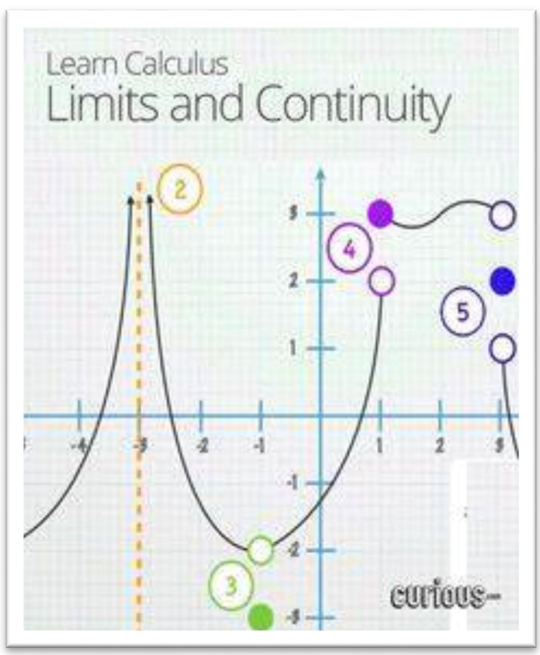
GOVERNMENT DEGREE COLLEGE SHADNAGAR

Ranga Reddy – Dist

Department of Mathematics

STUDY PROJECT – 2021 – 22

On



“Continuity Test with the help of

DESMOS GRAPHIC CALCULATOR”

Government Degree College - Shadnagar

Ranga Reddy (Dist)

Student Study Project

on

"Continuity Test with the help of DESMOS GRAPHIC CALCULATOR"

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Department of Mathematics

GDC - Shadnagar

Principal
GOVT. DEGREE COLLEGE
SHADNAGAR

Principal

GDC - Shadnagar

Government Degeer College Shadnagar

Ranga Reddy (Dist)

Certificate

This is to certify that BSc (MPC & MPCs) SEM III students has successfully completed a Study Project on **"Continuity Test with the help of DESMOS GRAPHIC CALCULATOR "** for the academic year 2021 - 22 under the Supervision of **T. Sri Krishna, Department of Mathematics**.

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Hence it is certified

Principal
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project : continuity test with help of Desmos graphic calculator.

Definition : let S be an aggregate $f: S \to R$ be a function and $a \in S$ f is said to be continuous at 'a' from left.

if given $\varepsilon > 0$ there exists $\delta > 0$ such that $x \in S$ $a - \delta < x < a$

$|f(x) - f(a)| < \varepsilon$

f is said to be continuous at 'a' from right if given $\varepsilon >$ there exists $\delta > 0$ such that $x \in S$, $a < x < a + \delta$

$|f(x) - f(a)| < \varepsilon$

f is said to be continuous at 'a'

if given $\varepsilon > 0$ there exists $\delta > 0$ such that $x \in S$.

$a - \delta < x < a + \delta \Rightarrow |f(x) - f(a)| < \varepsilon$

Definition : (limit notation of continuity at a point)

let : $f: S \to R$ be a function and $a \in S$ be a limit point of S. f is said to be continuous at 'a' from left if $\lim_{x \to a-} f(x) = f(a)$ or $f(a-0) = f(a)$

f is said to be continuous at 'a' from right. if $\lim_{x \to a+} f(x) = f(a)$ or $f(a+0) = f(a)$

f is said to be continuous at 'a' if $\lim_{x \to a} f(x) = f(a)$.

note : let $f: S \to R$ be a function and $a \in S$ be a limit point of S If f is continuous at 'a' then three conditions true.

1. f must be defined at 'a', $f(a)$ has exists
2. $\lim_{x \to a} f(x)$ must exists.

