in (a,b)$ such that $f(c)=0$

proof: $f:[a,b] \longrightarrow R$ is continuous function.

case 1: let $f(a)<0$ and $f(b)>0$

$$S = \{x \in [a,b] / f(x)<0\}$$

we know that $f(a)<0 \Rightarrow a \in S$

$\therefore$ S is nen-emptyset

we know that $f(b)>0 \Rightarrow b \in S$

$\therefore$ b is upper bound of S

S is a non-empty set and bound above

let sup of $S = c$

we know that f is continuous on $[a,b]$

f is continuous at $x=a$ and $f(a)<0$ $\exists$ $\delta>0$ such that $x \in (a, a+\delta) \Rightarrow f(x)<0$

$c \in (a,b)$ $f(c)=0$

let $x \in [a, a+\delta) \Rightarrow x \in S \Rightarrow a+\frac{\delta}{2} \leq \sup S = 0$

$\therefore a \neq c$

let f is continuous at b and $f(b)>0$

$\exists$ $\delta>0$ such that $x \in (b-\delta, b) \Rightarrow f(x)>0$

let $x \in (b-\delta, b]$

$\Rightarrow x \in S$

$\Rightarrow b - \frac{\delta}{2}$ sup $S = c$

$\therefore b \neq c$

$\therefore c \in (a, b)$

let To prove that $f(c) = 0$

let $f(c) < 0$. by nbd property $\exists\ \delta > 0$ such that $f(x) < 0$ where $x \in (c-\delta, c+\delta)$ for some $x \in [c, c+\delta]$

$x > c$ and

we have $f(x) < 0$

$\neq f(c) \neq 0$

let $f(c) > 0$, by nbd property $\exists\ \delta > 0$ such that $f(x) > 0$ where $x \in (c-\delta, c+\delta)$

we know that sup $S = c$

$\exists\ \delta \in S$ such that $d \in (-\delta, c)$

$\therefore f(d) < 0$

$\neq f(c) \ngtr 0$

$\therefore f(c) = 0$ where $c \in (a, b)$

<u>case</u> 2: $f(a) > 0$ and $f(b) < 0$

let $F(x) = -f(x)\ \forall\ x \in (a, b)$

$f(a) > 0 \Rightarrow -f(a) < 0 \Rightarrow F(a) < 0$

$f(b) < 0 \Rightarrow -f(b) > 0 \Rightarrow F(b) > 0$

when $F(a) < 0$ and $F(b) > 0$ by case 1

$\exists\ c \in (a, b)$ such that $F(c) = 0$

$-F(c) = 0 \Rightarrow f(c) = 0$

$\therefore$ If $f(a) > 0$ and $f(b) < 0$ Then $\exists\ c \in (a, b)$ such that $f(c) = 0$.

Dialectic creation and resolving of the problem of the existence of root - application of Bolzano theorem in the analysis (the fixed point theorem in topology).

A ―――――― B

professor: George who lives in the village A. started on foot to go to the village B of his grandmother. He started at 7am and arrived at 11 am. He slept at his grandmother's and the other day started from the village B at 7 am and arrived in the village A at 11 am. The way he walked both days was ran-dom. Sometimes slowly, sometimes stable, some times quickly and some times stoppetd to drink water or to admire a beautiful flower.

- student: And then;

- professor: The story of George ends here.

what would you answer if I told you that there is a point of the route on which george was at the same time both days;

- student: Thinks ... strange ! How is that possible ?

- professor: Imagine now that while George begins from village A to village B of his grandmother, his twin brother starts from the village B in the village of A.

Note that both walk on their own way, and that both cover the path at exactly the same time.

A ————————— E

- student: I understand, some time they meet at an intermediate random point C and their clocks show the same time... between 7 and 11

A ————————— r ——— B

- professor: Good jab I you just solved a problem that is resolved by applying the theorem of Bolzano.

Let us now recall the theorem of Bolzano:

If one function f is continuous in the interval $[a,b]$ and $f(a)-f(b)<0$, then there is at least one point $\xi \in (a,b)$ such that $f(\xi)=0$

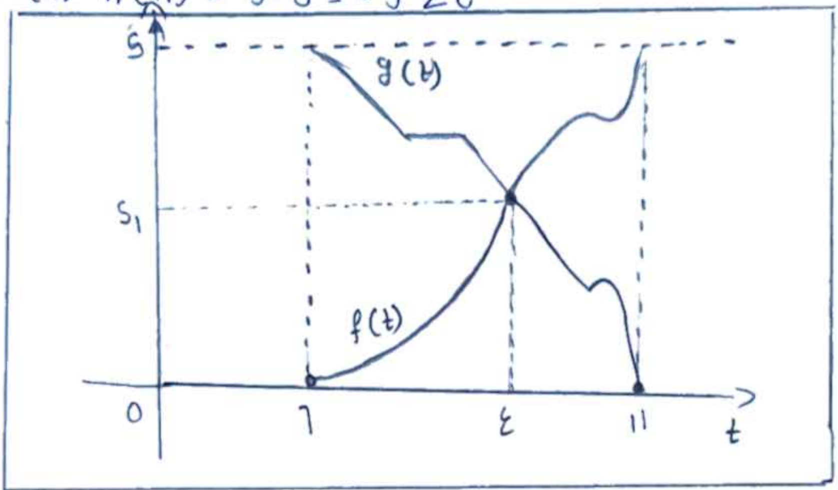
- student: I fail to associate it.
- professor: Let us suppose that the distance between the two villages is s and the functions $f(t)$ and $g(t)$ with $t \in (7,11)$ which express the distance that George has traveled the first and the second day respectively at time t.

Then we get $f(7)=0$, $g(7)=s$, $f(11)=s$, and $g(11)=0$.
so assuming the function h with $h(t)=f(t)-g(t)$

we have the following: The function h is continuous in the interval $[7, 11]$ as a difference of continuous functions also:

$h(7) = f(7) - g(7) = 0 - S = -S$ and $h(11) = f(11) - g(11) = S - 0 = S$

so $h(7) \cdot h(11) = S \cdot S = -S^2 < 0$



Student: so that conditions of the theorem of Bolzano are fulfilled!

- professor: very well.
- Student: Therefore there is at least a point ε_1 with $\varepsilon_1 \in (7, 11)$ such that $h(\varepsilon_1) = 0$ or $f(\varepsilon_1) - g(\varepsilon_1) = 0$ or $f(\varepsilon_1) = g(\varepsilon_1) = S_1$

which means that there is some point (time) ε_1 where George is located at the same place the first and second day!

- professor: Exactly.

By the same logic we can prove that: every given time there is at least one pair of diametrically opposite points of the equator of the earth at the same temperature.!