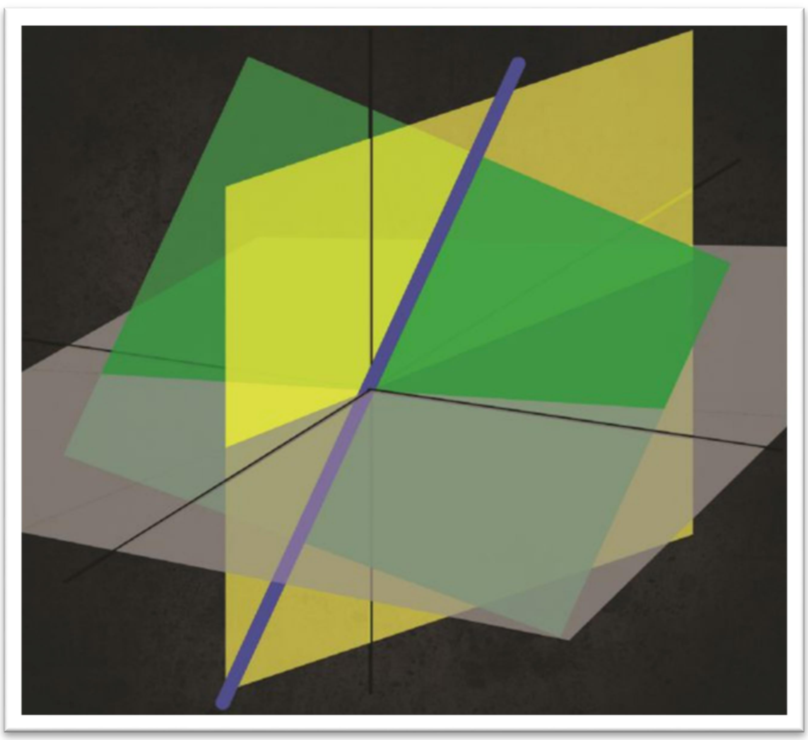
GOVERNMENT DEGREE COLLEGE
SHADNAGAR

Ranga Reddy – Dist

Department of Mathematics

STUDY PROJECT – 2021 – 22

On



"Difference between Col A and Nul A"

Government Degree College - Shadnagar

Ranga Reddy (Dist)

Student Study Project

on

"Difference between Col A and Nul A"

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Principal

GDC - Shadnagar

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Ranga Reddy (Dist)

Certificate

This is to certify that BSc (MPC & MPCs) SEM V students has successfully completed a Study Project on **"Difference between Col A and Nul A"** for the academic year 2021 - 22 under the Supervision of **T. Sri Krishna, Department of Mathematics**.

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Hence it is certified

Principal
GOVT. DEGREE COLLEGE
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Principal

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Null Space :-

Null Space : the null space of an $m \times n$ matrix A, is denoted by Null A, is the set of all solutions of the homogeneous equation $AX = 0$.

$\text{Nul } A = \{x : x \text{ is in } R^n \text{ and } Ax = 0\}$

The Nul A is a set of all x in R^n that are mapped into the zero vector of R^n with the linear transformation $T(x) = Ax$.

Caluclation of Nul A Matrix :-

The Null Space of an $m \times n$ matrix A is a Subspace of R^n.

Proof :- $\text{Nul } A = \{x : x \text{ is in } R^n \text{ and } Ax = 0\}$

To prove that Nul A is Subspace of R^n.

we know that $A0 = 0$, therefore

$0 \in \text{Nul } A$.

Therefore Nul A is non empty subset of R^n.

Let $u, v \in$ Nul A $\Rightarrow Au = 0, Av = 0$

To prove that $u+v, cu \in$ Nul A.

Let $A(u+v) = Au + Av = 0 + 0 = 0$

Therefore $u+v \in$ Nul A

Let $A(cu) = c(Au) = c0 = 0$

Therefore $cu \in$ Nul A.

Therefore Nul A is Subspace of R^n.