prove that $f(x) = x \sin \frac{1}{x}$ if $x \neq 0$ and $f(0) = 0$ is continuous at $x = 0$

sol: The given function $f(x) = x \sin \frac{1}{x}$ and $f(0) = 0$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} x \sin \frac{1}{x}$$

$$= \lim_{x \to 0} x \lim_{x \to 0} \sin \frac{1}{x}$$

$$-1 \leq \sin x \leq 1 \Rightarrow -1 \leq \sin \frac{1}{x} \leq 1$$

$\sin \frac{1}{x}$ is bounded function

$$\lim_{x \to 0} x = 0$$

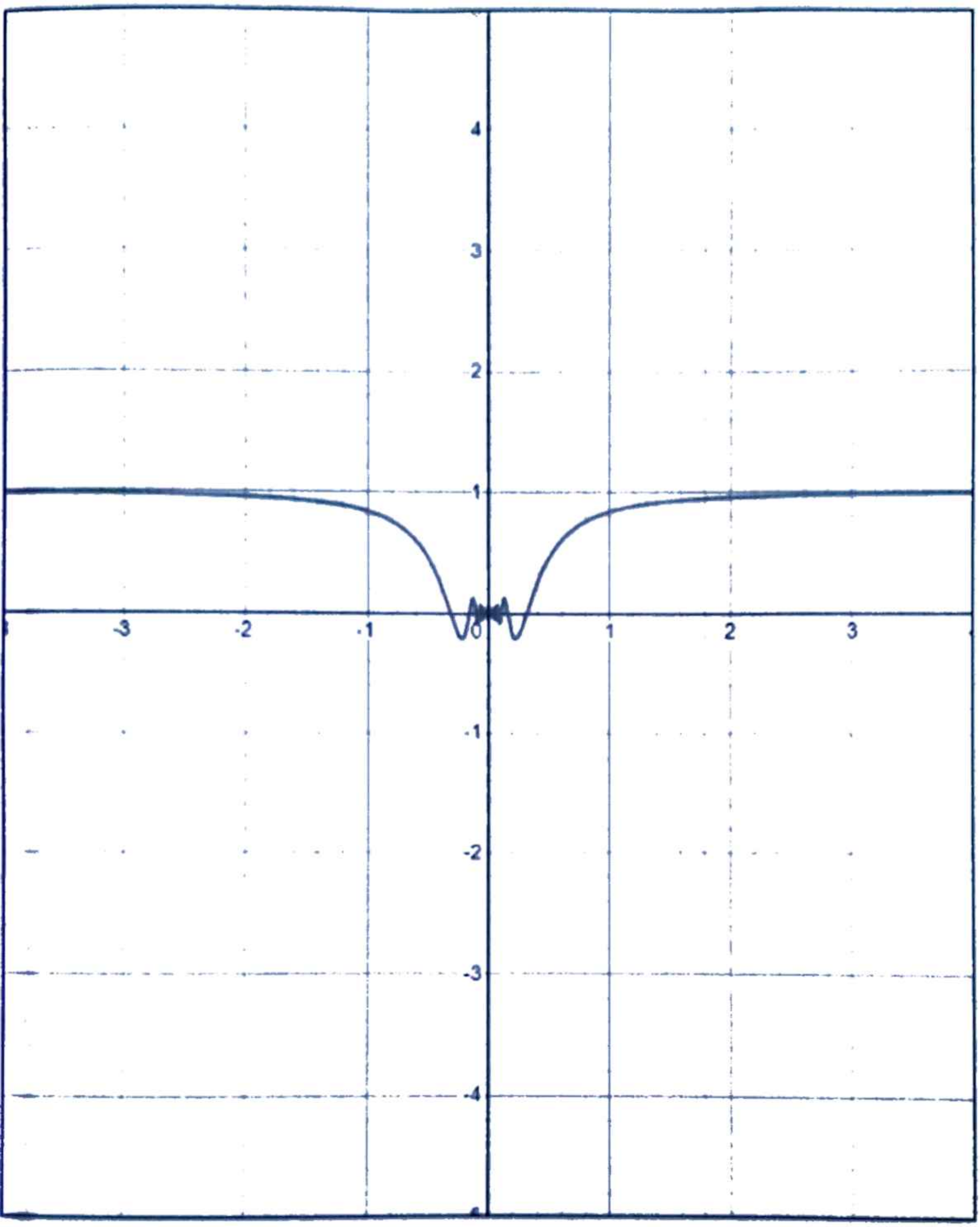
$$\lim_{x \to 0} f(x) = 0$$

we know $f(0) = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$f(x) = x \sin \frac{1}{x}$ is continuous at $x = 0$

