“Differential Equations and its Applications”

JIGNSA STUDENT STUDY PROJECT

In

MATHEMATICS

Submitted to the

Commissioner Collegiate Education

Government of Telangana

By

1. **Mohd. Shoaib Chand**
2. **N. Bhanu prakash**
3. **Muskan Begum**
4. **Asma sameera**
5. **Afroz Fathima**

Under the guidance of

G. RANGA REDDY

&

SUREKHA

DEPARTMENT OF MATHEMATICS

SCNM GOVERNMENT DEGREE COLLEGE, NARAYANPET

NARAYANPET DISTRICT, T.S. 509210

Sri Chittem Narsireddy ... Degree College
NARAYANPET -509 210
Mahabubnagar - Dist. (T.S)

RESEARCH ADVISORY COMMITTEE

SCNM GDC NARAYANPET

Academic Year 2021-2022

1. G. RANGA REDDY
2. SUREKHA.

(Research Supervisor)

Approved by:

Chairperson: ______________________________
(Dr. Mercy Vasantha)

Jignasa Coordinator: ______________________________
(Dr. Md. Riyaz Khan)

Members:

1. ______________________________
(E. Narayana Goud)

2. ______________________________
(Shaik Shujath Ali)

3. ______________________________
(Dr. Md. Shafiq Ahmad)

4. ______________________________
(M. Uday Kumar)

5. ______________________________
(Dr. Md. Riyaz Khan)

CERTIFICATE

This is to certify that the project work entitled **"Differential Equations and its Applications"**

This work has been carried out by **Mohd. Shoaib Chand, N. Bhanu prakash, Muskan Begu, Asma sameera, Afroz Fathima** of Department of Zoology, SCNM Govt. Degree College-Narayanpet is a record of original research work done by aforementioned students being submitted to Commissioner of Collegiate Education Government of Telangana.

Supervisors
(G.Ranga Reddy)
(Surekha)

Jignasa coordinator
(Dr. Riyaz Khan M)

Principal
(Sri Mercy Vasantha)

PRINCPIAL
Sri Chittem Narsireddy
Memorial Govt. Degree College
Narayanpet-509 210
NARAYANPET-Dist. T.S.

CONTENTS

I. INTRODUCTION

A. Introduction to Differential Equation

A differential equation is an equation relating some function f to one or more of its derivatives. An example is

(1) It is obvious that this particular equation involves a function f together with its first and second derivatives. Any given differential equation may or may not involve f

or any particular derivative of f. But, for an equation to be a differential equation, at least some derivative of f must appear.

we already perceive a fundamental new paradigm: When we solve an algebraic equation, we seek a number or perhaps a collection of number, but when we solve a differential equation we seek one or more functions. Many of the laws of nature – in physics, in chemistry, in biology, in engineering, and in astronomy – find their most natural expression in the language of differential equations. Putting in other words, differential equations are the language of nature.

Applications of differential equations also abound in mathematics itself, especially in geometry and harmonic analysis and modeling. Differential equations occur in economics and systems science and other fields of mathematical science. It is not difficult to perceive why differential equation arises so readily in the sciences. If is a given function, then the derivative df /dx can be interpreted as the rate of change of f with respect to x. In any process of nature, the variables involved are related to their rates of change by the basic scientific principles, that govern the process that is, by the laws of nature. When this relationship is expressed in mathematical notation, the result is usually a differential equation.

1. Ordinary Differential Equation

An ordinary differential equation (ODE) is a differential equation in which the unknown function (also known as the dependent variable) is a function of a single independent variable. In the simplest form, the unknown function is a real or complex valued function, but more generally, it may be vector-valued or matrix-valued: this corresponds to considering a system of ordinary differential equations for a single function. Ordinary differential equations are further classified according to the order of the highest derivative of the dependent variable with respect to the independent variable appearing in the equation. The most important cases for applications are first-order and second-order differential equations. For example, Bessel's differential.

(2) Linear differential equation

The important thing to note about linear differential equations is that there are no products of the function, y(t), and its derivatives and neither the function or its derivatives occur to any power other than the first power. The coefficients and g(t) can be zero or non-zero functions, constant or non-constant functions, linear or non-linear functions. Only the function, y(t), and its derivatives are used in determining if a differential equation is linear.

II. REVIEW OF LITERATURE

1. Modelling cancer growth and therapies (project): This project will investigate some of the cancer growth models and effects of drug or other therapies. The project involves ODEs/PDEs, and or data science approaches. You will learn a combination of analytical and numerical skills.

2. Mathematical Pharmacology (project) Mathematics: pharmacology is the mathematical behinds models used in pharmacology. Most projects involve ODEs and usually you will learn a combination of analytical and numerical skills.

3. Dynamics in Game Theory (project) : it can involve designing an app or programme to illustrate some of the ideas behind dynamic games.