Find a Spanning set for the null space of the matrix, where $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

Sol:- The given matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

$$[A \ 0] = \begin{pmatrix} -3 & 6 & -1 & 1 & -7 & 0 \\ 1 & -2 & 2 & 3 & -1 & 0 \\ 2 & -4 & 5 & 8 & -4 & 0 \end{pmatrix}$$

R_1 interchanging R_2

$$= \begin{pmatrix} 1 & -2 & 2 & 3 & -1 & 0 \\ -3 & 6 & -1 & 1 & -7 & 0 \\ 2 & -4 & 5 & 8 & -4 & 0 \end{pmatrix}$$

$R_2 \to R_2 + 3R_1$; $R_3 \to R_3 - 2R_1$

$$= \begin{bmatrix} 1 & -2 & 2 & 3 & -1 & 0 \\ -3+3 & 6-6 & -1+6 & 1+9 & -7-3 & 0+0 \\ 2-2 & -4+4 & 5-4 & 8-6 & -4+2 & 0-0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 2 & 3 & -1 & 0 \\ 0 & 0 & 5 & 10 & -10 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{bmatrix}$$

$R_2 \to R_2(1/5)$

$$[A\ O] = \begin{bmatrix} 1 & -2 & 2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$; $R_3 \to R_3 - R_2$

$$= \begin{bmatrix} 1 & -2 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 2x_2 - x_4 + 3x_5 = 0 \quad ; \quad x_3 + 2x_4 - 2x_5 = 0$

$x_1 = 2x_2 + x_4 - 3x_5 \quad ; \quad x_3 = -2x_4 + 2x_5$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2x_2 + x_4 - 3x_5 \\ x_2 \\ -2x_4 + 2x_5 \\ x_4 \\ x_5 \end{bmatrix}$$

$$x = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Let $u = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; v = \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \; w = \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

$x =$ Span of $\{u, v, w\}$

$\therefore$ Nul $A =$ Span of $\{u, v, w\}$.

Column Space :-

The Column Space of an $m \times n$ matrix A, is the set of all linear combinations of the column of A. If $A = \{a_1, a_2, \ldots a_n\}$

$$\text{Col } A = \text{Span}\{a_1, a_2, a_3, \ldots \ldots, a_n\}$$

we know that $\text{Span}\{a_1, a_2, \ldots a_n\}$ is Subspace of R^m.

Therefore column space of an $m \times n$ matrix A is subspace of R^m.

Col A is denoted by $\text{col } A = \{b : b = Ax \text{ for some } x \text{ in } R^n\}$.

Col A is the range of the linear transformation $T(x) = Ax$.

Let $A = \begin{bmatrix} 10 & -8 & -2 & -2 \\ 0 & 2 & 2 & -2 \\ 1 & -1 & 6 & 0 \\ 1 & 1 & 0 & -2 \end{bmatrix}$ and $w = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix}$.

Determine w is in Column A. Is w is

in Nul A.

Sol:- $A = \begin{bmatrix} 10 & -8 & -2 & -2 \\ 0 & 2 & 2 & -2 \\ 1 & -1 & 6 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix}$ and $w = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix}$

$$[A \ w] = \begin{bmatrix} 10 & -8 & -2 & -2 & 2 \\ 0 & 2 & 2 & -2 & 2 \\ 1 & -1 & 6 & 0 & 0 \\ 1 & 1 & 0 & 2 & 2 \end{bmatrix}$$

R_1 interchanging R_3

$$= \begin{bmatrix} 1 & -1 & 6 & 0 & 0 \\ 0 & 2 & 2 & -2 & 2 \\ 10 & -8 & -2 & -2 & 2 \\ 1 & 1 & 0 & 2 & 2 \end{bmatrix}$$

$R_2 \to R_2(1/2)$; $R_3 \to R_3 - 10R_1$

$R_4 \to R_4 - R_1$

$$= \begin{bmatrix} 1 & -1 & 6 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 \\ 10+10 & -8+10 & -2-60 & -2-0 & 2-0 \\ 1-1 & 1+1 & 0-6 & 2+0 & 2-0 \end{bmatrix}$$

$$[A \quad \omega] = \begin{bmatrix} 1 & -1 & 6 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & 2 & -62 & -2 & 2 \\ 0 & 2 & -6 & 2 & 2 \end{bmatrix}$$