$f(x) = x \sin 1/x.$



prove that $f(x) = \frac{1}{1-e^{1/x}}$ has an ordinary discontinuous at $x=0$

The given function $f(x) = \frac{1}{1-e^{1/x}}$

we know that $\lim_{x \to 0-} e^{1/x} = 0$; $\lim_{x \to 0+} e^{-1/x} = 0$

(i) left continuty at $x=0$

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0-} \frac{1}{1-e^{1/x}}$$

$$= \frac{1}{1-0} = 1/1 = 1$$

(ii) Right continuty at $x=0$

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0+} \frac{1}{1-e^{1/x}}$$

$$\lim_{x \to 0+} \frac{e^{-1/x}}{e^{-1/x}(1-e^{1/x})}$$

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0+} \frac{e^{-1/x}}{(e^{-1/x} - e^{-1/x} e^{-1/x})}$$

$$= \lim_{x \to 0+} \frac{e^{-1/x}}{(e^{-1/x}-1)}$$

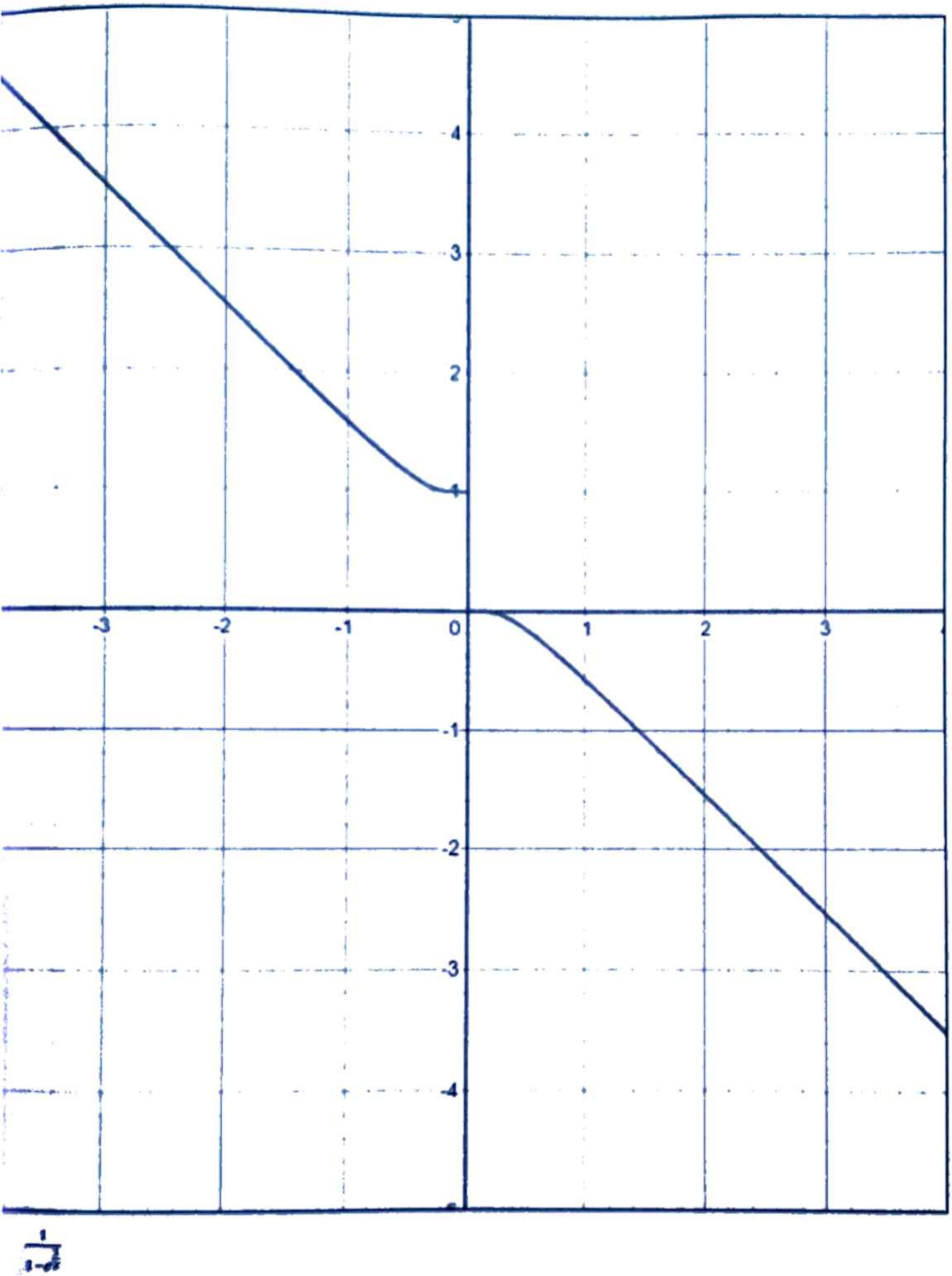
$$= \frac{0}{0-1} = 0$$

$$\lim_{x \to 0+} f(x) = 0$$

$$\lim_{x \to 0-} f(x) \neq \lim_{x \to 0+} f(x)$$

$f(x)$ is dis continuous at $x=0$

$$f(x) = \frac{1}{1-e^{1/x}}$$



Discuss the continuity of $f(x) = \frac{xe^{1/x}}{1+e^{1/x}}$ when $x \neq 0$ and $f(0) = 0$ at the origin.

$$f(x) = \frac{xe^{1/x}}{1+e^{1/x}} \text{ when } x \neq 0 \text{ and } f(0) = 0$$

we know that $\lim_{x \to 0-} e^{1/x} = 0$

$$\lim_{x \to 0+} e^{-1/x} = 0$$

(i) left continuity at $x = 0$

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0-} \frac{xe^{1/x}}{1+e^{1/x}}$$

$$= \frac{0 \cdot 0}{1+0} = \frac{0}{1} = 0$$

(ii) Right continuity at $x = 0$

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0+} \frac{xe^{1/x}}{1+e^{1/x}}$$