4. A mathematical model related to DNA copying (project) : This project focuses on a mathematical model for the interaction between DNA and RNAP (the polymer essential in DNA reproduction. The model is a PDE, but we will focus on solutions that can be analysed by using ODE techniques. One of the techniques is matching of two of three phase portraits, such that they nicely align at the area of interaction between the DNA and RNAP. This is a simple, but very efficient technique. You will learn this technique and other skills in this project that will involve a combination of analytical and numerical (Matlab) work. Mathematical modeling of desertfication (project or literature review) It has been observed that fertile areas can become

deserts due to droughts, over-cultivation or similar processes. Once this has happened, it is very hard to get the area back to being fertile again. Some mathematical modelling has been done in the literature and a possible explanation involving bi-stable steady states has been observed. This project/literature review will look at the relevant papers, explore the literature and in case of a project will analyse the models further.

5. Mickens, R. (2001) this paper is an introduction to non standard finite difference methods, which are useful to construct differential equations. In his paper, he described exact finite difference scheme, also rules for constructing non standard scheme with its application.

III. METHODOLOGY

The differential equation is an equation in which dependent variable independent variable and derivative of differential equation occur simultaneously. Many physical and engineering problems when formulated in the mathematical language give rise to partial differential equations. Besides these, partial differential equations also play an important role in the theory of Elasticity, Hydraulics etc. Since the general solution of a partial differential equation in a region R contains arbitrary constants or arbitrary functions, the unique solution of a partial differential equation corresponding to a physical problem will satisfy certain other conditions at the boundary of the region R. These are known as boundary conditions When these conditions are specified for the time $t = 0$, they are known as initial conditions. A partial differential equation together with boundary conditions constitutes a boundary value problem. In the applications of ordinary linear differential equations, we first find the general solution and then determine the arbitrary constants from the initial values. But the same method is not applicable to problems involving partial differential equations. Most of the boundary value problems involving linear partial differential equations can be solved by the method of separation of variables. In this method, right from the beginning, we try to find the particular solutions of the partial differential equation which satisfy all or some of the boundary conditions and then adjust them till the remaining conditions are also satisfied. A combination of these particular solutions gives the solution of the problem.

A. Separation of Variables

In this method, we assume the solution to be the product of two functions, each of which involves only one of the variables. The following examples explain this method.

Separation of Variables

1. Check for any values of y that make g(y)=0.These correspond to constant solutions.
2. Rewrite the differential equation in the form

   $$dyg(y)=f(x)dx.$$

   $$dyg(y)=f(x)dx.$$

3. Integrate both sides of the equation.
4. Solve the resulting equation for y if possible.
5. If an initial condition exists, substitute the appropriate values for xx and y into the equation and solve for the constant

2.Newton's Law of Cooling Derivation

For small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed.

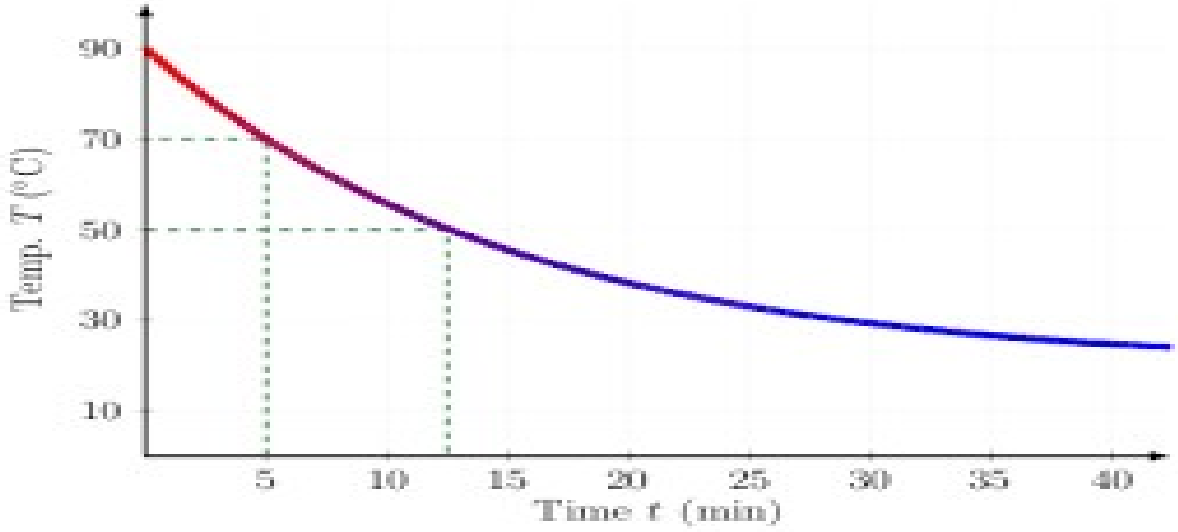
$dQ/dt \propto (q - q_s)]$, where q and q_s are temperature corresponding to object and surroundings.