$R_1 \to R_1 + R_2$; $R_3 \to R_3 - 2R_2$; $R_4 \to R_4 - 2R_2$

$$[A \quad \omega] = \begin{bmatrix} 1+0 & -1+1 & 6+1 & 0-1 & 0+1 \\ 0 & 1 & 1 & -1 & 1 \\ 0-0 & 2-2 & -62-2 & -2+2 & 2-2 \\ 0-0 & 2-2 & -6-2 & 2+2 & 2-2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 7 & -1 & 1 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & -64 & 0 & 0 \\ 0 & 0 & -8 & 4 & 0 \end{bmatrix}$$

$R_3 \to R_3(-1/64)$; $R_4 \to R_4(1/4)$

$$[A \quad \omega] = \begin{bmatrix} 1 & 0 & 7 & -1 & 1 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 \end{bmatrix}$$

$R_1 \to R_1 - 7R_3$; $R_2 \to R_2 - R_3$; $R_4 \to R_4 + 2R_3$

$$[A\omega] = \begin{bmatrix} 1-0 & 0-0 & 7-7 & -1-0 & 1-0 \\ 0-0 & 1-0 & 1-1 & -1-0 & 1-0 \\ 0 & 0 & 1 & 0 & 0 \\ 0+0 & 0+0 & -2+2 & 1+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_1 \to R_1 + R_4$; $R_2 \to R_2 + R_4$

$$[A\ \omega] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Rank of A = NO of non zero rows = 4

Rank of $[A\ \omega] = 4$

$\therefore$ Rank of A = Rank of $[A\ \omega] = 4$

The equation are Consistent we get A

$$A \cdot w = \begin{bmatrix} 10 & -8 & -2 & -2 \\ 0 & 2 & 2 & -2 \\ 1 & -1 & 6 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 20-16-0-4 \\ 0+4+0-4 \\ 2-2+0+0 \\ 2+2+0+4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 8 \end{bmatrix} \neq 0$$

$$Aw \neq 0$$

$\therefore w \notin \text{Nul } A.$

$\therefore$ The given equation is

$w \in \text{Col } A.$

Calculation of dim of Col A & Nul A :-

Determine the dim of Col A and Nul A for the matrix $A = \begin{bmatrix} 1 & 2 & -4 & 3 & -2 & 6 & 0 \\ 0 & 0 & 0 & 1 & 0 & -3 & 7 \\ 0 & 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Sol:- Given matrix

$$A = \begin{bmatrix} 1 & 2 & -4 & 3 & -2 & 6 & 0 \\ 0 & 0 & 0 & 1 & 0 & -3 & 7 \\ 0 & 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

A matrix order $= 4 \times 7$

column $=1$, col $=4$, col $=5$, col $=7$.

$\therefore$ dim col A $=$ no. of pivot column $= 4$

3 columns are not pivot column.

$\therefore$ The homogeneous $AX=0$ contains 3 free variables.

$\therefore$ dimension

dim col A $= 3$

$\therefore$ dim Nul A $=$ no. of free variables

$\therefore$ dim Nul A $= 3$.

Differance between Col A and Nul A :-

Nul A	Col A
1. Nul A is Subspace of R^n.	1. Col A is a Subspace of R^m.
2. For finding vector v, it is easy to tell if v is in Nul A. Just compute Av.	2. Given a Specific vector v. it my take time to tell if v is in Col A. Row operations on $[A\ v]$ are required.
3. for finding vector in Nul A, row operations on $[A\ 0]$ are required.	3. Given a Specific is easy to finding vector in Col A. The column of A are displayed.
4. Nul A = {0}, iff the equation $Ax=0$ has only the trivial Solution.	4. Col A $= R^m$. iff the equation $Ax=b$ has a solution for every b in R^m.
5. Nul = {0} iff the linear transformation $x \to Ax$ the trivial Solution.	5. Col A $= R^m$ iff the linear transformation $x \to Ax$ maps R^n onto R^m.