$$= \lim_{x \to 0+} \frac{xe^{1/x}}{e^{1/x}(e^{-1/x}+1)}$$

$$= \lim_{x \to 0+} \frac{x}{(1+e^{-1/x})}$$

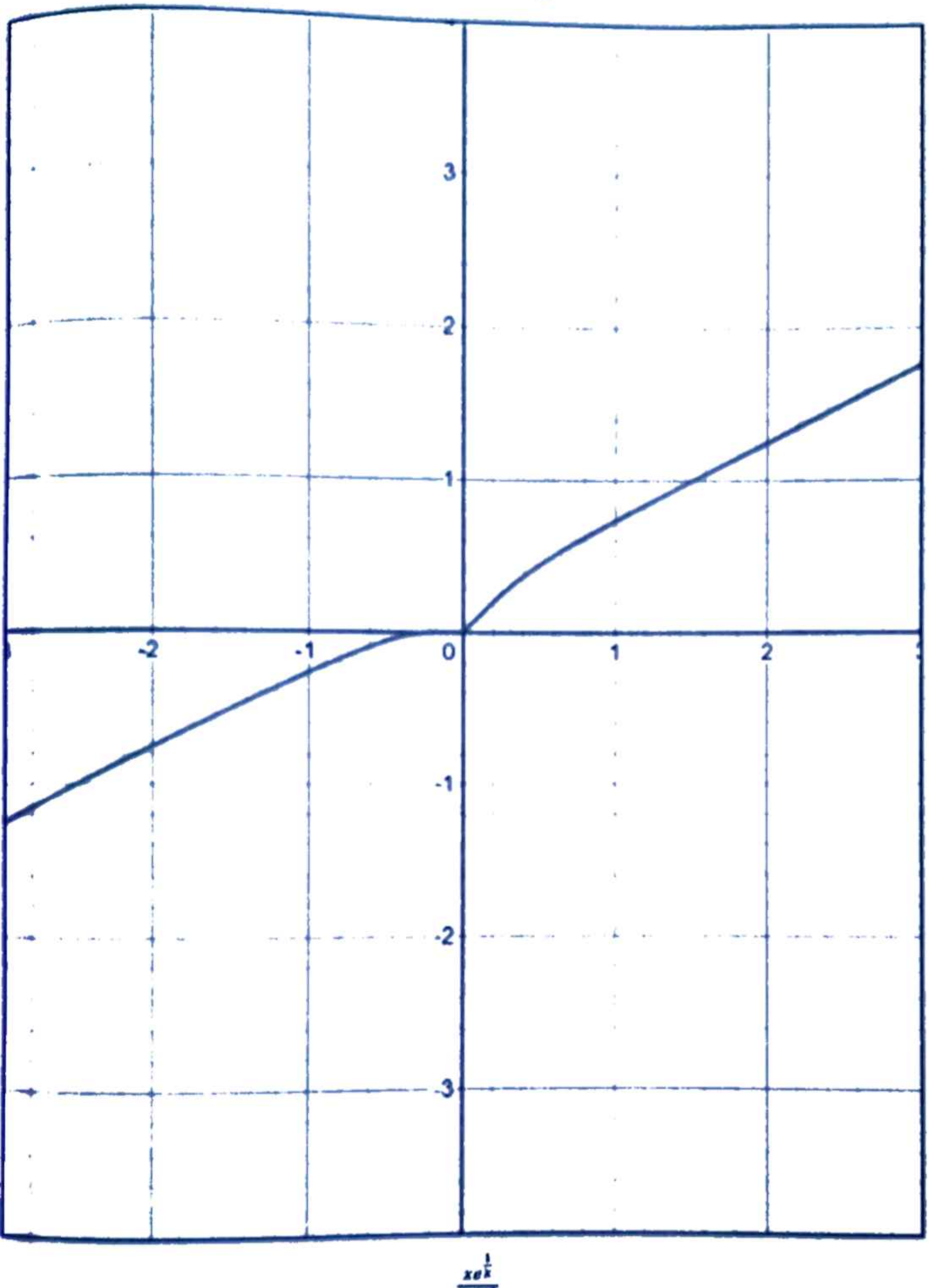
$$= \frac{0}{0+1} = \frac{0}{1} = 0$$

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0+} f(x) = 0$$

$$= f(0) = 0$$

The function is continuous at $x = 0$

$$f(x) = \frac{x e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}$$



$$\frac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}$$

Discuss the continuity of $f(x) = \dfrac{x(e^{1/x} - e^{-1/x})}{(e^{1/x} + e^{-1/x})}$ when $x \neq 0$ and $f(0) = 0$ at the origin.

$$f(x) = \frac{x(e^{1/x} - e^{-1/x})}{(e^{1/x} + e^{-1/x})}$$

we know that $\lim\limits_{x \to 0-} e^{1/x} = 0$ and $\lim\limits_{x \to 0+} e^{-1/x} = 0$

(i) left continuity at $x = 0$

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0-} \frac{x(e^{1/x} - e^{-1/x})}{(e^{1/x} + e^{-1/x})}$$

$$= \lim_{x \to 0-} \frac{x e^{1/x}(1 - (e^{-1/x})^2)}{e^{1/x}(1 + (e^{-1/x})^2)}$$

$$= \lim_{x \to 0-} \frac{x(1 - (e^{-1/x})^2)}{(1 + (e^{-1/x})^2)}$$

$$= \lim_{x \to 0} f(x) = \frac{0(1-0)}{1+0} = \frac{0}{1} = 0$$

(ii) Right continuity at $x = 0$

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0+} \frac{x(e^{1/x} - e^{-1/x})}{(e^{1/x} + e^{-1/x})}$$

$$= \lim_{x \to 0+} \frac{x e^{-1/x}((e^{1/x})^2 - 1)}{e^{1/x}((e^{1/x})^2 + 1)}$$