From above expression , $dQ/dt = -k[q - q_s)]$ . . .. (1)

This expression represents Newton's law of cooling. It can be derived directly from Stefan's law, which gives,

$k = [4e\sigma\times\theta^3{}_o/mc]\ A$ . . . . . (2)

Now, $d\theta/dt = -k[\theta - \theta_o]$



where,

q_i = initial temperature of object,

q_f = final temperature of object.

$\ln (q_f - q_0)/(q_i - q_0) = kt$

$(q_f - q_0) = (q_i - q_0)\, e^{-kt}$

$q_f = q_0 + (q_i - q_0)\, e^{-kt}$ . . . . . . (3).

⇒ **Check:** Heat transfer by conduction

Methods to Apply Newton's Law of Cooling

Sometime when we need only approximate values from Newton's law, we can assume a constant rate of cooling, which is equal to the rate of cooling corresponding to the average temperature of the body during the interval.

i.e. dθ\dt = k(<q> – q_0) . . . . (4)

If q_i and q_f be the initial and final temperature of the body then,

<q> = (q_i + q_f)/2 . . . . . (5)

Remember equation (5) is only an approximation and equation (1) must be used for exact values.

Limitations of Newton's Law of Cooling

- The difference in temperature between the body and surroundings must be small
- The loss of heat from the body should be by radiation only
- The major limitation of Newton's law of cooling is that the temperature of surroundings must remain constant during the cooling of the body
-

2. Growth and Decay.

Exponential Growth and Decay:

Differential Equations Observations about the exponential function In a previous chapter we made an observation about a special property of the function y = f(x) = e x namely, that dy/ dx = e x = y so that this function satisfies the relationship dy/ dx = y. We call this a differential equation because it connects one (or more) derivatives of a function with the function itself. In this chapter we will study the implications of the above observation. Since most of the applications that we examine will be time-dependent processes, we will here use t (for time) as the independent variable. Then we can make the following observations:

1. Let y be the function of time: y = f(t) = e t Then dy /dt = e t = y With this slight change of notation, we see that the function y = e t satisfies the differential equation dy/ dt = y

2. Now consider y = e kt . Then, using the chain rule, and setting u = kt, and y = e u we find that dy/ dt = dy /du du/ dt = e u · k = kekt = ky.

So we see that the function y = e kt satisfies the differential equation dy/ dt = ky.

3. If instead we had the function y = e −kt we could similarly show that the differential equation it satisfies is dy/ dt = −ky

4. Now suppose we had a constant in front, e.g. we were interested in the function y = 5e kt . Then, by simple differentiation and rearrangement we have dy dt = 5 d dte kt = 5(kekt) = k(5e kt) = ky. So we see that this function with the constant in front also satisfies the differential equation dy/ dt = ky.

5. The conclusion we reached in the previous step did not depend at all on the constant out front. Indeed, if we had started with a function of the form y = Cekt where C is any constant, we would still have a function that satisfies the same differential equation

6. While we will not prove this here, it turns out that these are the only functions that satisfy this equation. A few comments are worth making: First, unlike algebraic equations, (whose solutions are numbers), differential equations have solutions that are functions. We have seen above that depending on the constant k, we get either functions with a positive or with a negative exponent (assuming that time t > 0).

Where do differential equations come from. The process of going from initial vague observations about a system of interest (such as planetary motion) to a mathematical model, often involves a great deal of speculation, at first, about what is happening, what causes the motion or the changes that take place, and what assumptions might be fruitful in trying to analyze and understand the system. Once the cloud of doubt and vague ideas settles somewhat, and once the right simplifying assumptions are made, we often find that the mathematical model leads to a differential equation. In most scientific applications, it may then be a huge struggle to figure out which functions would be the appropriate class of solutions to that differential equation, but if we can find those functions, we are in position to make quantitative predictions about the system of interest. In our case, we have stumbled on a simple differential equation by noticing a property of functions that we were already familiar with. This is a lucky accident, and we will exploit it in an application shortly. In many cases, the process of modeling hardly stops when we have found the link between the differential equation and solutions. Usually, we would then compare the predictions to observations that may help us to refine the model, reject incorrect or inaccurate assumptions, or determine to what extent the model has limitations. A simple example of population growth modeling is given as motivation for some of the ideas seen in this discussion.