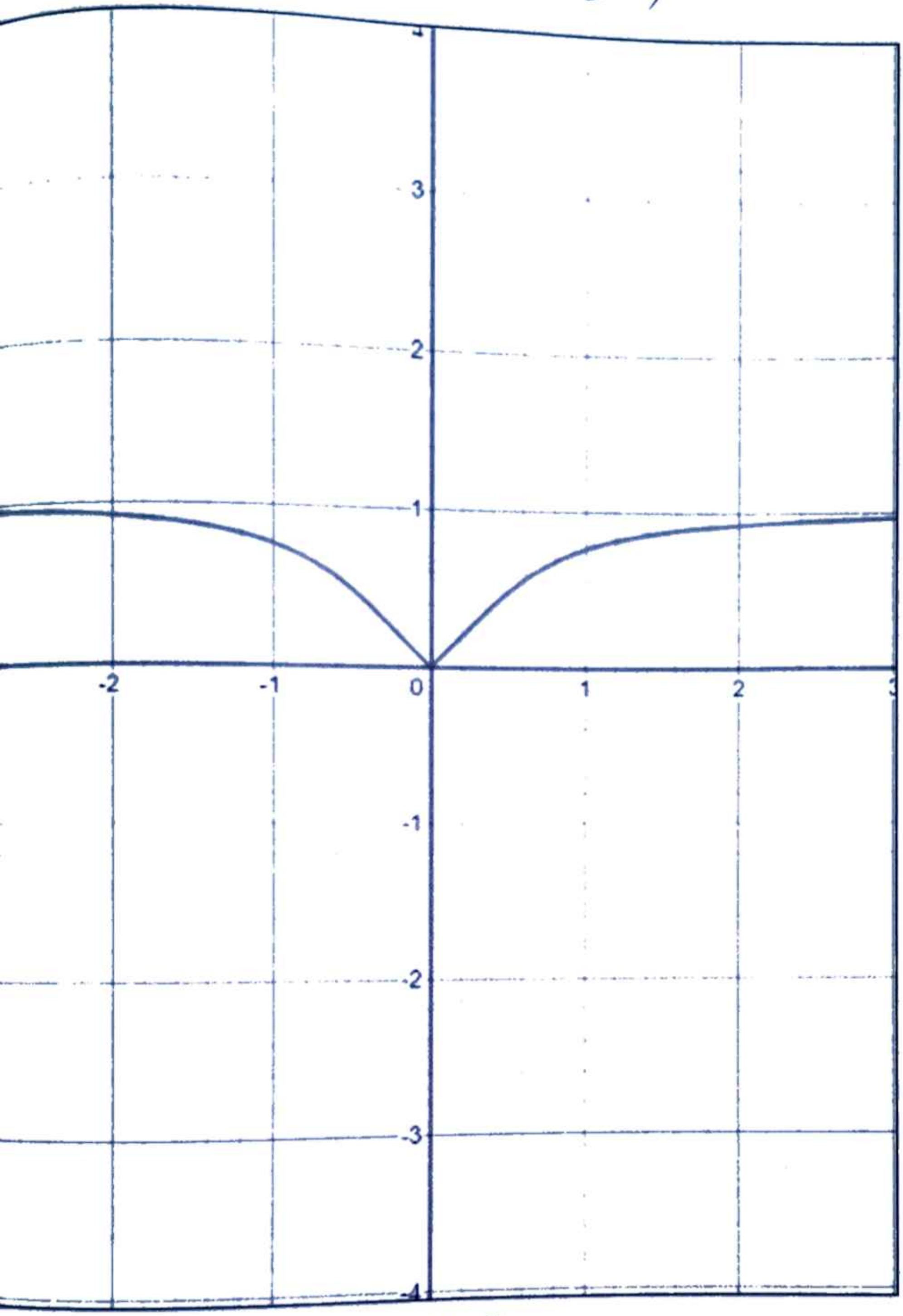
$$= \lim_{x \to 0+} \frac{x((e^{1/x})^2 - 1)}{((e^{1/x})^2 + 1)}$$

$$\lim_{x \to 0+} f(x) = \frac{0(0-1)}{(0+1)} = \frac{0}{1} = 0$$

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0-} f(x) = 0 = f(0)$$

$f(x)$ is continuous at $x = 0$

$$f(x) = \frac{x\left(e^{1/x} - e^{-1/x}\right)}{e^{1/x} + e^{-1/x}}$$



$$x\left(e^{\frac{1}{x}}-e^{-\frac{1}{x}}\right)$$

what value of k, $f(x) = \dfrac{(x-1)}{e^{1/(x-1)}+1}$ if $x \neq 1$ and $f(x) = k$ if $x=1$ is continuous at $x=1$

$$f(x) = \frac{(x-1)}{e^{1/(x-1)}+1} \text{ if } x \neq 1 \text{ and } f(x) = k \text{ if } x = 1$$

we know that $\lim\limits_{x \to 1+} e^{-1/(x-1)} = 0$

$\lim\limits_{x \to 1-} e^{1/(x-1)} = 0$

) left continuity at $x=1$

$$\lim_{x \to 1-} f(x) = \lim_{x \to 1-} \frac{(x-1)}{e^{1/(x-1)}+1}$$

$$= \frac{0}{0+1} = \frac{0}{1} = 0$$

$$\lim_{x \to 1-} f(x) = 0$$

) right continuity at $x=1$

$$\lim_{x \to 1+} f(x) = \lim_{x \to 1+} \frac{(x-1)}{e^{1/(x-1)}+1}$$

$$= \lim_{x \to 1+} \frac{(x-1)\, e^{-1/(x-1)}}{(e^{1/(x-1)}+1)\, e^{-1/(x-1)}}$$