IV. RESULTS OF EXPERIMEN

Crime scene

A detective is called to a scene of a crime where a dead body has just been found she arrives on the scene at 10: 23 pm and begins her investigation immediately , the temperature of the body is taken and is found to be 80 degree foreign heat. The detective checks the programmable thermo stat. and finds that the room has been kept at a constant 68 degree foreign heat for the past 3 days. After evidence from crime scene is corrected , the temperature of the body is taken once more and found to be 78.5 80 degree foreign heat . this last temp reading was taken exactly one hour after the first one. The next day the detective is asked by another investigator, "WHAT TIME DID OUR WICTIM DIE"? . assuming that the victims body temperature was normal(98 .680 degree foreign heat) prior to death, what is her answer to this question ?

Newton's law of cooling can be used to determine the victim's time of death.

Example 1 : A population growth at the rate of 5% per year . How long does it takes for the population to double?

ANSWER:

Let the initial population be p0 and let the population after t years say p

Which is a Separable differential equation

Let population at time t be x(t)
Then, because the growth of population depends on the current population:
$\Rightarrow$d/dx(t) α x(t)
The rate of increase in population is 5%=0.05.
This implies:
$\Rightarrow$d/dx(t)=0.05x(t)
$\Rightarrow$x(t)/dx(t)=0.05dt
Integrating both sides:
$\Rightarrow\int$x(t)d(x(t))=$\int$0.05d(t)
$\Rightarrow$ln(x(t))=0.05t+c............(x)
Let initial population be P, that is, let x(0)=P.
Then, ln(x(0))=0.05x0+c putting t=0
$\Rightarrow$lnP=c
Again, let after time tx, the population becomes 2P.
Thus, relation(x) as follows:
ln(x(t))=0.05t+lnP
Becomes:ln(x(t)x)=0.05tx+ln(P)
$\Rightarrow$ln(2P)=0.05tx+lnP
$\Rightarrow$ln2+lnP=20tx+lnP
$\Rightarrow$tx=20ln2
After 20 ln2 years, the population doubles.

V. SUMMARY AND CONCLUSION

In this project, many of the laws of nature – in physics, chemistry, biology, engineering and astronomy find their most natural expression in the language of differential equations. In other words, differential equations are the language of nature. Applications of differential equations also abound in mathematics itself, especially in geometry and harmonic analysis and modeling. Differential equations occur in economics and systems science and other fields of mathematical science. Many physical and engineering problems when formulated in the mathematical language give rise to partial differential equations.

Abstract world of mathematical concept, which is where model is built. We when manipulate the model using techniques or computer aided numerical computation. Finally we re enter the real world , taking with us the solution to the mathematical problems, which is translated into a useful solution to the real problems. The application of first order differential equation in temperature have been studied the method to separation of variables Newton's law of cooling were used to find the solution of the temperature problems that requires the use of first order differential equation and these solution are very useful in mathematics, biology, physics especially in analyzing the problems involving temperature which requires the use of Newton's law of cooling.

It has been observed that differential equations can describe any phenomena and the given conditions are completely solvable to find various results. In chapter 1 we are giving some introductory concept of differential equation and some basic concepts related to differential equation like degree order homogenous and non homogenous differential equation with governing equation. using separation of variable and finally we are solving Laplace equation in polar form. After considering number of problems based on different types of differential equations, we found that the solutions of all differential equations satisfied the phenomena of theoretical basis. The various category of differential equations described in the phenomena are genuine. The deflection u(x, y, t) of a rectangular membrane, square membrane and an infinitely long plane uniform plate is bounded by two parallel edges and an end at right angles to them etc. was found which showed a very good agreement

VI. REFERENCES

1] S. Abbasbandy, A new application of He's variational iteration method for quadratic Riccati differential equation by using Adomian's polynomials, Journal of Computational and Applied Mathematics, 207 (2007), 59-63.

[2] S. Abbasbandy, M. Otadi and M. Mosleh, Numerical solution of a system of fuzzy polynomials by fuzzy neural network, Information Sciences, 178, (2008), 1948-1960.

[3] S. Abbasbandy and A. Taati, Numerical solution of the system of nonlinear Volterra integro-differential equations with nonlinear differential part by the operational Tau method and error estimation, Journal of Computational and Applied Mathematics, 231, (2009), 106-113. [4] M. A. Abdou and A. A. Soliman, Variational iteration method for solving Burger's and coupled Burger's equations, Journal of Computational and Applied Mathematics, 181, (2005), 245-251. [5] E. M. Abulwafa, M. A. Abdou and A. A. Mahamoud, The solution of nonlinear coagulation problem with mass loss, Chaos, Solitons and Fractals, 29, (2006), 313-330.

[6] G. Adomian, Solving frontier problems of physics: The decomposition method, Kluwer Academic Publishers, Boston, 1994.

[7] A. Ahmadian, S. Salahshour and C. S. Chan, A Runge–Kutta method with reduced number of function evaluations to solve hybrid fuzzy differential equations, Soft Computing, 19, (2015), 1051-1062.