$$= \lim_{x \to 1+} \frac{(x-1)\, e^{-1/(x-1)}}{(1+e^{-1/(x-1)})}$$

$$\lim_{x \to 1+} f(x) = \frac{0 \cdot 0}{1+0} = \frac{0}{1} = 0$$

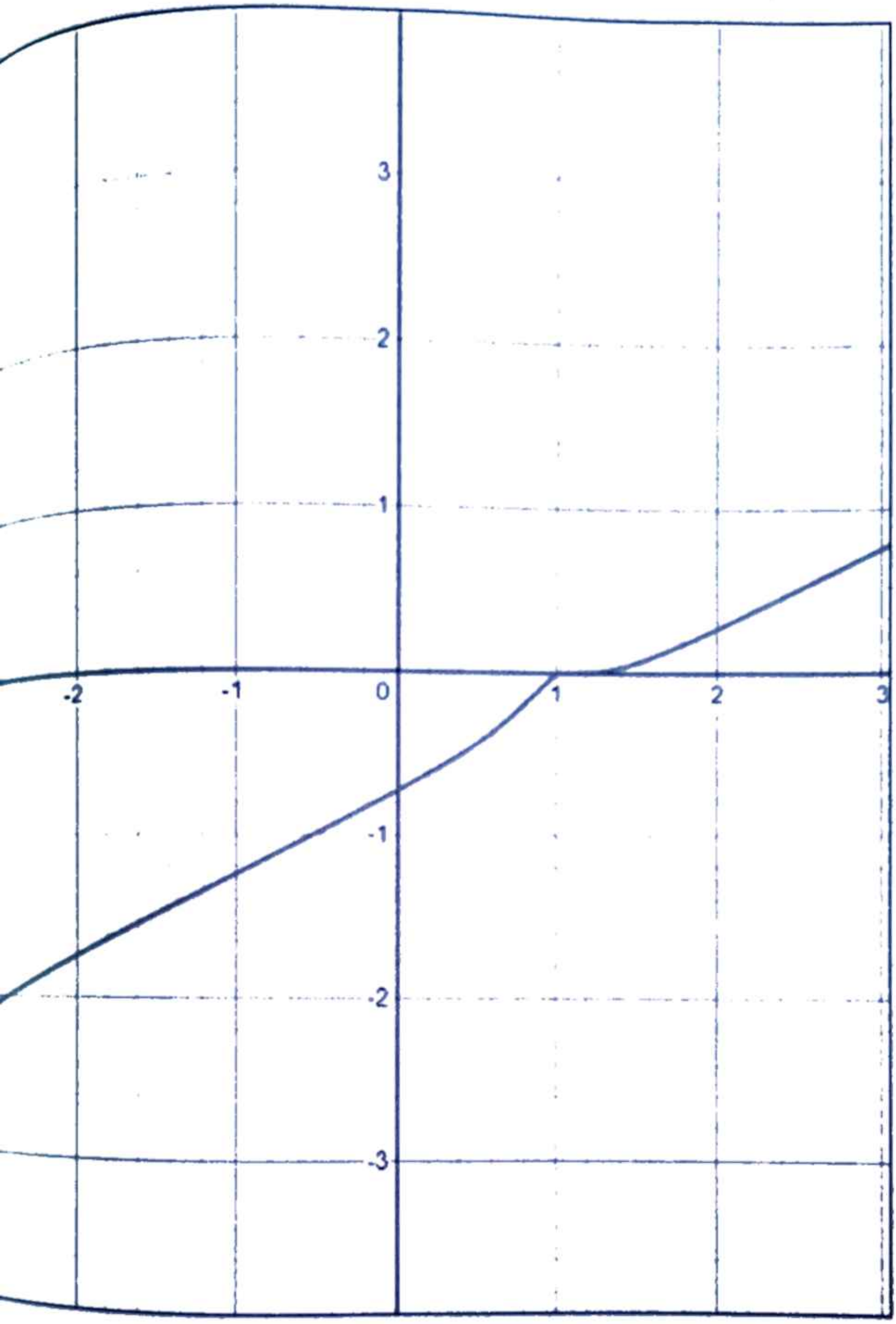
we know that $f(x)$ is continuous at $x=1$

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0-} f(x)$$

$$= f(1)$$

$$k = 0$$

$f(x) = \frac{x e^{1/x}}{1+e^{1/x}}$ $\qquad$ $f(x) = \frac{(x-1)}{e^{1/(x-1)}+1}$



$$\frac{(x-1)}{e^{\frac{1}{x-1}}+1}$$